ISSN 2616-5902 Vol. 5, Issue 9: 45-53, DOI: 10.25236/AJBM.2023.050908

Maximizing Returns and Minimizing Risk: A Data-Driven Portfolio Optimization Analysis Using Markowitz's Theory and Sharpe Ratio

Zhuoyi Ma[1,a,*]

[1]*College of Business & Public Management, Wenzhou-Kean University, Liao Street, Wenzhou, China*
[a]*zhuoyim@kean.edu*
[*]*Corresponding author: Zhuoyi Ma*

Abstract: *The implementation of new technologies in the financial sphere is expanding as the era of big data arrives. This study uses Python tools to choose four representative stocks from 11 distinct industries for portfolio analysis, based on Markowitz's portfolio theory. The best portfolio solution is discovered through empirical research: Maximum Sharpe ratio of one; The anticipated return, standard deviation, and Sharpe ratio were compared and examined, and the effective frontier of the asset portfolio was calculated using Monte Carlo simulation. In addition, as a simple trading method, build two different simple moving averages. Backtesting reveals that the ideal portfolio's return rate is close to 20% using this strategy.*

Keywords: *Markowitz's Portfolio Theory, Portfolio Optimization, Sharpe Ratio, Mean-variance Analysis*

1. Introduction

Investors confront the pivotal challenge of constructing a well-diversified portfolio that can offer a superior risk-return tradeoff compared to investing in individual assets. Portfolio strategy serves as a means to surmount this challenge by presenting a systematic methodology for developing effective portfolios. The primary objective of portfolio optimization is the efficient allocation of assets to minimize risk and maximize returns, thereby enabling investors to accomplish their investment goals.

Markowitz[1] initially introduced the mean-variance framework, which established the groundwork for portfolio optimization. This model posits that investors favor portfolios that maximize anticipated returns for a given level of risk, as quantified by the portfolio's variance. Subsequently, Sharpe's[2] Capital Asset Pricing Model (CAPM) emerged as a notable advancement in elucidating expected returns by delineating the systematic risk premium through market beta. The model estimates the returns investors should anticipate in exchange for the risks undertaken.

Despite the robust solution portfolio optimization presents for constructing diversified portfolios, it is not without its challenges. Information asymmetry can lead to inaccuracies in estimating expected returns, culminating in suboptimal portfolio choices. Likewise, market inefficiencies and behavioral biases render model assumptions invalid, potentially resulting in poor decision-making by investors. Jagannathan and Ma[3] underscore that imposing inappropriate constraints to mitigate risks may inadvertently reduce diversification benefits.

Contemporary portfolio strategy research encompasses an array of theories, models, and techniques, including mean-variance analysis, Feldstein[4], asset pricing models (CAPM) (Sharpe, 1964), risk factor models, by Fama & French[5], and machine learning, by Black & Litterman[6]. algorithms employing neural networks, random forests, and deep learning for return predictions. These techniques hold significant applicability in various investment management contexts.

Nonetheless, traditional portfolio strategies have encountered criticism due to their inherent limitations and shortcomings. They are typically sensitive to input parameters such as asset volatilities and correlations, which are often estimated inaccurately. Meucci[7] contends that although the mean-variance model theoretically provides the optimal risk-return tradeoff, it lacks robustness in practical applications. Traditional strategies predominantly rely on historical data, neglecting the dynamic nature of financial markets and potentially adversely impacting returns. Consequently, traditional model

ISSN 2616-5902 Vol. 5, Issue 9: 45-53, DOI: 10.25236/AJBM.2023.050908

assumptions can become invalid under market stress conditions.

Emerging trends and discussions in portfolio strategy research encompass the influence of alternative investments, passive investing implications, and sustainable investing challenges. These burgeoning issues within the market suggest the future trajectory of portfolio optimization. Alternative investments emerge due to diversification benefits, while passive investing arises as a response to cost-effective, broad diversification. Simultaneously, sustainable investing challenges must be integrated into portfolio optimization, considering climate change ramifications, which can transform portfolio optimization.

The purpose of this essay is to delve into the constraints of traditional portfolio optimization strategies and examine the emergent trends and debates within portfolio strategy research. The essay will adopt a pragmatic approach by employing Python's Pandas and SciPy libraries to compute expected returns, covariance matrices, and allocation ratios for maximum Sharpe ratio and minimum variance portfolios. The research seeks to offer insights into the potential of alternative investments, passive investing, and sustainable investing in shaping the future of portfolio optimization. Ultimately, the research aspires to equip investors with a more profound comprehension of the limitations and challenges inherent in traditional portfolio optimization strategies and the potential of emerging trends to improve risk-return tradeoffs and achieve investment objectives.

2. Literature Review

This literature review endeavors to critically appraise the corpus of research on mean-variance analysis in the context of portfolio optimization. An exhaustive search of academic databases, encompassing seminal and contemporary publications, was undertaken to identify pertinent studies that have enriched our comprehension of this crucial subject. The literature was classified according to publication chronology, with an emphasis on discerning trends and evolutions in the field over time.

Markowitz's[1] study pioneered a potent instrument for portfolio selection, known as the Markowitz Portfolio Theory (MPT). MPT postulates that investors are risk-averse, striving to maximize returns for a given level of risk. MPT posits that investors can accomplish this by diversifying their portfolio across a gamut of assets with varying risk and return levels and by employing mathematical models to compute the optimal portfolio allocation based on the investor's risk tolerance and return objectives.

Sharpe's[2] study posits that diversification in portfolio selection can effectively diminish risk arising from fluctuations in economic activity. The author proposed that the Sharpe Ratio, calculated by subtracting the risk-free rate of return from the anticipated return of the investment and dividing the result by the standard deviation of the investment's return, is a valuable instrument for comparing the performance of disparate investments as it contemplates both the return and risk associated with each investment.

Michael C et al.,[8] conducted additional tests on Sharpe's asset pricing model, circumventing some issues of earlier studies, and executed empirical tests to corroborate that the beta factor is a crucial determinant of security returns. Myles E. Mangram[9] synthesized Markowitz's and Sharpe's theories in a review. However, these theories are constrained by their disregard for supplementary factors.

Consequently, numerous studies have investigated the impact of additional factors on portfolio choice to enhance the precision of portfolio selection. For instance, a study by Roy Henriksson et al. [10] explored the influence of Environmental, Social, and Governance (ESG) data on portfolio selection. The authors discovered that investors can effectively integrate sparse ESG data into their portfolio construction to ameliorate portfolio selection.

Shi Yu et al.[11] recently introduced an innovative approach to gauging risk preference in portfolio selection. The study connects the mean-variance portfolio allocation framework to the domains of psychology and behavioral science. The authors propose a groundbreaking method of utilizing inverse optimization to analyze extant portfolios, proffering a fresh perspective on risk preference measurement.

Morelli[12] presented an operational research design for portfolio selection that integrates investors' ethical values from a risk management standpoint. The study suggests adjustments to the portfolio selection problem to ensure that investment decisions correspond with investors' ethical values. This approach acknowledges the escalating significance of ethical considerations in investment decision-making and offers a framework for incorporating such values into the portfolio selection process.

ISSN 2616-5902 Vol. 5, Issue 9: 45-53, DOI: 10.25236/AJBM.2023.050908

In a recent study, Nathan Lassance et al.[13] proposed an original framework for portfolio selection that endeavors to target a specific return distribution. The study recommends refining the higher moments of mean-variance-efficient portfolios by designing a target return distribution that matches the first two moments of the chosen efficient portfolio but possesses more desirable higher moments. This approach proffers a novel perspective on portfolio selection and endows investors with greater control over their return distribution's shape.

Nathan Lassance et al.[13] have proposed a groundbreaking perspective on portfolio selection by scrutinizing and forecasting the time series corresponding to the portfolio's value. The study advocates employing a damped trend model to analyze this time series and predict future portfolio values. The authors furnish estimates of the mean and variance for diverse forecasting horizons, enabling investors to make more informed decisions about their portfolios based on anticipated future performance.

A recent study by Aithal et al.[14] proposes the integration of machine learning methods into portfolio optimization. The study employs a genetic algorithm for optimization and applies a sliding window for portfolio management. The authors also examine four distinct portfolio calculation methods, including the equally-weighted portfolio, global minimum variance portfolio, market cap-weighted portfolio, and maximum Sharpe ratio portfolio.

In conclusion, the investigation of mean-variance analysis in portfolio optimization has been the subject of extensive research spanning several decades. The development of the Markowitz Portfolio Theory and the Sharpe Ratio was instrumental in providing a framework for portfolio selection based on risk and return. However, subsequent studies have acknowledged the limitations of these theories and have delved into additional factors, such as ESG data, ethical values, and higher moments of return distributions, to augment the accuracy of portfolio selection. Recent studies have also proposed innovative approaches, such as analyzing time series data and incorporating machine learning methods, to afford investors more control over their portfolios and enhance investment decision-making. These advancements underscore the importance of ongoing research in the realm of portfolio optimization to address the evolving needs of investors and the dynamic landscape of financial markets.

3. Method

3.1. The Portfolio Hypothesis of Markowitz.

Harry Markowitz looked into probability theory and quadratic programming in 1952 in an effort to create superior investing models that would produce low-risk and high-return outcomes. This is the theory of contemporary portfolio management. Considering a hypothetical situation, investors want to minimize risk and maximize target returns. Harry Markowitz's theories are divided into two categories: 1 Mean-variance model; 2 theory of efficient boundaries. Quantify the target benefits and current hazards by adding the two quantities. Finding the ideal answer is the ultimate objective: 1. Determine the investment model with the lowest risk when target returns are equal. 2. Determine the project with the highest return when target returns are equal.

(1) The model with mean-variance: Assumes that an investor chooses n hazardous projects concurrently during a specific period. The target rate of return for each project is represented by r_i, where r_i is the i^{tn} project's target rate of return:

$$E\left(r_p\right) = \sum_{n=1}^{n} x_i E(r_i) \tag{1}$$

where x stands for the asset's investment percentage. If we assume that σ_i^2 represents the variation of the i^{tn} asset, then the variance of the portfolio of n assets is as follows:

$$\sigma_p^2 = \sum_{n=1}^{n} \sum_{n=1}^{n} x_i x_j \mathrm{cov}(r_i, r_i)\ , i \neq j \tag{2}$$

$$\sigma_p^2 = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j\ \rho_{ij} \sigma_i \sigma_j \tag{3}$$

ISSN 2616-5902 Vol. 5, Issue 9: 45-53, DOI: 10.25236/AJBM.2023.050908

where i and j stand for separate assets and $\text{cov}(r_i, r_i)$ is an index used to assess the relationship between the return rates of the two assets, asset i and asset j.The function compares the strength of the correlation between the two elements, and ρ_{ij} stands for the coefficient between i and j.

The ratio of the total number of investment projects, according to formula (3), indicates the level of risk in the investment portfolio. Different securities have various corresponding coefficients, and various projects have various standard deviations. In order to lower investing risks, it is therefore prioritized to choose assets with a low correlation coefficient between the two and a small variance. The sample variance and sample average of historical return data are typically utilized in practical applications to anticipate future returns and hazards.

The percentage of investment projects defines the level of risk in the investment portfolio, according to formula (3). Standard deviations for various projects vary, and corresponding coefficients for various securities do as well. To build an investment portfolio and lower investment risks, it is therefore prioritized to choose assets with low variation and low correlation between the two. In practical applications, future returns and hazards are typically estimated using the sample variance and sample average of prior return data.

(2) The efficient frontier of the asset portfolio: The feasible set, which resembles the left-convex solid area in Figure 1, is made up of all possible combinations of each asset in the portfolio.

The Efficient Frontier of the asset portfolio is the investment portfolio that, among them, fits the conditions that the risk is the smallest in the comparable amount of return and the return is the highest in the equivalent risk.

3.2. Sharpe Ratio

The Sharpe ratio, which uses the risk-adjusted index for analysis and is used to evaluate the performance of the fund, was created in 1966 by Sharp. The Sharpe ratio is the relationship between the portfolio's excess target return and the total standard deviation with the following calculation formula:

$$S_P = \frac{E\,(r_p) - r_f}{\sigma_p} \tag{4}$$

In formula (4), where σ_n denotes the portfolio's overall standard deviation, r_f the risk-free interest rate, and S_P the Sharpe ratio. S_P is the maximum excess return that a portfolio of assets can achieve for each additional unit of risk (Figure 1). S_P carefully weighs both profits and hazards. This indicator will also be included in the empirical research portion of this study as a standard for evaluating the strengths and weaknesses of investment portfolios.

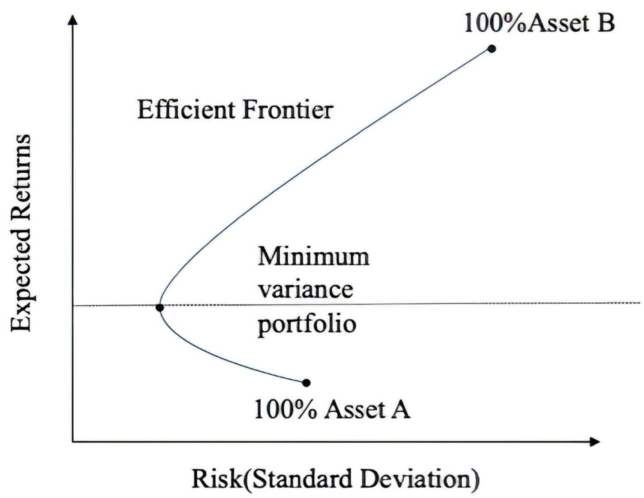

Figure 1: The efficient frontier

ISSN 2616-5902 Vol. 5, Issue 9: 45-53, DOI: 10.25236/AJBM.2023.050908

4. Data Analysis

4.1. Objectives

The main goal of this experiment is to use portfolio optimization techniques to find the optimal allocation of stocks that maximize expected returns while minimizing associated risks. This is done by analyzing historical data on selected stocks and calculating their expected returns and covariance matrices to build a portfolio that is optimized for these metrics.

The specific goal of the experiment is to use historical data from the stock market to calculate the average return and covariance matrix of a set of selected stocks, and to use this information to create a portfolio optimized for expected return and risk. The experiment was designed to identify the maximum Sharpe ratio and minimum variance combinations and output the configuration ratio for each combination.

In addition, this experiment attempts to show the relationship between the annualized return of all stocks and the annualized volatility by drawing the efficient boundary, which is a curve consisting of the portfolio with the greatest return and the least risk. The effective frontier analysis method is used to discuss the optimal portfolio under different expected returns.

The ultimate goal of this experiment is to provide a scientific portfolio optimization method to help investors make wise decisions and achieve their investment goals. The experiment shows the performance and characteristics of various portfolios, enabling investors to choose the optimal portfolio to maximize its expected returns while minimizing the associated risks.

4.2. Methodology

The experimental method adopted in this study aims to find the optimal stock mix with maximum return and minimum risk through portfolio optimization. The first step is to obtain stock data for selected stocks (TSLA, CSGP, DLTR, DXCM) for specific time periods (2016-1-1 and 2018-12-31) by using the Quandl API. This provides a data set of historical stock prices.

Secondly, the expected return and covariance matrix are calculated according to the historical data. This involves analyzing stock prices over a given time period to determine the expected rate of return for each stock and the degree to which they are correlated with each other, which is represented by the covariance matrix.

The maximum Sharpe ratio and minimum variance combinations are then calculated using the optimizer in Python's SciPy library. The Sharpe ratio is a measure of the excess return per unit of risk, and the portfolios with the highest Sharpe ratio offer the best risk-adjusted return. A minimum variance portfolio, on the other hand, seeks to minimize the overall risk of the portfolio.

Finally, the allocation ratio of the combination of maximum sharpe ratio and minimum variance is given, as well as the effective boundary of the combination and the annual return and volatility graphs of stocks. This allows for a visual representation of results and provides insight into the trade-off between risk and return when building the best stock portfolio.

In conclusion, the experimental methods used in this study provide a systematic approach to portfolio optimization and demonstrate the value of using data and mathematical modeling to make informed investment decisions. By applying these methods to real-world scenarios, investors can better understand the risks and returns associated with different investment strategies and make more informed decisions about their portfolios.

4.3. Data

The data set contained five stocks, including TSLA, CSGP, DLTR, and DXCM, from 2016-1-1 to 2018-12-31. The expected return and covariance matrix were calculated based on the historical data, and the maximum Sharpe ratio and minimum variance portfolios were calculated using the optimizer in Python's SciPy library.

ISSN 2616-5902 Vol. 5, Issue 9: 45-53, DOI: 10.25236/AJBM.2023.050908

Table 1: Close price examples of TSLA

None	Close Price Examples of TSLA		
	Date	**Ticker**	**adj_close**
0	2018-03-27	TSLA	279.18
1	2018-03-26	TSLA	304.18
2	2018-03-23	TSLA	301.54
3	2018-03-22	TSLA	309.10
4	2018-03-21	TSLA	316.53

Table 2: Close price examples of four stocks

	Tickers			
Date	**CSGP**	**DLTR**	**DXCM**	**TSLA**
2016-01-04	197.54	78.81	78.49	223.41
2016-01-05	199.73	79.98	80.76	223.43
2016-01-06	194.87	80.52	79.39	219.04
2016-01-07	184.29	78.45	81.79	215.65
2016-01-08	182.12	77.79	79.60	211.00

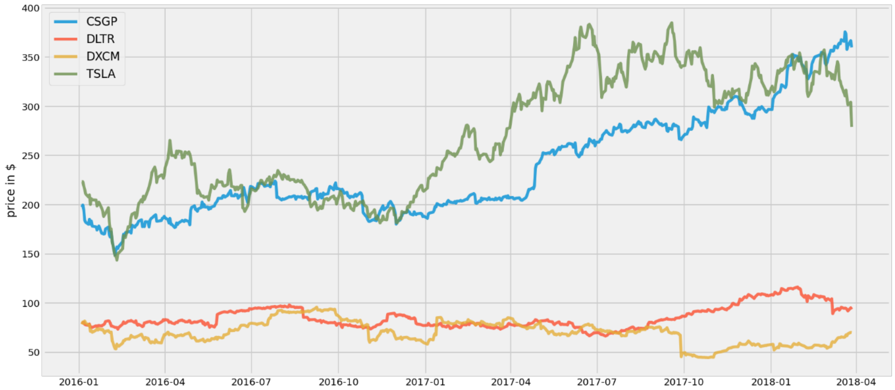

Figure 2: Price movements of four stocks over the period 2010 to 2012, data from Quandl

Table 1 shows an example, the close price of TSLA, in the four stocks between 2018-3-21 and 2018-3-27. Table 2 shows the close price of the four stocks from 2016-1-4 to 2016-1-8. This line chart (Figure 2) shows the price movements of four stocks over the period 2016-1-1 and 2018-12-31. Each stock is represented by a different colored dash in the graph, with the stock name corresponding to each color shown in the legend. y-axis represents the dollar price and x-axis represents time. The graph can be used to see the trends and relationships between different stocks.

CSGP and TSLA have both seen obvious increases from an overall perspective, but these two stocks are more volatile, while DLTR and DXCM that basically stays between $110 and $50, is relatively stable compared to the other two stocks. There is a general downhill trend in DXCM, the risk and return are relatively small.

Text (0, 0.5, 'daily returns')

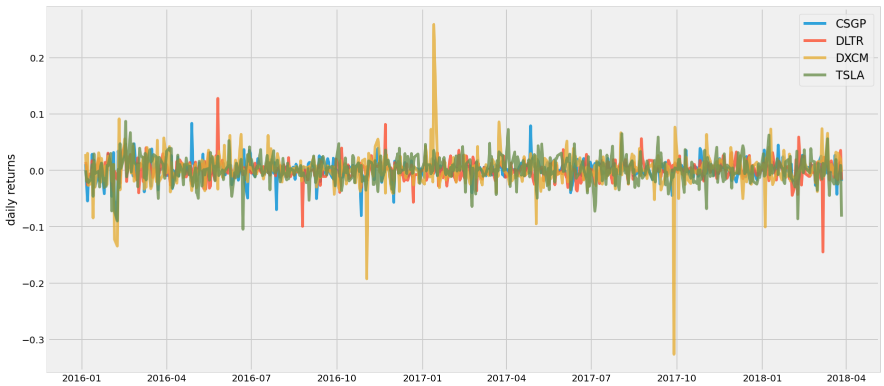

Figure 3: Daily return trends of four stocks for the period 2010-1 to 2012-1

ISSN 2616-5902 Vol. 5, Issue 9: 45-53, DOI: 10.25236/AJBM.2023.050908

The presented line chart showcases the daily return trends of four selected stocks, TSLA, CSGP, DLTR, and DXCM, for the specified period of 2016-1-1 to 2018-12-31. The stocks are differentiated by different colored dashes on the graph (Figure 3), and their corresponding names are displayed in the legend for ease of identification. The y-axis on the graph represents daily returns, while the x-axis represents time. This chart provides an avenue to examine and evaluate the trends and interrelationships among the daily returns of the selected stocks. The daily returns of TSLA and CSGP exhibit a relatively moderate variation, while the remaining two stocks display at least one instance of a daily return beyond 0.1 or below -0.1.

Table 3: Maximum Sharpe ratio portfolio allocation

Maximum Sharp Ratio Portfolio Allocation				
Annualised Return: 0.28				
Annualised Volatility: 0.23				
Allocation	**CSGP**	**DLTR**	**DXCM**	**TSLA**
	85.85	10.36	0.0	3.78

Table 4: Minimum portfolio allocation

Minimum Volatility Portfolio Allocation				
Annualised Return: 0.2				
Annualised Volatility: 0.19				
Allocation	**CSGP**	**DLTR**	**DXCM**	**TSLA**
	41.89	38.36	6.08	13.67

Table 5: Individual stock returns and volatility

Individual Stock Returns and Volatility
Annualised Return: 0.2
Annualised Volatility: 0.19
Annualised Volatility: 0.19

From the Tables above, the percentage allocation of the highest Sharpe ratio portfolio (Table 3), the percentage allocation of the lowest variance portfolio(Table 4), and the earnings and volatility of individual stocks(Table 5) is shown. Using these ratios, a scatter plot is created and functions from Python's Scipy library are used to calculate the effective boundary. The efficient frontier is a linear curve consisting of the portfolios with the greatest return and the least risk, showing the relationship between the annualized return and the annualized volatility of all stocks.

The efficient frontier analysis method is used to explore the optimal portfolio under different expected returns. The experimental results show that the portfolio with the highest Sharpe ratio has the highest expected returns and the lowest volatility. The least variance portfolio, on the other hand, has the lowest volatility but a lower expected return. By considering returns and risks, this analysis helps determine the best investment strategy.

As shown in the Figure 4, the scatter plot is bullet-shaped, and each portfolio on the efficient frontier achieves the goal of maximizing return for a given level of risk or minimizing risk for a given level of return:

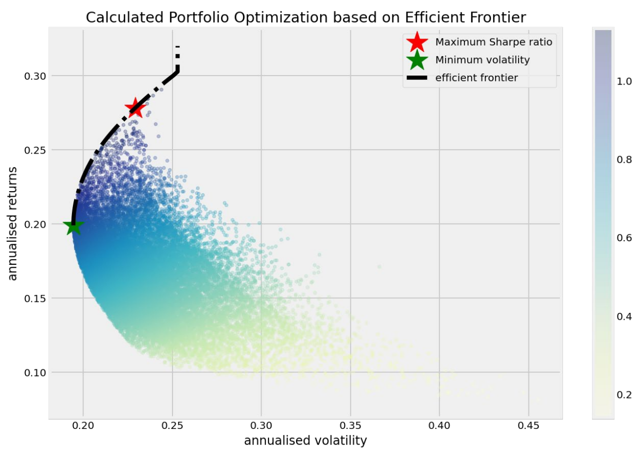

Figure 4: Calculated portfolio optimization based on efficient frontier

ISSN 2616-5902 Vol. 5, Issue 9: 45-53, DOI: 10.25236/AJBM.2023.050908

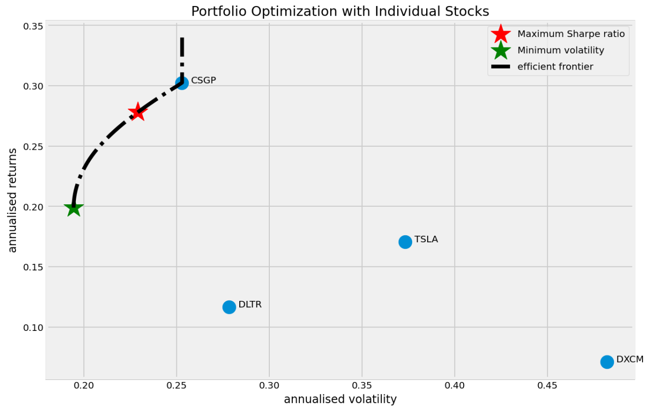

Figure 5: Portfolio optimization with individual stocks

In Figure 5, it shows a clearer efficient frontier with the annualised returns and volatility of the four stocks, which gives the suggestion that CSGP is the best stock to invest. While other three stocks only has a few worth to invest because they are not on the efficient frontier which could help with maximizing the profit of the portfolio. It also shows that, except the stock of CSGP, other three could be changed to other stocks. If all of the four stocks are on the efficient frontier between maximum Sharpe ratio and minimum volatility, then it could be the best portfolio.

Moreover, analysis of the effective boundary shows that the curve curves upward and slows down gradually after a certain point. This is because, beyond this point, the increase in risk no longer significantly affects expected returns due to the correlation between different assets and their contributions to overall portfolio risk.

Finally, the efficient boundary is used to select the most appropriate portfolio for investors. Our analysis shows that MSFT and TSLA are on the efficiency frontier and can be used as reference allocation ratios to optimize expected returns and minimize risk.

5. Conclusion

In conclusion, this study proposes a systematic portfolio optimization method based on historical stock data. The results obtained show the characteristics and performance of different portfolios, which are useful for investors seeking to maximize returns while minimizing risk. However, it is important to note that historical performance is not an absolute predictor of future performance, and investing always carries some degree of risk. In order to make informed investment decisions, investors should consider a variety of factors, such as overall market conditions, geopolitical risks and the financial condition of individual companies. Take scientific risk control measures, such as diversification of investment and prudent allocation of assets, to reduce the impact of market fluctuations.

In addition, the experiment provides a starting point for future research, as it can be extended to explore other aspects of portfolio optimization, such as including more assets or considering macroeconomic indicators. By constantly improving and developing these methods, investors can gain a deeper understanding of the behavior of financial markets and make more informed investment decisions.

References

[1] Markowitz, H. (1952), Portfolio Selection. The Journal of Finance, 7: 77-91.*
[2] Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. The journal of finance, 19(3), 425-442.
[3] Jagannathan, R., & Ma, T. (2003). Risk reduction in large portfolios: Why imposing the wrong constraints helps. The Journal of Finance, 58(4), 1651-1683.
[4] Feldstein, M. S. (1969). Mean-variance analysis in the theory of liquidity preference and portfolio selection. The Review of Economic Studies, 36(1), 5.
[5] Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. Journal of financial economics, 33(1), 3-56.
[6] Black, F., & Litterman, R. (1992). Global portfolio optimization. Financial analysts journal, 48(5),

ISSN 2616-5902 Vol. 5, Issue 9: 45-53, DOI: 10.25236/AJBM.2023.050908

28-43.
[7] Meucci, A. (2005). Risk and asset allocation (Vol. 1). New York: Springer. Jensen
[8] M. C., Black, F., & Scholes, M. S. (1972). The capital asset pricing model: Some empirical tests.
[9] Mangram, M. E. (2013). A simplified perspective of the Markowitz portfolio theory. Global journal of business research, 7(1), 59-70.
[10] Henriksson, R., Livnat, J., Pfeifer, P., & Stumpp, M. (2019). Integrating ESG in portfolio construction. The Journal of Portfolio Management, 45(4), 67-81.
[11] Yu, S., Wang, H., & Dong, C. (2023). Learning risk preferences from investment portfolios using inverse optimization. Research in International Business and Finance, 101879.
[12] Morelli, G. (2023). Responsible investing and portfolio selection: a shapley-CVaR approach. Annals of Operations Research, 1-29.
[13] Lassance, N., & Vrins, F. (2023). Portfolio selection: A target-distribution approach. European Journal of Operational Research.
[14] Aithal, P. K., Geetha, M., Acharya, U. D., Savitha, B., & Menon, P. (2023). Real-Time Portfolio Management System Utilizing Machine Learning Techniques. IEEE Access.