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Direct Iterative Algorithm for Limit Lower Limit Analysis of Axisymmetric Structures

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Abstract: *Axisymmetric structure is a common structural form, and its stability analysis is the key to ensure structural safety. However, for complex axisymmetric structures, the conventional limit analysis method has the problems of low computational efficiency and low computational accuracy. The research in this paper will utilize the direct iterative algorithm to improve the computational accuracy and efficiency. The algorithm transforms the limit analysis problem into a series of easy-to-solve sub-problems by defining a suitable iterative function, and gradually converges to the limit state of the structure through gradual approximation. Experimental results show that the algorithm can achieve the highest prediction accuracy of 95.5% and the fastest computation can be realized within 0.1 seconds. In addition to the advantages in prediction accuracy and computational efficiency, the direct iterative algorithm shows good stability of the algorithm under different material properties and structural conditions.*

Keywords: *Axisymmetric Structures, Ultimate Lower Bound Analysis, Direct Iterative Algorithm, Computational Efficiency, Ultimate Load Carrying Capacity*

1. Introduction

Due to the good force characteristics and wide application prospects, axisymmetric structures have become a hot spot of structural research in recent years. Finite element calculation is an effective calculation method. Although the traditional finite element method has been generally accepted and widely used, its numerical computation and solution become very cumbersome due to its special geometry, loading and other factors. This constrains the efficiency of the research and also reduces the accuracy of the research results, especially for those engineering designs and safety evaluations that require rapid response. In addition, current finite element programs often consume huge computational resources and time when solving large or complex axisymmetric structures, which makes it difficult to meet the engineering demands. In addition, human errors will be added in the calculation process, which reduces the credibility of the research results. Developing effective analysis algorithms is of great significance to ensure the safety and effectiveness of engineering.

This paper adopts direct iteration and establishes a new solution method to reduce the complexity of the solution. By optimizing the iterative method, the algorithm is fast and stable. The key to this algorithm is to avoid the complex pre- and post-processing process, thereby greatly improving the calculation efficiency. In addition, this paper also conducts a rigorous convergence analysis of each step to ensure the effectiveness and accuracy of the method. This paper will use a combination of numerical simulation and theoretical analysis to further verify the effectiveness and applicability of the algorithm under different engineering conditions.

2. Related Works

In many disciplines such as geotechnical engineering, materials and fluid dynamics, the study of the ultimate performance of a very important type of axisymmetric structure has always been a hot topic of concern. Keawsawasvong S used the cylindrical foundation as the research object and conducted an analysis and experimental study on it using the finite element method [1]. Hu G explored the structural optimization and fertilization performance of a non-axisymmetric venturi nozzle [2]. Wang D investigated the collision performance and multi-objective optimization of a honeycomb-filled

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thin-walled energy absorber with axisymmetric thickness [3]. Farrokh M extended Carrera's unified formula for analyzing the mechanical and thermal buckling of an axisymmetric FG circular plate [4]. Li F investigated the hypersonic quadratic instability of Görtler vortices on axisymmetric configurations in the boundary layer [5]. Baskaran P analyzed the aerodynamic propulsion performance of an axisymmetric machine body for boundary layer inhalation applications [6]. Sun Rui proposed an axisymmetric adaptive lower bound finite element method based on the Mohr-Coulomb criterion and second-order conic planning technique [7]. Lin Yuanxiang analyzed the lower limit of stability of a layered road structure under moving simple harmonic loading [8]. Wang Xiaogang proposed a limit analysis method for slope stability based on rigid block discretization[9]. Yang Xiaohua analyzed the stability of landslides in expansive soils using homogenization theory and upper limit analysis [10]. These studies provide a valuable theoretical and practical basis for limit analysis of axisymmetric structures.

Although the existing research has gained some progress in the limit state study of axisymmetric members, there are still many defects. The existing solution algorithms have the problem of slow solution speed when solving complex loads and boundary conditions, which cannot be well adapted to practical needs. Secondly, some methods in some researches are due to the simplification of the model, as well as the consideration of some subjective factors for the selection of parameters, which has a large impact on the accuracy of the calculation. In order to solve the above problems, this paper is based on the direct iterative method of limit lower bound analysis for axisymmetric structures, and by improving the iterative strategy and modeling in the algorithm, the solution speed and accuracy of the algorithm have been significantly improved.

3. Methods

3.1 Mathematical Model of Direct Iterative Algorithm

The limit analysis of axisymmetric structure is solved by direct iterative algorithm. The mathematical model is constructed based on the basic principles of material mechanics and structural mechanics, considering the mechanical behavior of axisymmetric structures under static loading. It is assumed that the material follows linear elasticity until yielding, after which it behaves as nonlinear plasticity [11]. The stress-strain relationship of the material can be expressed as:

$$\sigma = E(\varepsilon) \tag{1}$$

Where σ denotes the stress, E is the elastic modulus of the material, and ε denotes the strain. Further, to ensure the mechanical equilibrium of the structure, we introduce the equilibrium equation, which describes the relationship between internal forces and external loads:

$$\nabla \cdot \sigma + f = 0 \tag{2}$$

Where $\nabla \cdot$ denotes the scattering operation and f denotes the volumetric force, the equilibrium equation is one of the conditions that must be satisfied during the iterative solution process, and an accurate description of the yielding behavior of the material is essential for limit analysis. In this paper, the Mohr-Coulomb yield criterion is used to define the yield condition of the material:

$$f(\sigma) = \sqrt{J_2} - \sqrt{\frac{2c\cos(\phi)}{3}} \leq 3 \tag{3}$$

Where J_2 is the second invariant, c is the internal friction of the material, while ϕ is the friction angle, and the yield condition ensures that the nonlinear behavior of the material as it reaches its limit state is correctly modeled. The iterative process is centered on the gradual approximation of the limit state of the structure. We update the stress and displacement fields by means of the following iterative formulation:

$$\begin{aligned} \sigma^{(k+1)} &= \sigma^{(k)} + \Delta\sigma \\ u^{(k+1)} &= u^{(k)} + \Delta u \end{aligned} \tag{4}$$

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In equation (4), k denotes the current iteration step, while $\Delta\sigma$ and Δu represent the increments of stress and displacement, respectively.

3.2 Algorithmic Framework

This paper details a direct iterative algorithmic framework for solving limit analysis problems for axisymmetric structures. The algorithmic framework aims to progressively approximate and finalize the ultimate load carrying capacity of the structure through a series of ordered steps [12]. The algorithmic framework is shown in Figure 1:

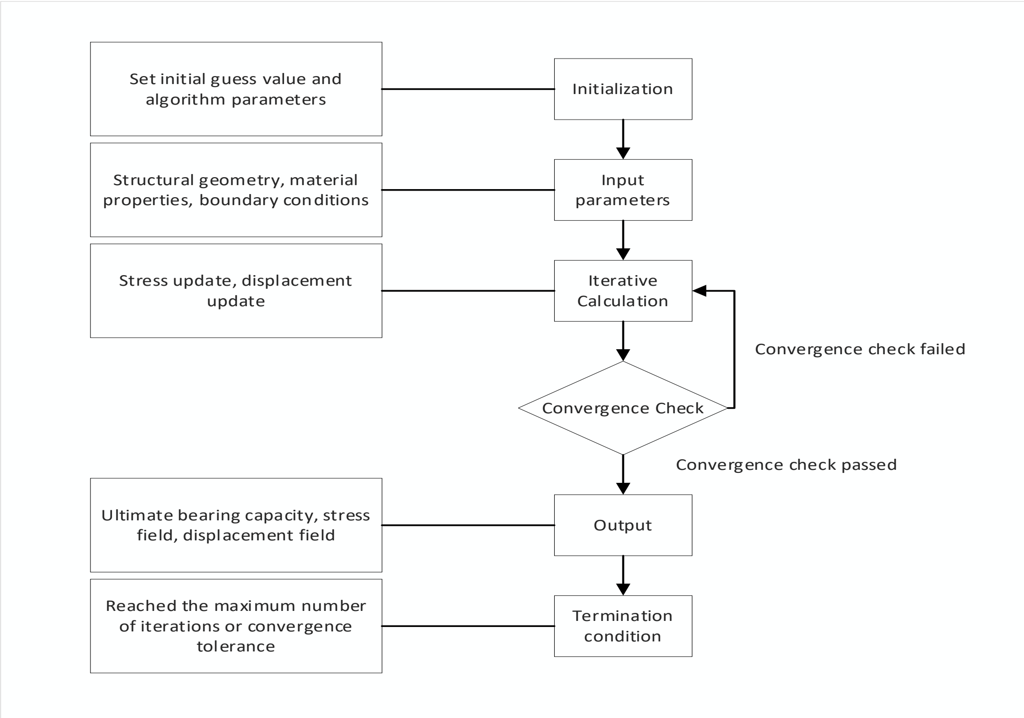

Figure 1: Direct iterative algorithm framework

The key to the algorithm is the use of an iterative method, i.e., the stresses and displacements within the structure are modified by successive modifications until the full equilibrium and yield state is reached. The method includes initialization, iterative operations, convergence checking and output of results. In the initialization process, reasonable initial guess solutions will be selected, including initial displacements u_0 and initial stresses σ_0. The method is based on iteratively obtaining the displacement field, finding the stress at each material point from the material's intrinsic equation, and checking the yield state of each material point. If the yield condition is not met, it is corrected according to the equilibrium equation and the yield criterion, and then it is repeated until the specified number of iterations is reached or until it reaches a state of convergence. At the end of each iteration, it is necessary to enter to check whether the change in displacement between two consecutive iterations is less than the preset tolerance, and to check whether the change in stress between two consecutive iterations is less than the preset tolerance. If applicable, the change in the test indicator (e.g., the load carrying capacity of the structure) is checked to see if it is lower than a predetermined tolerance allowable value. When the method gets converged, the final stress field, the displacement field and the final load carrying capacity can be obtained [13-14].

3.3 Algorithm Implementation Details

Since the algorithm is highly capable in numerical operations, matrix operations and graphical display, the method is carried out in MATLAB. The software uses MATLABR2021a, which employs the Parallel Computing Toolbox to realize parallel computing, thus greatly improving the operational efficiency of the algorithm. The information related to stress, strain, displacement, etc. of each stacked layer is effectively processed by utilizing an array of structural bodies. Matrices are used to represent the geometrical parameters and material properties of the structure, and vectors are used to store key

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parameters of the iterative process, such as convergence tolerance and number of iterations. The core of the algorithm lies in the implementation of the material principal model, the equilibrium equation solver, the yield condition checking and the iterative update strategy. In order to further improve the execution efficiency of the algorithm, multiple subspace partitioning techniques will be used to partition the subspaces and achieve mutual independence and parallelization among the subspaces, while a dynamic load balancing mechanism will be used to ensure that the computational resources within each subspace can be maximized.

3.4 Simulation Experiments and Collection of Receipts

In terms of data collection, rigorous testing methods were adopted, and a large number of experiments were conducted, and compared with traditional theoretical and analytical methods. Combined with experimental data, the convergence and solution efficiency of the direct iteration algorithm were evaluated. Through numerical simulation, the mechanical performance of the axisymmetric connection under different working conditions was studied, and its calculation accuracy was evaluated. In the experiment, shafts with different radii, different heights and different materials were used, and different restrictions and external loads were set. Then, a systematic study was carried out on the number of iterations, calculation time, and storage capacity from the aspects of geometry, materials, boundary conditions, and external loads.

4. Results and Discussion

4.1 Ultimate Load Carrying Capacity

This paper has set parameters, the specific settings are shown in Table 1:

Table 1: Parameter Settings

Parameter name	Settings	Parameter range	Influence
Initial guess value	0	(0,1]	Convergence speed
Learning rate	0.01	(0,1]	Convergence
The maximum number of iterations	1000	[1,∞)	Computing costs
Time step	0.1	(0,∞)	Stability
Regularization parameter	$\lambda = 0.01$	(0,∞)	Overfitting control

This paper adopts the direct iterative algorithm method to solve the ultimate bearing capacity problem of the shaft under various material properties and external pressure conditions. Through a series of numerical simulations, the calculation results are compared with the test results to verify the accuracy and reliability of the algorithm. These comparison results are summarized in Table 2:

Table 2: Ultimate bearing capacity

Structure type	Elastic modulus (gpa)	Internal friction (kpa)	Friction angle (°)	Pressure 1(kpa)	Pressure 2(kpa)	Predicted ultimate load capacity (kn)	Experimental data carrying capacity (kn)
A	70	30	30	100	150	3500	3536
	70	40	35	120	180	3600	3587
	80	30	30	100	150	4100	4071
	80	40	35	120	180	4200	4225
B	60	25	25	80	120	2800	2834
	60	35	30	100	150	2850	2868
	90	25	25	80	120	3300	3283
	90	35	30	100	150	3400	3438
C	50	20	20	60	90	2500	2487
	50	35	25	70	105	2700	2725
	60	20	20	60	90	3000	3035
	60	35	25	70	105	3300	3271

As can be seen from Table 2, the predicted ultimate load carrying capacity is in general agreement with the experimental results, proving the feasibility of the analytical method applied in this paper. From the data, it can be seen that structures with higher modulus of elasticity usually have higher

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ultimate load carrying capacity, and the increase in internal friction is also associated with the increase in load carrying capacity. In addition, it can be seen that the experimental data are very close to the load carrying capacity predicted by the algorithm, and the comparison between the ultimate load carrying capacity predicted by the algorithm and the experimental data shows a high degree of consistency, and the algorithm is capable of reliably predicting the load carrying capacity of different structures under different material properties and pressure conditions.

4.2 Accuracy Analysis

Accuracy data was collected by comparing the algorithm's predicted ultimate load carrying capacity with a series of tightly controlled experimental results. Figure 2 illustrates the algorithm's predicted accuracy data collected in this paper:

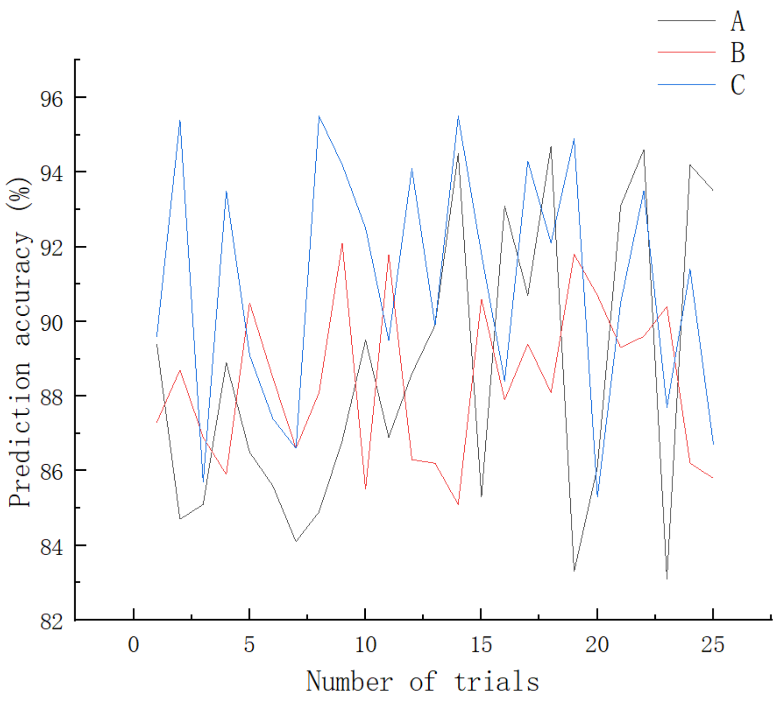

Figure 2: Algorithm prediction accuracy

In the data in Figure 2, it can be seen that the accuracy of the algorithms remains above 80% for all the structure types considered, with its great potential as a reliable tool. The highest algorithmic prediction accuracy is able to reach 95.5%, a result that emphasizes the high accuracy of the algorithms for this type of structure; the lowest algorithmic prediction accuracy is also 83.1%, which is well above acceptable engineering standards. These data validate the effectiveness of the direct iterative algorithm in the analysis of the lower limit of the limit of axisymmetric structures and also reflect that the algorithm has a very good ability to generalize across different structure types.

4.3 Computational Efficiency Analysis

In order to fully evaluate the effectiveness of the direct iterative algorithm in the limiting limit analysis of axisymmetric structures, this paper selects more existing research methods for comparison and analysis. These methods include the traditional Finite Element Method (FEM), Monte Carlo Simulation (MCS) based on random sampling, and Gradient Projection (GP) Method in optimization. Figure 3 synthesizes the results of the computational speed comparison of these methods in the analysis of the lower limit of the limit of axisymmetric structures.

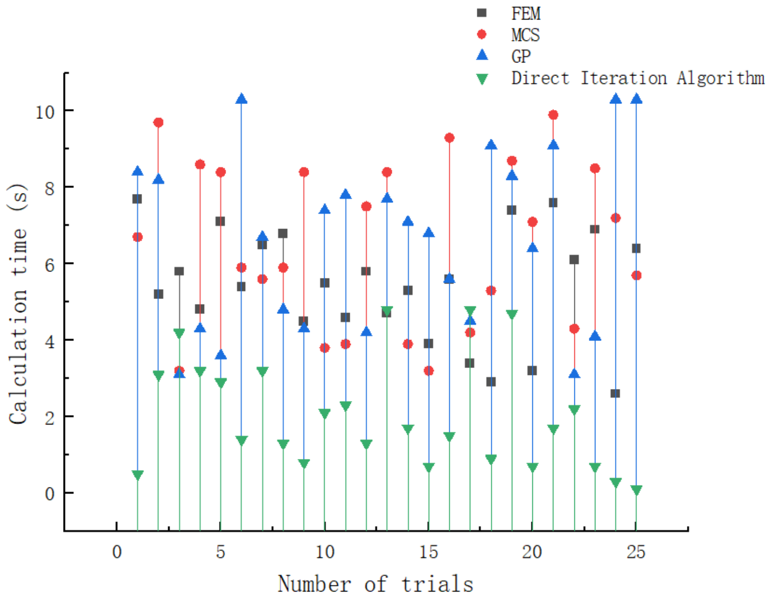

Figure 3: Calculation speed

It is obvious from Figure 3 that overall the computation time of the direct iterative algorithm is significantly lower than that of the FEM, MCS and GP. The direct iterative algorithm demonstrates superior computational speed in most cases, and the algorithm is able to avoid cumbersome pre- and post-processing. The fastest computation speed is 0.1 seconds, which is very suitable for engineering applications that require high response speed. However, it can also be noted that in some of the experimental cases the computational time of the direct iteration algorithm is relatively long. This may be due to the complexity of some of the experimental objects themselves, or the increase in the number of iterations required when dealing with certain specific situations. According to the specific situation, the performance bottleneck of the algorithm should be studied and improved to speed up the computational efficiency of the algorithm, so that the algorithm can satisfy the computational accuracy and thus better meet the engineering requirements.

Although this paper has proved that the direct iterative algorithm has significant advantages in terms of accuracy and computational speed, its computational speed is not fast enough in some experiments. In order to solve this difficulty, this topic will further adjust the parameters of the algorithm according to the actual situation of different complexity from three perspectives: the number of iterations, the reduction of redundant operations, and the acceleration of convergence speed.

5. Conclusion

In this study, a direct iterative algorithm was successfully developed and validated for predicting the ultimate load carrying capacity of axisymmetric structures under different material properties and external loads. Through careful setting and optimization of the algorithm parameters, this paper demonstrates the significant advantages of the algorithm in terms of accuracy and computational efficiency. The direct iterative algorithm performs well in the analysis of limit lower bounds for axisymmetric structures, showing significant advantages in both accuracy and computational efficiency. In some complex cases, the computation time of the algorithm is relatively long. To address this problem, future work will focus on further optimization of the algorithm, adjusting the parameters, exploring parallel computation methods, investigating problem simplification strategies, and validating specific examples.

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