Dynamic solution of high pressure tubing

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Abstract: *In the real environment, the pressure of high-pressure oil pipe is not always stable due to the process of oil in and out. We created a simulation experiment: to set the time for each opening of the one-way valve to change the pressure in the high-pressure tubing, ensuring that the pressure remains relatively stable under the initial pressure condition 100Mpa. By using the simulated conditions, we obtained the relationship between elastic modulus E and pressure P with high fitting degree, and deduced the relationship between pressure P and density temperature. Since the tubing size V is constant, we established the approximate dynamic balance model of fuel mass in high pressure tubing according to the mV mass. Since the opening time of the one-way valve is unknown, the simplified operation method is used to find the approximate solution and then the dichotomy method is used to find the results in this range. Our contribution is that we proposed a method to control the pressure steadily and rapidly in simulation. For each search result, the iterative method is used to obtain the mean pressure of the injection pipe after the pressure is stable, and the objective function is established to make the average pressure reach 150Mpa. Finally, we present the exact solution with MATLAB simulation.*

Keywords: *high pressure tubing, 100Mpa, MATLAB*

1. Introduction

Since the industrial revolution, the emergence of fuel engines has solved a large area of traffic and transportation problems, made human life have undergone earth-shaking changes, and greatly improved the process and efficiency of social development [1]. The key to the composition of the fuel engine-the fuel supply system composed of high-pressure fuel pipes is the heart of the fuel engine, and fuel entry and injection is the basis of many fuel engines. The entry and discharge of fuel will affect the change of pressure in the cavity of high-pressure tubing, resulting in errors in the amount of fuel ejected, which in turn affects the overall working efficiency of the engine.

2. Related works

Question a of the Chinese Undergraduate Mathematical Modeling Contest in 2019 is given on the basis of the high pressure common rail pipe of diesel engine. Through modeling and calculation, it is required to give the control scheme to keep the pressure in the high pressure oil pipe stable. Zhou et al [2] put forward some simulation experiments for the control scheme in different situation. Cai et al [3] gave the solution in controlling the intervals of check valve. In our works, we present a method for a nonlinear control scheme to achieve the goals and give 3D simulation in the relationships of time, intervals and pressure.

3. Model building

Because it is difficult to calculate the pressure directly, and it is noticed that the total size of the tubing is constant, it is considered that the problem should be transformed into the problem of maintaining the dynamic balance of the quality of the oil, that is, to make the difference between the fuel intake and the fuel injection close to zero. The fuel quality difference formula is listed as follows:

$$\Delta m = m_{enter} - m_{gush} \tag{1}$$

According to the traffic definition, the formula is transformed into a time-related function:

$$\Delta m = \int_0^T \rho_{in} Q_{\mathrm{in}} dt - \int_0^T \rho_{out} Q_{out} dt \tag{2}$$

Since the relationship between fuel intake and injection quantity and time is not directly given in the question, it is noted that the data between pressure and elastic modulus can be approximately fitted, which can be obtained by MATLAB quadratic fitting.

$$E=0.02893P^2+3.077P+1572 \tag{3}$$

Check R^2 and get:

$$R^2=0.9991 \tag{4}$$

It can be seen that this fitting degree is high. It is also known that the pressure change of fuel is proportional to the density change and the proportional coefficient is $\frac{E}{\rho}$, and the differential equation model is established as follows:

$$dp=\frac{E}{\rho}d\rho \tag{5}$$

Since quadratic fitting is known, E is a quadratic function of P as an independent variable, denoted as (P)E, and the integral of both sides of the differential equation is carried out:

$$\int_{P_0}^{P}\frac{1}{E(P)}dp=\int_{\rho_0}^{\rho}\frac{1}{\rho}d\rho \tag{6}$$

Where into, P_0 is the initial pressure of the high-pressure tubing 100MPa, ρ_0 is the fuel density 0.850mg/mm^3, and the opening time of each check valve is set as x. Based on the question, the relationship function between the check valve switching state and time within a period can be established as follows:

$$flag(t)=\begin{cases}0, t\in[0,x]\\ 1, t\in(x,10+x]\end{cases}, \text{ Where 1 is on and 0 is off} \tag{7}$$

Because the calculation of oil intake is more complicated, and because the flow formula in and out of high pressure tubing is known:

$$Q=CA\sqrt{\frac{2\Delta P}{\rho}} \tag{8}$$

Consider the use of high-pressure tubing flow formula using differential equations to express. According to the calculation, the fuel injection rate can be obtained as a piecewise function as follows:

$$v_{\text{out}}=\begin{cases}100\text{t}\left(0\le t<0.2\right)\\ 20\left(0.2\le t\le 2.2\right)\\ -100t+24\left(2.2<t\le 2.4\right)\end{cases} \tag{9}$$

Organize and establish a set of equations:

$$\begin{cases}\Delta m=\int_0^T\rho_{in}Q_{in}dt\text{-}\int_0^T\rho_{out}Q_{out}dt\\ \int_{P_0}^{P}\frac{1}{E(P)}dp=\int_{\rho_0}^{\rho}\frac{1}{\rho}d\rho\\ Q=CA\sqrt{\frac{2\Delta \text{P}}{\rho}}\end{cases} \tag{10}$$

The iterative model is established as follows:

$$P^{(k+1)} = P^{(k)} + \frac{0.0001\times P^{(k)3} - 0.001082\times P^{(k)2} + 5.474\times P^{(k)} + 1532}{\rho^{(k)}} \times \frac{m^{(k+1)} - m^{(k)}}{v} \tag{11}$$

Take the step length Δt = 0.001t ms, that is, because the pressure in the high-pressure tubing is required to increase from 100 MPa to 150MPa, the available range is [0.2,0.8], and the 3D figure is drawn by MATLAB as follows:

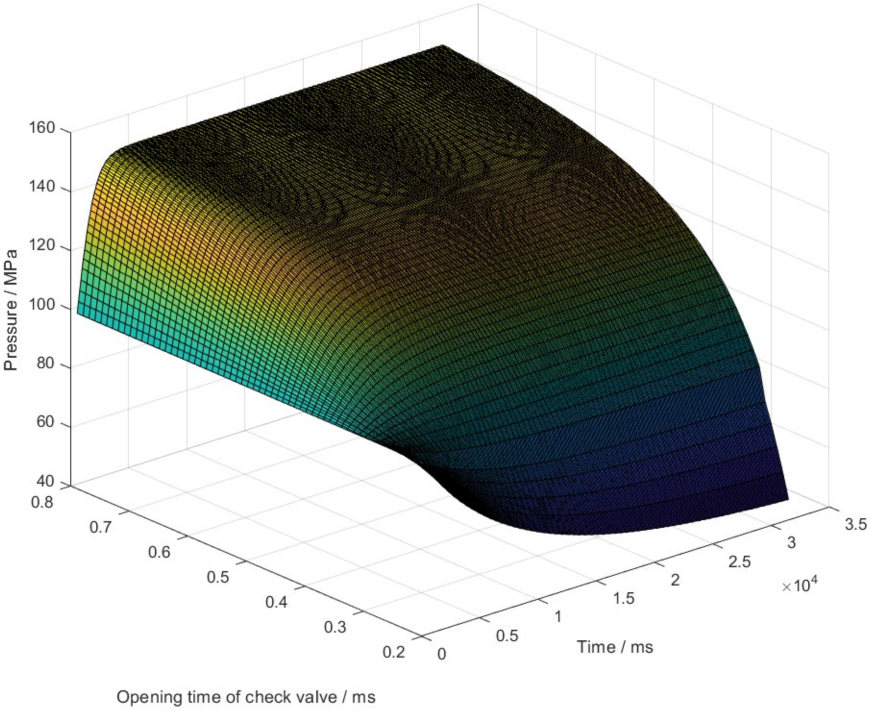

Figure 1: Three-dimensional schematic diagram

Worth mentioning is, considering the whole topic background (by reducing the pressure variation in the tubing to reduce the error of jet fuel quantity so as to improve the efficiency of engine working), notice that the pressure fluctuations periodic relationship, to make the fuel injection quantity more smoothly, more intuitive picture, select all the peaks and troughs of periodic fluctuation is worth midpoint for drawing. It can be solved by analyzing the diagram model.

4. Model solving

According to the equations of the model, the equations are solved step by step.

The relationship of fuel change has been obtained as follows:

$$\Delta m = \int_0^T \rho_{in} Q_{in} dt - \int_0^T \rho_{out} Q_{out} dt \tag{12}$$

Because the calculation is complicated, it is very difficult to obtain the analytical solution directly by mathematical method. Therefore, firstly, the approximate solution of the model is considered according to the time in which the one-way valve is opened and closed at one time.

Assumptions in and out of the high pressure oil pipe lumen inside and outside pressure constant, i.e. $P_{outer} = 160\ MPa$, $P_{inside} = 100 MPa$, ρ_{in} liquid density, ρ_{out} is constant, integral solution injection rate v Q_{out} can be made easily, according to the diagram, the one-way valve within an open closed within the length of the fuel injection quantity is the function of the fuel injection rate curve and time axis formed by the area, namely the $Q_{out} = 44mm^3$.

Flow calculation formula:

$$Q_{enter} = CA\sqrt{\frac{2\Delta P}{\rho}} \tag{13}$$

Where, $\Delta P = P_{外} - P_{内} = 60MPa$ is the pressure difference between the two sides of the small hole, $C = 0.85$ is the flow coefficient, and A is the area of the small hole (mm^2), which can be easily solved by putting it into the formula.

$$t \approx 2.4ms \tag{14}$$

That is, the total opening time of the one-way valve is about 2.4ms.

Assuming each check valve open time of XMS, because of the need to shut down after a one-way valve every open 10 ms (for an intermittent), open the valve a record for a one-way valve with intermittent duty cycle, and the fuel injector work ten times per second and injection time of 2.4 ms at work, namely the adjacent two work began at 100 ms, so remember this one work cycle for fuel injector, each working cycle contains one-way valve open and intermittent process repeatedly and injector of an injection process.

Because the data range of the opening time of the one-way valve is unknown, the approximate value x* of the opening time of the one-way valve is obtained by using the approximation method before calculating the opening time of the one-way valve.

In the approximate calculation, the total time of the one-way valve opening and intermission is $(x^* + 10)ms$. In one working cycle of the injector 100ms, there are about 10 working cycles of the one-way valve. Therefore, the approximate equation is considered to be established:

$$(10 + x)*10 \approx 100ms \tag{15}$$

$$x*10 \approx 2.4ms \tag{16}$$

Determine the range [0.2, 0.8], step size dt = 0.001s

In order to obtain the exact solution, consider using the iterative method to solve the differential equation here:

For a given x, take the fixed step length dt= 0.001ms. For the time t, set it as k iterations, then t+dt is 1+k iterations. Iterate each dt, and the state of each variable of the system at any time t can be deduced.

For the m iteration of the mass of the tubing, there are $dm = m^{(k+1)} - m^{(k)}$.

For the P iteration of the pressure of the tubing, there are $dP = P^{(k+1)} - P^{(k)}$.

The quadratic fitting results are known in (3). By bringing the results into the equation (1), the relationship between density and pressure can be obtained by establishing an elementary model.

The iterative formula can be established as follows:

$$m^{(k+1)} = m^{(k)} + y \times \pi \times 0.7^2 \times \sqrt{2 \times (160 - p^{(k)}) \times 0.8711} \times dt - QB \times (m^{(k)} / v) \times dt$$

$$p^{(k+1)} = p^{(k)} + ((0.0001 \times {p^{(k)}}^3 - 0.001082 \times {p^{(k)}}^2 + 5.474 \times p^{(k)} + 1532) / p^{(k)} \times ((m^{(k+1)} - m^{(k)}) / v)$$

Where y is the function established each time the one-way valve works.

Matlab was used for programming, and the time interval dt=0.001ms was used for gradual iteration. For a given x, the pressure of the high-pressure tubing was obtained after a certain time t. According to the figure, it could be found that the pressure of the high-pressure tubing fluctuated to some extent, so the objective function was established to optimize the final result.

Objective function: $\frac{\sum_{t=a}^{b} P(t,x)}{b-a+1} = 100$, Where $t \in [a,b]$ is the time period corresponding to the injection state after the pressure is stable:

Constraints: $x \in [0.2, 0.8]$

It can be seen intuitively from the three-dimensional diagram that the pressure finally tends to a steady value. Therefore, first of all, consider to find out the stationary value that is closest to the 150MPa required by the topic. Considering that the background of the whole problem is to reduce the error of fuel injection by reducing the change of pressure in the tubing so as to improve the engine efficiency, so the average pressure in the fluctuation descending section is taken, and the average pressure is compared with the 150MPa to be obtained in the question, the pressure value with the minimum error is obtained and the corresponding opening time of the one-way valve is obtained by inverse solution.

Using Matlab, we got the final answer x =0.754ms. That is, when the opening time of the check valve is 0.754ms, the pressure in the pipe is 150MPa.When x=0.754ms, the function graph of t and pressure P in the pipe is drawn as follows:

According to the figure above, it can be found that after t=4000 ms, the pressure in the tube gradually stabilizes at 150MPa.

If it is required to stabilize at 150 MPa: after 2 s adjustment process. According to the three-dimensional diagram, it can be directly observed that the size of x is positively correlated with the rate of convergence, that is, the larger x is, the faster the rate of convergence is. Therefore, consider building the image as follows:

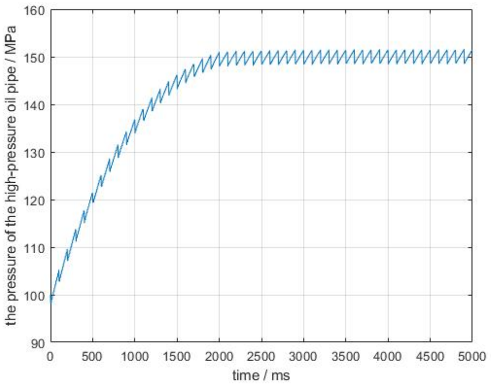

Figure 2: Simulation diagram

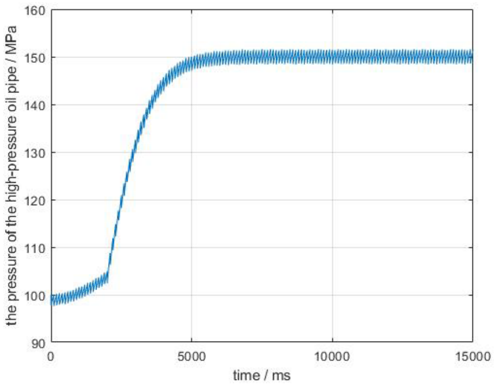

Figure 3: Simulation diagram-first-order function

5. Conclusion

In combination with the actual background, the high-pressure tubing system is an electronic control system, which can be intervened manually. In order to stabilize the system at 150 MPa after the adjustment process of 10 s, function X is established to make its independent variable t. When t=0, x=0.288; when t=0ms, x=0.288; when t=10000ms, x=0.754. Finally, the first function $x=4.66*10^{-5}t+0.288$ is determined. That is to say, when $x=4.66*10^{-5}t+0.288$, it is stable at 150 MPa after meeting the adjustment process of 10 s.

References

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