

Research on the 3D Geometry of Sand Cooks Caused by Ocean Wave Erosion

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ABSTRACT. Sandcastle by the sea is a popular leisure activity among tourists of all ages. Most people who have piled sandcastle by the sea have encountered the situation of sandcastle being washed away by the sea. So, whether it is possible to build a sand castle as stable as possible, so that it can stand out from the crowd of sand castle in the face of wave erosion and even rainfall, has become an interesting and practical problem. In view of the above, our team uses the time series theory and the computer-generated "Wave-Sandcastle Model" program. Under the guidance of rigorous theoretical derivation, we use the computer program to simulate the real situation.

KEYWORDS: Sand castle, Computer simulation, Wave erosion, 3-Dimensional Geometric Shape

1. Introduction

Wherever there are recreational sandy ocean beaches in the world, there seem to be children (and adults) creating sandcastles on the seashore. Visitors created sandcastles, while sandcastles serve as recreation pleasing visitors back. One typically forms an initial foundation of a sandcastle consisting of a single, nondescript mound of wetted sand, and then proceeds to cut and shape this base into a recognizable 3-dimensional geometric shape upon which to build the more castle-defining features. Even if built roughly the same size and at roughly the same distance from the water on the same beach, castles do not react the same way to waves and tides. Therefore, we have studied the 3D geometric shape changes of sand castles under seawater erosion, so as to provide guidance for the construction of solid sand castles.

2. Model Assumptions

The waves and tides on the beach will impact the sand castle foundation. Because the contact between the foundation and sea water will turn the part of sandpile from semisolid and plastic state enter into the fluid state, instantly. [?] So

every impact will make part of the sand into the flow state, and leave the foundation with the waves under the effect of the ebb viscous force; The other part of the loss is due to direct mechanical friction. We always assume that this part of loss is only related to the size of the wave.

As waves on sea shore erode shrink to 10-20 cm thick, compared with a sand castle of several meters, the difference is not big. So we consider the geometry of two-dimensional scour resistance, and then combine the nature of sandpile to extend to three-dimensional figure.

A simplified two-dimensional model is as follows: Assume that our primary substrate is a completely dry sand pile S_0 ; The wave comes in random size, and its unit force has peak value W_f and valley value W_g .

3. Model Assumptions

The foundation model in figure 1 is the bottom shape of sand castle overlooking the sea. The bottom of the figure is farthest from the sea water, and the top is closest to the sea water. The corresponding 3D view is shown in the right figure below.

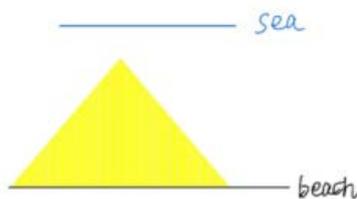


Figure. 1 2-D example



Figure. 2 Imagine Location

To simulate the effect of waves on Sandcastle, it is necessary to build a wave model first. Because waves around the world are related geographical location, surrounding environment and other factors, we have conducted some research on the general form of waves and found that the water flow under the wave crest is affected by waves along the wave propagation direction the current under the trough is affected by the opposite direction of wave propagation, so the flow direction is always changing. The circular motion path formed with the change of water depth is a circular track, which is called the track velocity of wave [1].

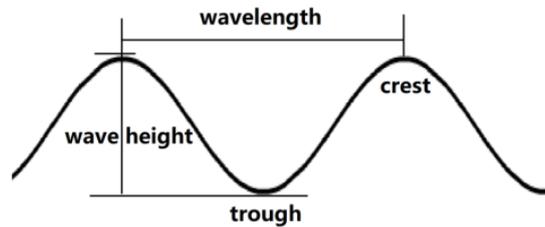


Figure. 3 Wave crest and trough

The track velocity decreases exponentially with the increase of water depth and the decrease of wave length. That is, with the decrease of water depth, the track velocity increases exponentially and approaches to infinity; the track velocity approaches to infinity, which means that the circle track of circular motion path formed with the change of water depth increases. [2] It is understood as a circle with radius approaching to infinity, so the wave length increases and the wave height decreases. In this way, when the waves rush to a coast, the water depth tends to 0, and it can be approximately considered that the wave shape tends to a straight line.

According to the fact that the waves don't wash evenly on the beach in the actual situation, we use the computer to generate certain noise, simulate the waves. And design the probability of noise occurrence in the form of negative correlation according to the size of the noise. So as to get the simulation sea wave model. Combined with the sandcastle model, we get the "wave sand castle foundation two-dimensional model". Effects of waves on different positions of sand fort are simulated carefully, and the erosion effect of sand fort with different shapes under the action of simulated waves is observed [3].

The wave erosion results corresponding to several basic geometric shapes are given in Table 1:

Table 1 Wave erosion results corresponding to several basic geometric shapes

parameter	Triangle Foundation	Rectangulai Foundation	Convex Foundation
origin shape			
Sea Water Scour Onece			
Sea Water Scour 10 Times Sand Loss Rate	 46.6%	 52%	 %32.8%
Sea Water Scour 20 Times Sand Loss Rate	 80%	 95.62%	 %67.52%

In fact, after a lot of theoretical verification and computer simulation, we found that the sand castle with bell curve (normal distribution density curve) section is the best shape to keep the sand castle stable. The 3D view of sandcastle is shown below [4].

4. Rationality Analysis and Conclusion

When tides come, we explore the pressure δP . At the sand-water-contacting moment, pressure inside sandcastle foundation considered none. Pressure at the junction

$$\begin{aligned} \Delta m \cdot v_0 &= \Delta F \Delta t - \rho g h \Delta S \Delta t \\ \Rightarrow \rho &= \frac{\Delta F}{\Delta S} = \frac{\Delta m}{\Delta S \Delta t} v_0 + \rho g h \\ &= p v_0^2 + p g h \\ \Delta P &= p - 0 = p v_0^2 + p g h \end{aligned}$$

Suppose that all the sand wetted by the sea water is taken away by the action of the sea water when the tide is low, so:

$$\left\{ \begin{aligned} y_0 - y &= \int_0^t v_1 dt \\ v_1 &= \frac{k \Delta P}{\eta y} = \frac{k p}{y y} = \frac{k(p v_0^2 + \rho g h)}{\eta y} \\ \Rightarrow \frac{dy}{dt} &= -\frac{k x}{y y} \\ \Rightarrow y &= \sqrt{y_0^2 - \frac{2 k n}{y} t} \end{aligned} \right.$$

If we suppose origin foundation as a triangle base, that is,

$$y_0 = ax - b$$

Then we get:

$$\begin{aligned} \therefore y &= \sqrt{y^2 - \frac{2kp}{y}t} \\ \cdot \frac{4}{\sqrt{2}} &= \frac{1}{\sqrt{y^2 - 2y - t}} \cdot \left(-\frac{y}{y}\right) \end{aligned}$$

Assuming the optimal foundation shape $y_0 = f(x)$, the number of sand removed per unit time should be minimized, that is, $\left|\frac{dy}{dt}\right|$ will be minimized. By fitting and observing the data, we consider when:

$$y_0 = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

is satisfied, $\left| \frac{dy}{dt} \right|$ reaches its minimum value.

5. Conclusion

In this paper, we study the geometric shape change of sand castle under seawater erosion, which extends from two-dimensional to three-dimensional. And by using the method of model analysis and computer simulation, the conclusion is scientific and accurate, which provides guiding significance for the construction of solid sand castle.

References

- [1] A. I. H. O. Thirapong Pipatpongsa, Sokbil Heng, "Statics of loose triangulai embankment under nadai's sand hill analogy," Journal of the Mechanics and Physics of Solids, 2010.
- [2] . L.-k. .-c. LI Yuanfu, fu, "An experimental study on the self-organized criticality of sand pile model with one-grade slope," JOURNAL OF SOUTHWEST JIAOTONG UNIVERSITY, vol. 35, no. 2, 2000.
- [3] S. Bingjun, "Numerical analysis on slope seepage field and stability under rainfall infiltration," Faculty of Civil Engineering and Mechanics Kunming University of Science and T echnology, 2013.
- [4] J. Y. ZHANG Wo-hua, WU Chang-can, "Numerical modeling for beach erosion," The Ocean Engineeing, vol. 20, no. 2, 2002.