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Break Personal Bests: A Comprehensive Study on Strategies of Cycling

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Abstract: *With the expansion of sports industry, more and more athletes begin to use mathematical models to study sports process, in order to improve sports performance. To characterize the abilities of cyclists, this paper defines the Power Profile and plot the radar diagrams to describe sprint abilities, anaerobic capacity,* VO_2 *max capacity and FTP (Function Threshold Power) of different riders. In order to optimize the output power of cyclists in different stages of the race, an effective Cycling Model was established to provide the relationship between the rider's position on the course and the power the rider applies.*

Keywords: *Cycling Model, Radar Diagrams, Differential Equations*

1. Introduction

It has been a trend to use Mathematical Models to improve athletes' performance since 1970s[1]. American math scientist Joseph B. Keller published an article called A theory of competitive running[2], which proposed a model that could represent the characteristics of different runners and was a great guide to the sport of running at that time, thus the athletes greatly improved their performance [3]. Andrew M. Jones et al. described the power curve into intermittent high-intensity exercise and used existing two-parameter models (hyperbolic relationship between power and duration) to make better predictions for athlete fatigue monitoring[4]. Morton extended the two-parameter model to a three-parameter model and developed a new power curve model [5]. Dr. Andrew Coggan et al. used a three-parameter model to analyze the strengths and weaknesses of different types of riders through statistical data and obtained the corresponding power curve[6]. In addition to this, there is also a lot of research on how to distribute power wisely so that the rider can win.

Our team believe that there are many similarities between running and cycling, for example, they both test the ability to combine aerobic and anaerobic exercise, and they are both competitive sports, so we modified J. B. Keller's running model[2] appropriately and set more control parameters to reflect the various situations encountered in cycling, which is different from running.

Dajo Sanders et al. used maximal aerobic force, peak sprint force output, and an exponential constant describing the decay of force over time to establish force-duration relationships for each participant[7].

Consequently, a definition of Power Profile needs to be considered in our model, which is to describe the comprehensive ability of different kinds of riders. Use it to describe at least two types of riders and a Cycling Model also needs to be established to describe the relationship between the rider's position on the course and the power the rider applies.

On the definition of Power Profile, this paper plans to use the work of Hunter Allen, Andrew Coggan, PhD, and Stephen McGregor, PhD as a reference[8]. We should build relationships between the four parameters in their concept and the four parameters in our model to describe riders' abilities, thus define our Power Profile.

On the question of describing the relationship between the rider's position on the course and the power the rider applies, this paper establishes the Cycling Model to describe the ideal speed at every point of the course using the parameters we define in Power Profile. This model can provide a well-established cycling strategy to assist coaches to guide cyclists to have an excellent performance in the competition and break the best results.

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2. Analysis and Modelling

2.1 Power Profile in Cycling Model

According to Hunter Allen, Andrew Coggan, PhD, and Stephen McGregor, PhD's research on Training and Racing with a Power Meter, Power Profile tells us about athletes' abilities on cycling [8]. It contains four very important values telling us about: sprint abilities (5s), anaerobic capacity described by 1 minute maximum power, 5 minutes to tell us about VO_2 max capability and 20 minutes to describe the *FTP*. We abbreviate the above parameters as *SA, AC,* VO_2 and *FTP*.

Using the known data, we can easily plot the Figure 1 to describe the Power Profile of different athletes.

Figure 1: Power Profile of different athletes

However, in our model, it's not a best choice to consider the four values directly, so we try to build a connection between the four values and the four physiological parameters: the maximum force *F*, the rate of energy recovery ξ, the coefficient τ and the energy stored before the competition E_0.

After analyzing previous researches, we are able to build connections between three of the physiological parameters. The sprint ability of a rider corresponds to his explosive power to accelerate in a short time, which is related to the maximum impulse force *F*. It is known that the relationship is linear. Consequently, we did fitting in (1) and the *R-square* is 0.9457. Figure 2 shows the fitting result of *F* and *SA*.

$$F = 0.01957 \cdot SA + 0.8132 \tag{1}$$

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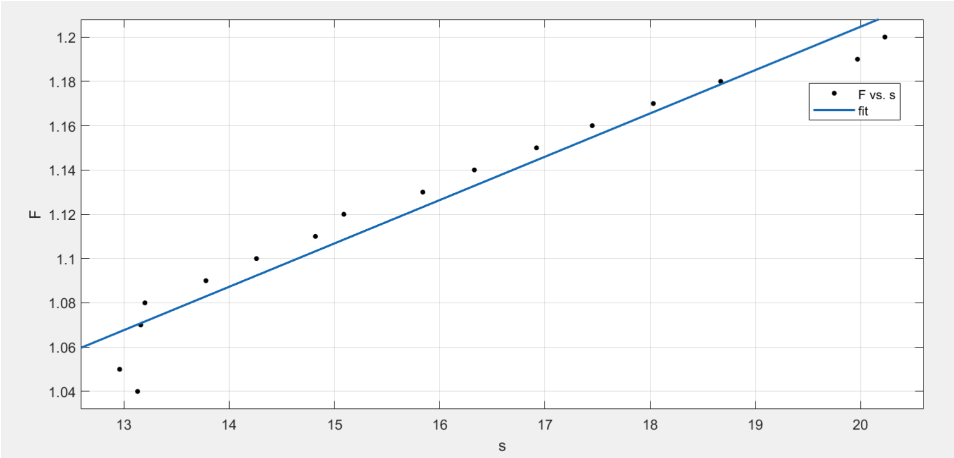

Figure 2: The fitting result of F and SA

In addition, we transform the value of *FTP* into the recovery coefficient ξ to describe the change in rider's energy. Also, VO_2 reflects the aerobic capacity of a rider, and we define τ to represent the internal and external resistance coefficient of a rider in riding. The relationships are indicated below.

$$\xi = 0.1467 \cdot FTP + 0.3006 \tag{2}$$

$$\tau = -4.929 \cdot VO_2 + 37.02 \tag{3}$$

Finally, we are able to establish our Power Profile. In our model, Power Profile is a combined-value to introduce the physical abilities of athletes, which depends on the individual only. Power Profile contains four values: the maximum impulse force F, the rate of energy recovery ξ, the coefficient τ and the energy stored before the competition E_0.

2.2 Cycling Model

After defining our Power Profile, we are ready to build a Cycling Model. Having analyzed the model given by J. B. Keller in 1973[2], we can divide the cycling course into three sections. In the primary section, riders try their best to accelerate. Then they complete the middle section at a relatively stable speed. Finally, before they finish the whole course, riders use up their energy and dash across the finish line with inertance.

2.2.1 Preliminary Modelling

According to the mathematical relationship between speed, time, and distance in physics, the distance of the course can be defined as follows.

$$D = \int_0^T v(t)dt \tag{4}$$

Then we need to analyze how to use the least time to cover the course. This question equals to how far the bicycle can travel within the given time. To simplify the question, we decide to establish a model for unit mass (m = 1). According to Newton's 2nd Law, we can describe the movement in the following model:

$$\begin{cases} \dfrac{dv}{dt} + \dfrac{v}{\tau} = f(t) \\ v(0) = 0 \end{cases} \tag{5}$$

where $f(t)$ represents the impulse force, $\frac{1}{\tau}$ represents the proportionality coefficient between velocity and internal, as well as external resistance when riding a bike. The faster a rider is, the smaller τ will be.

Here, the impulse force is limited by human abilities, which is related to the physiological structure, so the maximum impulse force of one person is fixed to a known figure, which can be calculated using Power Profile we have defined. We use F to represent it thus we have this restriction.

$$f(t) \leq F \tag{6}$$

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Then, we take the wind resistance into consideration, for we are going to discuss the influence of it in the following parts. As is known, wind resistance is proportional to the speed, so we define the proportionality coefficient as $\frac{1}{\lambda}$. The stronger the wind is, the bigger the resistance will be. We can get a new model below.

$$\begin{cases} \frac{dv}{dt} + \frac{v}{\tau} + \frac{v}{\lambda} = f(t) \\ v(0) = 0 \end{cases} \tag{7}$$

In our model, Power Profile, which is the maximum force F, the rate of energy recovery ξ, the coefficient τ and energy E_0, is referred to as the four physiological parameters for further study.

2.2.2 Upward Slopes and Sharp Turns

Considering the complexity of the course, we have to add parameters to describe nontrivial road grades and sharp turns.

To analyze the situation of riding uphill, we need to do force analysis. As for the purpose is to find a way to save as much time as possible, athletes are supposed to use impulse force as much as possible to accelerate. What is worth to mention is that the impulse force during the race is limited by both the road condition and the energy consumption of a rider.

We use f(t) to represent the impulse force at the time point t and F_n to represent the support force given by the ground. When riding uphill, bicycle athletes need to overcome gravity with impulse force. Comparing with riding on flat land, they may use the same force but move slower. Here, θ represents the angle between the slope and flat ground. It is related to the course itself only, and can be referred to as θ(s), where s represents the distance the rider has travelled. Different athlete can reach different distance in the same time, so s can be a function of time t. Thus the angle is finally marked as $\theta(s$(t)) which satisfies following equations.

$$s = \int_0^t v(t)dt \tag{8}$$

We suppose the angle is negative when the bicycle runs downhill and zero when on flat land, so the function θ exists through the whole course.

In the case of sharp turns, we use r(s) to represent the curvature radius on a 3-dimension map. Similar to the angle θ, the curvature radius r is only related to the course itself thus can be a function of the time t.

In the case when the bike is taking turns at maximum speed. f_{max} represents the maximum fiction of this turn, which can be determined by friction coefficient μ. If we suppose the curvature radius on straight road is infinity, the highest speed on every single point of the course is fixed. The course, as a curve, is relatively smooth, so the function r(s) is smooth. As a result, the speed, which can be calculated using f_{max}, m(m = 1), and r(s), is smooth. Hence, to cover the course within the shortest time, athletes are supposed to travel at the highest speed on every point of the course theoretically, which is the minimum speed limited by the turning and the energy consumption.

After taking the ramp angle and the curvature radius into consideration, we improve the model as follows.

$$\frac{dv}{dt} + \frac{v}{\tau} + \frac{v}{\lambda} = f(t) - Gsin(\theta(s))$$

$$f_{max} = \frac{v^2}{r(s)}$$

$$s(t) = \int_0^t v(t)\, dt \tag{9}$$

$$v(0) = 0$$

$$f(t) \leq F$$

2.2.3 Energy Consumption

Athletes need energy to finish the course. During the competition, the energy consumed can be

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indicated as $f \cdot v$. ξ represents the rate of energy recovery. The model is as follows:

$$\begin{cases} \dfrac{dE}{dt} = \xi - f(t) \cdot v \\ E(0) = E_0 \end{cases} \tag{10}$$

where E_0 represents the energy stored in the beginning.

Therefore, there are three factors which limits the speed in total.

- The terrain change throughout the whole course.
- The impulse force an athlete can use when cycling.
- The energy stored during the competition.

3. Optimization of Model Results

Got inspired by the running model, we separate the course into three sections divided by time point T_1 and T_2. In the primary section, athletes accelerate with their maximum impulse force. In the middle section, athletes change their impulse force according to the terrain changes. In the final section, athletes exhaust their energy and dash across the final line with inertance.

3.1 Primary Section

We define the section with t ($0 \leq t \leq T_1$), when the athlete uses the maximum impulse force to accelerate. In this section, the velocity of the athlete has not reached the maximum speed limited by the terrain. Hence, we have the model below.

$$\begin{cases} \dfrac{dv}{dt} + \dfrac{v}{\tau} + \dfrac{v}{\lambda} = F \\ v(0) = 0 \end{cases} \tag{11}$$

This is a Cauchy problem for first-order linear differential equations. We can solve it to get v(t).

$$v(t) = \frac{F\tau\lambda}{\tau + \lambda}(1 - e^{-(\frac{1}{\tau} + \frac{1}{\lambda})t}) \tag{12}$$

Then we use (12) to solve (10), and we have the equation below.

$$\begin{cases} \dfrac{dE}{dt} = \xi - \dfrac{F^2\tau\lambda}{\tau + \lambda}(1 - e^{-(\frac{1}{\tau} + \frac{1}{\lambda})t}) \\ E(0) = E_0 \end{cases} \tag{13}$$

Solve this differential function, and we get the function E(t).

$$E(t) = E_0 + (\xi - \frac{F^2\tau\lambda}{\tau + \lambda})t + (\frac{F\tau\lambda}{\tau + \lambda})^2(1 - e^{-(\frac{1}{\tau} + \frac{1}{\lambda})t}) \tag{14}$$

Here, we need to assure that $\xi - \frac{F^2\tau\lambda}{\tau+\lambda} \leq 0$, because otherwise,

$$\lim_{t \to +\infty} E(t) = \infty$$

According to the assumption that $E(0) = E_0$, We can use Figure 3 to indicate the energy change over time.

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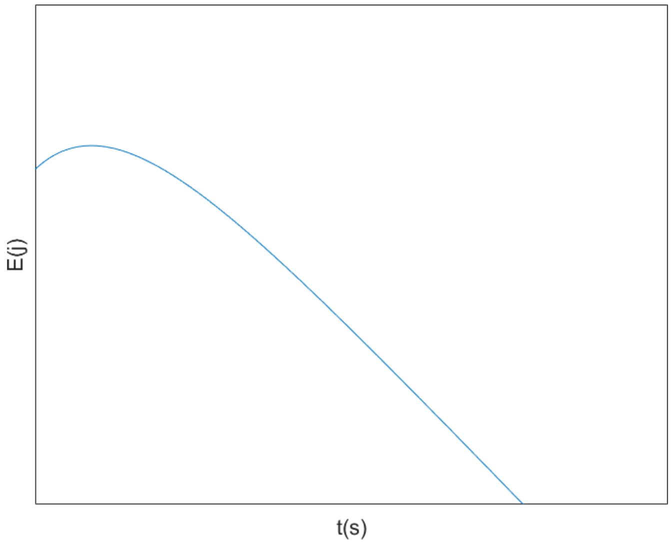

Figure 3: Energy change over time in the primary section

From Figure 3, we can indicate that the energy finely increases and then slowly decreases in the first section.

3.2 Final Section

In the final section, athletes have exhausted their energy and dash across the final line with inertance, that is to say E(t) = 0, ($T_2 \leq t \leq T_t$). For (10), we have the following functions.

$$\begin{cases} \frac{dE}{dt} = \xi - v(\frac{dv}{dt} + \frac{v}{\tau} + \frac{v}{\lambda} + Gsin(\theta(s))) \\ s(t) = \int_0^t v(t)\, dt \\ E(0) = E_0 \end{cases} \tag{15}$$

E(t) = 0, so $\frac{dE}{dt} = 0$. Consequently, we have (16).

$$\begin{cases} v(\frac{dv}{dt} + \frac{v}{\tau} + \frac{v}{\lambda} + Gsin(\theta(s)))T_2 = \xi T_2 \\ s(t) = \int_0^t v(t)\, dt \\ E(0) = E_0 \end{cases} \tag{16}$$

For θ(s) is a known function but s is related to t, we can use *MATLAB* to find the numerical solution, which can be the description of the relationship between velocity v and time t. Here, we leave θ(s) as a function to be determined. After finding the numerical solution, we can continuously find the relationship between velocity and time by fitting. In other words, we finally have this function.

$$v = v(t) \tag{17}$$

Furthermore, we use the same method to find the relationship between energy E and time t as follows.

$$E = E(t) \tag{18}$$

3.3 Middle Section

Finally, we are going to talk about the middle section in a competition. According to the results in the previous two sections and the equation (4), the total distance a rider can reach in particular time can be described with the equation below.

$$D(v(t)) = \int_0^{T_1} F(\frac{1}{\tau} + \frac{1}{\lambda})^{-1}(1 - e^{-(\frac{1}{\tau}+\frac{1}{\lambda})t})\, dt + \int_{T_1}^{T_2} v(t)\, dt + \int_{T_2}^{T_t} v(t)\, dt \tag{19}$$

Using (18), we find the first condition. Considering the centripetal force when making turns, we find the second condition. Consequently, the question is to figure out the maximum distance under the two conditions in (20).

$$\begin{cases} f_{max} = \dfrac{v^2}{r(s)} \\ E(T_2) = 0 \end{cases} \tag{20}$$

f_{max} represents the maximum friction according to the previous text, also influenced only by the course itself.

Then it can be seen as an extreme value problem which can use the *Lagrange multiplier method*. Using the calculated f(t) and v(t), we can easily obtain the power P(t) using (21) for the direction of f and v corresponds.

$$P(t) = f(t) \cdot v(t) \tag{21}$$

So far we have finished the model establishment.

4. Conclusions

In this paper, in order to better describe the comprehensive ability of different riders, we define the Power Profile and plot radar diagrams to describe sprint abilities, anaerobic capacity, VO_2 max capacity and *FTP* of different riders. Linear fitting is used to build relationships with the four physiological coefficients in our model. The fitting results give our definition of Power Profile.

Moreover, we have successfully built a Cycling Model to provide the relationship between the rider's position on the course and the power the rider applies. The Newton's 2nd Law helps to build the differential equations. We use Differential Geometry to describe the course.

As the sports industry grows, mathematical models are being used to help athletes break their personal bests. We hope that our research can be used to develop track-specific race guidelines for different types of cyclists of different genders to achieve the best theoretical performance.

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