

A new isosceles right triangle fuzzy pattern recognition method and the analysis of its feasible conditions

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Abstract: *This paper proposes a dependency type isosceles right triangle affiliation function for the problem that isosceles right triangles cannot be recognized accurately, relying on the principle of maximum affiliation. This function calculates the affiliation of isosceles right triangles by the previously obtained isosceles triangle and right triangle affiliation. The absolute reliability of the proposed function under certain reasonable conditions is verified through theoretical proofs, providing a new perspective in the field of triangular fuzzy pattern recognition.*

Keywords: *Isosceles right triangle; Fuzzy pattern recognition; Maximum affiliation principle; Dependency type isosceles right triangle affiliation function*

1. Introduction

Fuzzy pattern recognition as a sub-discipline of artificial intelligence has been developed rapidly in recent years. The identification of different objects in real-world problems (e.g., chromosomes, sickle platelets, etc.) can often be reduced to the identification of some geometric shapes, with triangles being one of the most basic geometric shapes. Some applications of triangle recognition are shown in Fig. 1. Triangles can be divided into two categories: typical triangles and atypical triangles. There is a very small probability of encountering absolute typical triangles in real life: isosceles triangles, right triangles, isosceles right triangles, and equilateral triangles. These triangles, if they exist, are not accurately identified due to conditions such as the accuracy of the measurement tool. Therefore, the problem of triangle type recognition can be summarized as fuzzy pattern recognition [1].

In 1965, Professor Zadeh [2] proposed fuzzy set theory in the United States, and fuzzy patterns were developed considerably. Fuzzy pattern recognition classifies the recognized objects by establishing an appropriate affiliation function and using the principle of maximum affiliation. Earlier studies on triangular fuzzy pattern recognition can be found in the literature [3]. In 1992, Zongyuan Zhang [4] devised several fuzzy triangle affiliation functions. In 2005, Xue Zhang [5] proposed a new recognition algorithm for the shortcomings of the current irregular triangle and quadrilateral recognition methods. In 2010, Xiangjun Cui et al [1] proposed an exponential affiliation function, which takes the triangle interior angle and difference relationship as the base and exponential variables. Later in 2020, Shanshan Cao et al [6] proposed an affiliation function construction method based on weighted average and sine theorem for isosceles triangles and equilateral triangles. In the same year, Mengyao Zhang et al [7] improved the triangle affiliation function and proposed a triangle pattern recognition method based on variable thresholds.

In this paper, a dependency type isosceles right triangle affiliation degree function is proposed based on the previous work. This function calculates the affiliation of isosceles right triangles based on the previously obtained affiliation of isosceles triangles and right triangles. The remainder of this paper is structured as follows: In Section 2, the basics of triangular fuzzy pattern recognition are introduced. Section 3 presents the new method proposed in this paper and gives a proof of the feasibility condition. The research work of this paper is summarized and prospected in Section 4.

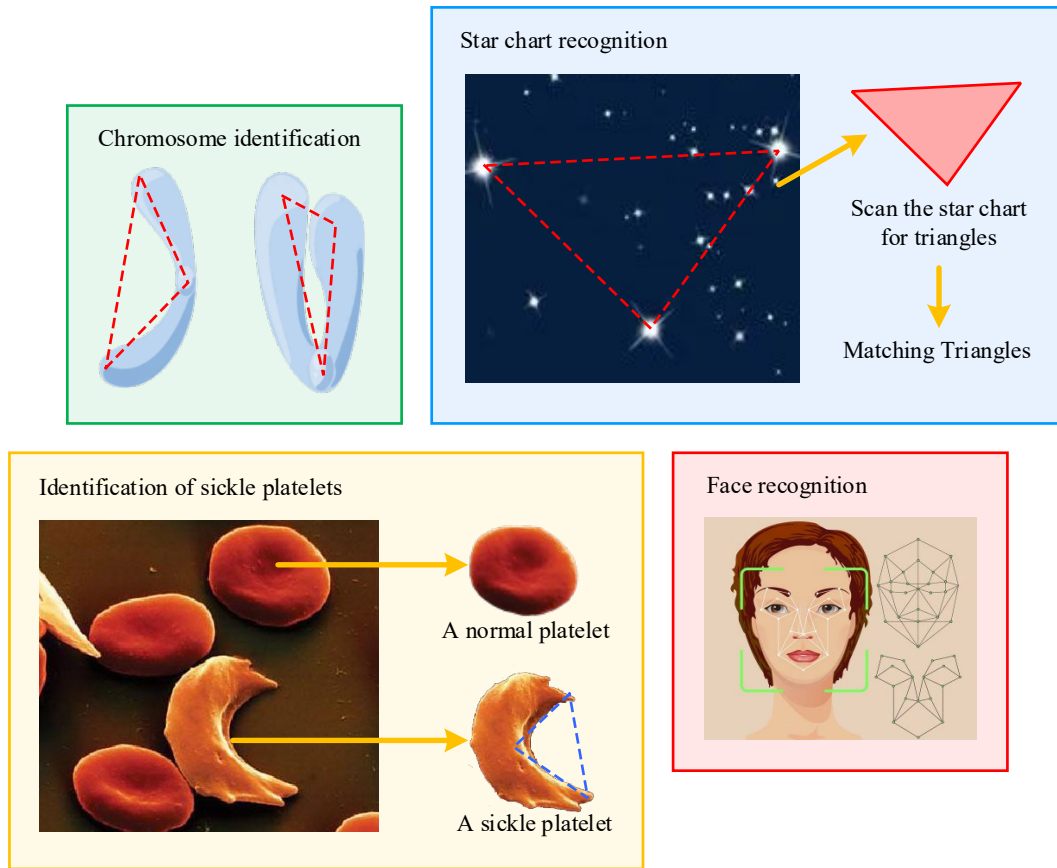


Figure 1. Some applications of triangle recognition.

2. Basic concepts

Definition 1 [8] A is said to be a fuzzy set on U if for any element x in the argument domain U , there is a number $A(x) \in [0,1]$ corresponding to it. At this point, $A(x)$ is called the affiliation of x to A . When x varies in U , $A(x)$ is a function called the affiliation function of A . The closer the affiliation $A(x)$ is to 1, the higher the degree of x belonging to A . The closer $A(x)$ is to 0, the lower the degree of x belonging to A .

Definition 2 [1] For a fuzzy pattern S on a given argument domain U , u_1, u_2, \dots, u_n is the n objects to be identified on U . If $S(u_i) = S(u_1) \vee S(u_2) \vee \dots \vee S(u_n)$, u_i is considered to be preferentially attributed to the fuzzy pattern S . This is the principle of maximum affiliation.

3. Dependency type isosceles right triangle affiliation function

This section gives the definition of the dependency type isosceles right triangle affiliation function and the analysis of the feasibility conditions.

3.1. Definition of the function

It may be useful to let the affiliation degree functions of equilateral, isosceles, right-angle, isosceles right-angle and atypical triangles be E, I, R, IR and T , respectively. The dependency type isosceles right triangle affiliation function proposed in this paper is defined as

$$IR = \begin{cases} 1 - |I - R|^2, & \max\{E, I, R, T\} \neq T \\ I \wedge R, & \max\{E, I, R, T\} = T \end{cases}$$

To prove the feasibility of the function, i.e., to prove that the affiliation obtained by the function under the condition of non-isosceles right triangles is less than the affiliation of a certain type of triangle (not containing isosceles right triangles). Moreover, under the condition of isosceles right triangles, the affiliation obtained by this function is greater than the affiliation of any type of triangle (excluding isosceles right triangles). Equilateral, isosceles, right-angled and atypical triangles can be represented as

$$\begin{cases} E = 1 - e \\ I = 1 - i \\ R = 1 - r \\ T = 1 - t \end{cases}$$

where $e, i, r,$ and t are any real numbers greater than 0 and less than 1.

3.2. Analysis of feasible conditions

The function relies on the previously calculated affiliations of equilateral, isosceles, right-angled and atypical triangles, so that if a proof of a feasible condition for the function is required, it needs to be assumed that other types of triangles can be identified properly, i.e., given the conditions for the following cases.

Case 1 The triangle to be identified approximates an equilateral triangle. The condition in this case is $e < \max\{i, r\} < t$;

Case 2 The triangle to be identified approximates an isosceles triangle. The condition in this case is $i < \min\{e, r\} < t$;

Case 3 The triangle to be identified approximates a right triangle. The condition in this case is $r < \min\{e, i\} < t$;

Case 4 The triangle to be identified approximates an atypical triangle. The condition in this case is $t < \min\{e, i, r\}$;

Case 5 The triangle to be identified approximates an isosceles right triangle. The condition in this case is $\max\{i, r\} < \min\{e, t\}$.

Obviously, all of the above conditions are reasonable. If you want to get the feasible condition of the method, i.e., prove the condition that $IR < E$ in case 1, $IR < I$ in case 2, $IR < R$ in case 3, $IR < T$ in case 4 and $IR > \max\{E, I, R, T\}$ in case 5. The proof is given below.

(1) Case 1

When $e < \max\{i, r\} < t$, $IR = 1 - |r - i|^2$. Then $\lambda < \log_{|r-i|} e$ can be obtained from $IR < E$.

(2) Case 2

When $i < \min\{e, r\} < t$, $IR = 1 - |r - i|^2$. Then $\lambda < \log_{|r-i|} i$ can be obtained from $IR < I$.

(3) Case 3

When $r < \min\{e, i\} < t$, $IR = 1 - |r - i|^2$. Then $\lambda < \log_{|r-i|} r$ can be obtained from $IR < R$.

(4) Case 4

When $t < \min\{e, i, r\}$, $IR = I \wedge R$. Obviously, $IR < T$ holds in any case.

(5) Case 5

When $\max\{i, r\} < \min\{e, t\}$, $IR = 1 - |r - i|^2$. Then $\lambda > \min\{\log_{|r-i|} i, \log_{|r-i|} r\}$ can be obtained from $IR > \max\{I, R\}$.

Although certain conditions need to be satisfied for the feasibility of this function, these conditions can be satisfied with an appropriate affiliation function, verifying the reliability of the method proposed in this paper. To facilitate the understanding of the proposed method, Fig. 2 shows the variation of $1 - |r - i|^\lambda$ with r and i when λ takes different values.

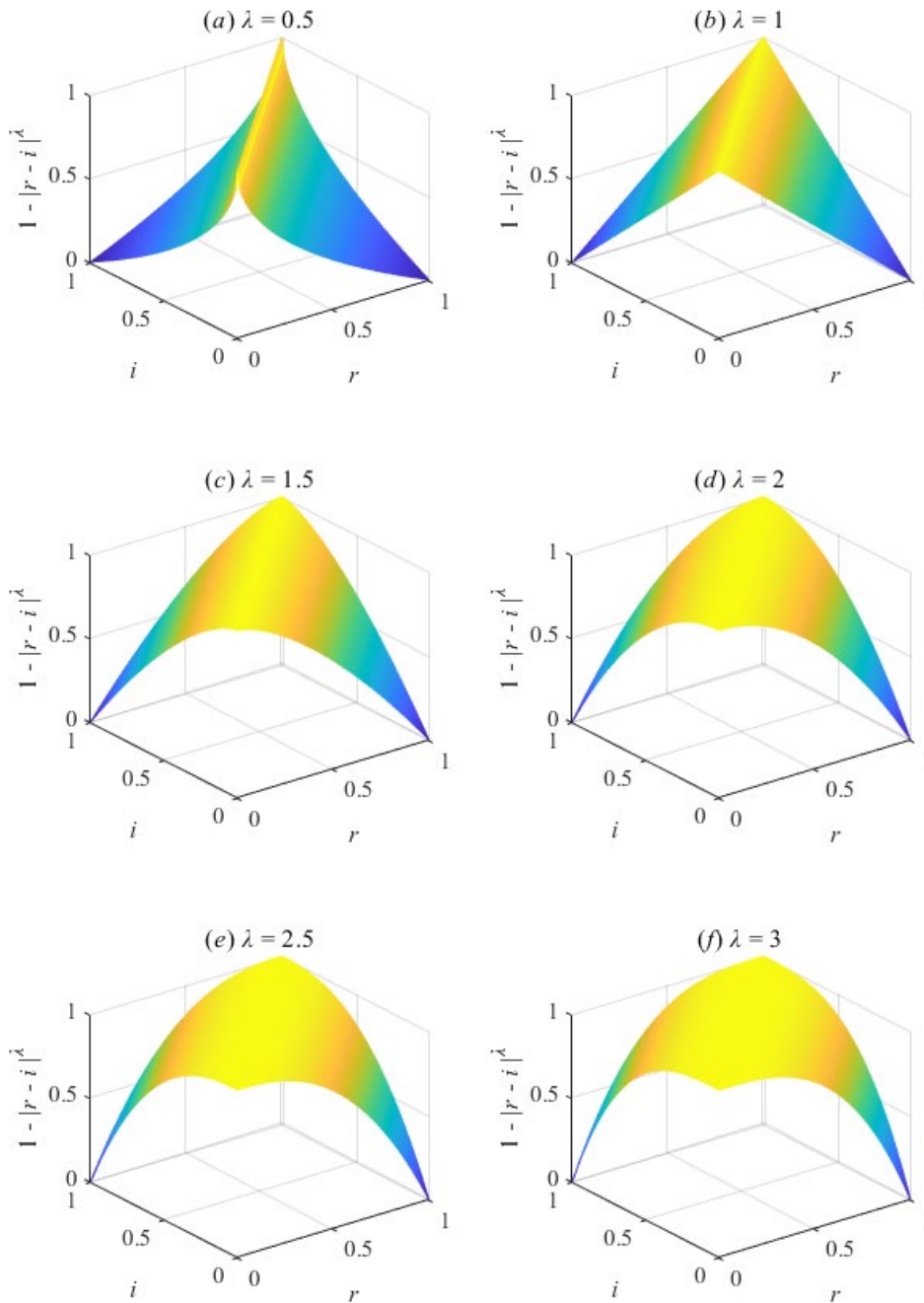


Figure 2. The variation of $1 - |r - i|^\lambda$ with r and i when λ takes different values.

4. Conclusions

In this paper, a dependency type isosceles right triangle affiliation degree function is proposed. The calculation of this function relies on the previously calculated affiliations of isosceles and right triangles. While providing a new affiliation function, this study also theoretically demonstrates its absolute reliability under certain reasonable conditions and provides a new vision for triangular fuzzy

pattern recognition.

This paper will continue to study the more universal recognition method of triangular fuzzy pattern recognition. Furthermore, the proposed methodological approach is attempted to be used in real-world problems.

Data availability

No data was used for the research described in the article.

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