

Curved Structure Motion Portray Based on Separation Axis Theorem and Iterative Algorithm

Yuqing Xia^{1,*}, Zhihan Gong¹, Fangyu Wei¹

¹*School of Mathematics and Statistics, Northwestern Polytechnical University, Xi'an, China*

*Corresponding author: xiayுqing@mail.nwpu.edu.cn

Abstract: This paper investigates the motion of curved architectures based on the separation axis theorem and iterative algorithm, aiming to model the collision and motion of curved architectures during the execution of complex motions. First, the study utilizes the nature of isometric curves to obtain the position coordinates of the head of the curved architecture at each moment, and fits the motion trajectories of the knuckle body and the tail through an iterative algorithm. In addition, in order to understand the velocity of the knuckle body and the tail, this paper applies the kinematics principle to decompose the combined velocity into radial and tangential velocities, which is then solved. Then, in terms of collision detection, based on the established position and velocity models, the right-angle coordinate system method and the separated axis theorem (SAT) are used to calculate the specific coordinates where the collision occurs. It is further analyzed that the geometric relationship of the turnaround path and the establishment of the velocity model are the key to ensure the validity of the model. Finally, the study also explores the maximum critical velocity of the head of the curved architecture, which is solved to derive the maximum traveling velocity by a step-by-step traversal.

Keywords: Iterative Algorithms, Collision Detection, Separated Axis Theorem (SAT), Geometric Optimization

1. Introduction

In this paper, iterative algorithms [1], collision detection techniques [2], and the Separation Axis Theorem (SAT) [3] are comprehensively applied to study the motion characteristics of the curve architecture [4] in a dynamic environment in depth by establishing mathematical models and algorithms. Firstly, for the motion trajectory of the curve architecture, this paper establishes a model based on geometric optimization, and calculates its position and velocity during the clockwise disking in of the isometric curve. Second, through iterative simulation [5], the coil-in cutoff moment of the curved architecture and its corresponding motion state are analyzed in the case of no collision. In addition, this paper also explores the minimum curvature distance problem of the curved architecture during the turnaround process and designs a corresponding path optimization algorithm [6]. Finally, the algorithms and models studied in this paper not only provide theoretical support for the motion of curved architectures, but also provide a reference for the analysis of dynamic systems in related fields.

2. Curve architecture position and velocity analysis

The curved architecture is known to consist of 223 sections, and the section lengths of the head, the body and the tail of the curved architecture and the section widths of the sections are known. Two neighboring knots are connected by inserting handles through two holes in the knots. This section calculates the position and velocity of the entire curved structure per second from the initial moment until 300 s when the structure is coiled clockwise along an equidistant curve at a constant velocity.

2.1 Position modeling

$$\begin{cases} w = \frac{S}{T} \\ T = \frac{2\pi r}{v} \\ E = \frac{rw}{v} \end{cases} \tag{1}$$

Variable descriptions: w is the linear motion speed; S is the distance of the moving point along the curve after one week of rotation; T is the time needed to rotate the circumference by one week; E is the linear motion distance corresponding to each radian.

By constructing the model of moving distance, the specific position of the head of the curve architecture at 0 s, 60 s, 120 s, 180 s, 240 s, 300 s can be clarified respectively, and then the position of the known curve can be fitted by the section length, i.e., the intersection point of the handles after the section and the nearest curve can be determined when the position of the front handles is known and the position of each section can be determined by iteration afterward.

2.2 Velocity modeling

$$v_r = \frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} = b \cdot \omega \tag{2}$$

$$v_\theta = r(\theta) \cdot \omega = b \cdot \theta \cdot \omega \tag{3}$$

$$r(\theta) = b\theta \tag{4}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = b\omega\sqrt{1 + \theta^2} \tag{5}$$

Explanation of variables: $r(\theta)$ is the radius of the curve at angle θ ; b is the curvature; θ is the angle of a point on the curve along the screw center; ω is the angular velocity; v_r is the radial velocity; v_θ is the tangential velocity; v is the combined velocity.

The combined velocity is decomposed into radial velocity and tangential velocity, and combined with the polar equation of the curve to derive the expressions for the radial velocity, tangential velocity and combined velocity of the point moving along the curve.

$$\begin{cases} r_i = b\theta_i \\ r_{i+1} = b\theta_{i+1} \\ v_1 = 1m/s \\ v_j = v_{j+1} \\ v_\theta/v_r = \theta \end{cases} \begin{matrix} (i = 1, \dots, 223, i \in N^+; \\ j \text{ is even number } j = 1, \dots, 446) \end{matrix} \tag{6}$$

According to the same knuckle front and rear handles have the same partial velocity along the knuckle direction:

$$v'_{r,k} = v'_{r,k+1} \quad (k \text{ is odd, } k = 1, \dots, 447) \tag{7}$$

Explanation of variables: v_j is the combined velocity ($j = 1, \dots, 447; j \in N^+$) during the j decomposition; $v_{r,k}, v_{\theta,k}$ is the velocity along the radial and tangential directions for the k decomposition; $v'_{r,k}, v'_{\theta,k}$ is the velocity along the knuckle and vertical knuckle for the k decomposition.

In this section of the model, you can establish the connection between the velocity along the tangential, radial and along the nodes, vertical nodes, combined with the nature of the curve, and according to the synthesis and decomposition of the velocity of the relevant kinematics principles and mathematical knowledge of the equations, the establishment of a multidisciplinary cross-mathematical model, i.e., to establish the relationship between the velocity of the two adjacent handles, based on the known curve architecture head before the handle velocity and the model established in the velocity equations for step-by-step recursion, so as to carry out the solution to obtain the speed of the handle before and after each section of the body.

2.3 Model solving

By solving the above model, the positions and velocities of the front handle of the head of the curved architecture, the front handle of the 1st, 51st, 101st, 151st, 201st section behind the head of the curved architecture, and the rear handle of the tail of the curved architecture at a specific moment were selected, of which the velocity data are shown in table 1 below.

Table 1: Velocity results

| Case | 0s | 60s | 120s | 180s | 240s | 300s |
|-------------------|----------|----------|----------|----------|----------|----------|
| Head (m/s) | 1.000026 | 0.991628 | 0.975030 | 1.026143 | 0.983570 | 1.008528 |
| Section 1 (m/s) | 0.996732 | 0.987760 | 0.970343 | 1.020653 | 0.976037 | 0.997910 |
| Section 51 (m/s) | 0.907135 | 0.883962 | 0.846272 | 0.876682 | 0.785675 | 0.740717 |
| Section 101 (m/s) | 0.824430 | 0.789202 | 0.735013 | 0.748743 | 0.622517 | 0.528358 |
| Section 151 (m/s) | 0.747224 | 0.701494 | 0.633284 | 0.632416 | 0.477418 | 0.343286 |
| Section 201 (m/s) | 0.674536 | 0.619506 | 0.538903 | 0.525060 | 0.345426 | 0.177274 |
| Tail (m/s) | 0.643825 | 0.584931 | 0.499357 | 0.480162 | 0.290732 | 0.109251 |

3. Curve architecture disk-in termination moment analysis

3.1 Collision analysis

By analyzing the process of the head of the curve architecture, it is found that the collision situation between the knots is complicated. However, since the length of the section at the head of the curve architecture is longer than that of the section body and the tail of the curve architecture, and the head of the curve architecture is at the forefront of the entire “curve architecture” queue, it can be assumed that the head of the curve architecture must collide with other sections before the section body, i.e., the initial collision occurs between the head of the curve architecture and the subsequent section body.

By iterating the program one by one every second to simulate the situation of the head of the curve architecture, it is found that the first collision occurs between the head of the curve architecture and the body of the 7th section, thus the location of the collision is determined, and then the termination time of the curve architecture is found, and the position and speed of the curve architecture are calculated. The collision schematic is shown in Figure. 1.

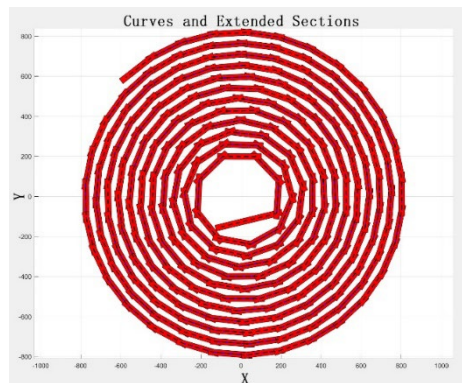


Figure 1: Schematic diagram of collision

3.2 Modeling: cartesian coordinate system methods

$t = t_0$ time, set the coordinates of the left front end and the right front section of the head of the curve architecture are $(a_1, b_1), (a_2, b_2)$; $t = t_0$ time, the equations of the straight line segments on both sides of the section colliding with the head of the curve architecture are respectively:

$$l_1 : A_1x + B_1y + C_1 = 0 \quad x \in [x_1, x_2] \tag{8}$$

$$l_2 : A_2x + B_2y + C_2 = 0 \quad x \in [x_3, x_4] \tag{9}$$

Using the point-to-straight-line distance formula:

$$d' = \frac{|Ax_0 + By_0 + Cz_0|}{\sqrt{A^2 + B^2}} \tag{10}$$

Calculate the distance d_1, d_2, d_3, d_4 from (a_i, b_i) to l_1, l_2 (and make sure that the vertical foot is on the straight line segment).

Let Δd be the width of the knuckle.

$$\Delta d = 300cm \tag{11}$$

If $d_1 < \Delta d$ and $d_2 < \Delta d$, or $d_3 < \Delta d$ and $d_4 < \Delta d$ then a collision has occurred.

With the program simulation data, the critical value of the collision can be derived, combined with the above collision analysis, that is, this collision is the first critical collision.

3.3 Modeling: separation Axis Theorem (SAT) Method

In the field of motion collision avoidance, the Separated Axis Theorem (SAT) is a common and effective theory, which is mainly used for the detection of whether a collision occurs or not. The separation axis theory determines whether a collision occurs by determining whether the projections of any two convex polygons at any angle both overlap. Abstracting the sections into a convex polygon, if two convex polygons do not intersect, then there must exist a line (axis) that separates these two polygons such that their projections on this line do not overlap. In this section, the planes of the sections can be viewed as convex polygons that satisfy the conditions for the application of the Separation Axis Theorem.

As in Figure. 2, if the projections of two quadrilaterals on both the x -axis and the y -axis overlap, a collision must occur; if the projections of two quadrilaterals on the x -axis and the y -axis have non-overlapping, no collision occurs.

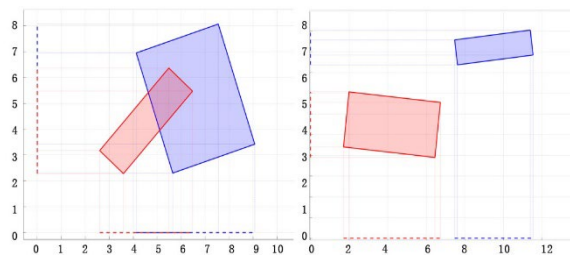


Figure 2: Collision visualization diagram

In this section, according to the Separation Axis Theorem, the projections of the head of the curve architecture as well as the body of the $i = 7$ section on the x -axis and y -axis, respectively, can be computed.

$$\begin{aligned} S_{ix} &= \Delta x_i \cos \theta_x \\ S_{iy} &= \Delta y_i \cos \theta_y \end{aligned} \tag{12}$$

According to the result of the program, if the projections of the head and the $i = 7$ node overlap on

the x, y -axis, then a collision occurs, and the termination moment of the coil-in of the curved architecture and the position and velocity of the curved architecture are derived from this solution.

3.4 Model solving

The result is the termination time: 399.32 s. The positions (in rectangular coordinates) and velocities of the front handle of the head, the front handles of the 1st, 51st, 101st, 151st, and 201st knuckle behind the head, and the rear handles of the rear of the curve are shown in table 2 below.

Table 2: Velocity solution results

| Case | Axis x (m) | Axis y (m) | Speed (m/s) |
|-------------------|------------|------------|-------------|
| Head (m/s) | -1.000628 | -1.524189 | 0.955900 |
| Section 1 (m/s) | 1.779196 | -0.851748 | 0.826976 |
| Section 51 (m/s) | 2.191996 | 3.702080 | 0.439422 |
| Section 101 (m/s) | 1.425236 | -5.566529 | 1.196668 |
| Section 151 (m/s) | 4.828354 | -4.918123 | 1.796560 |
| Section 201 (m/s) | -4.393069 | -6.532569 | 2.309180 |
| Tail (m/s) | -5.768968 | 5.921007 | 2.515371 |

4. Minimum curve pitch analysis

4.1 Modeling

For the head of the curve architecture to successfully turn around in a known given turnaround space, it is necessary to ensure that the head of the curve architecture successfully enters the turnaround space, i.e., no collision occurs before entering the turnaround space. In the solution process, the non-collision between the sections of the curve architecture during the traveling process can be taken as a known condition, and then solve for the minimum curvature distance of the equidistant curve. Then how to judge that no collision occurs between the sections becomes an urgent problem. Here, the separation axis theorem (SAT) can be introduced to determine whether the sections collide by judging whether the projections of the four edges of each section coincide, so as to solve for the minimum curvature distance.

4.1.1 Calculate the Normal Vector of One Edge of a Knuckle Edge

Let the two vertices of the edge be (x_1, y_1) and (x_2, y_2) , the vector representation of the edge be $(x_1 - x_2, y_1 - y_2)$, and the normal vector be $(y_2 - y_1, x_1 - x_2)$.

4.1.2 Calculate the Maximum and Minimum of the Projection on the Normal Vector for each edge of the section

$$\begin{cases} Temp1 = (y_2 - y_1)^2 + (x_1 - x_2)^2 \\ Temp2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 \\ Dot_x = Temp2 \cdot (y_1 - y_2) / Temp1 \\ Dot_y = Temp2 \cdot (x_1 - x_2) / Temp1 \\ Dot = Dot_x \cdot (y_1 - y_2) + Dot_y \cdot (x_1 - x_2) \end{cases} \quad (13)$$

Variable Description: Dot is the projection of either side on the normal vector; Dot_x is the projection of either side in the x direction on the normal vector; Dot_y is the projection of either side in the y direction on the normal vector.

Calculate the maximum and minimum values of each side Dot of the nodal quadrilateral. Compare the regions of values of the projections of the two nodal edges: if there is an intersection, the separation axis of the next edge is calculated; if there is no intersection, an axis is found such that the projections of the two nodal quadrilaterals on this axis do not have a region of intersection, i.e., there is no collision.

4.2 Model solving

A program is written using the Separation Axis Theorem in several iterations to find the critical value before a collision occurs, i.e., the smallest curvilinear distance for which there are no intersecting regions in the projections at the edges of the turnaround region. The programmed solution to find the minimum curvature distance that would allow the front handle of the head of the curved architecture to be coiled along the corresponding curve into the turnaround space is: 45.667934cm.

Figure. 3 below depicts the simulated image of the curved architecture successfully entering the turnaround space with the minimum curve pitch.

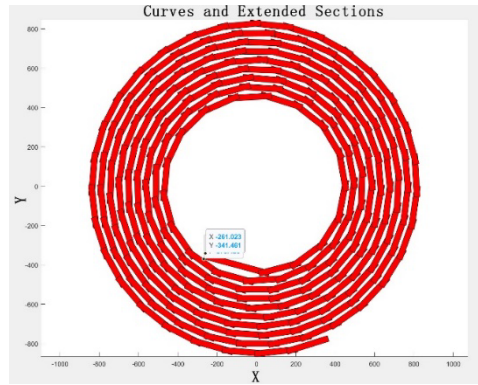


Figure 3: Simulated image of entry into the turnaround space at minimum curvature distance

5. Turnaround path analysis

5.1 Turning path

Set the endpoint of the disk-in curve $D(x_D, y_D)$, the endpoint of the disk-out curve $A(x_A, y_A)$, and the coordinates of the point of tangency of the two circles $O(x_O, y_O)$. Since the two arcs are tangent, the slopes are equal:

$$\left. \frac{dy_D}{dx} \right|_{x=x_0} = \left. \frac{dy_A}{dx} \right|_{x=x_0} \tag{14}$$

Let the equation of the tangent at the endpoint of the disk-in curve be m_1 and the equation of the tangent at the endpoint of the disk-out curve be m_2 :

$$\begin{cases} m_1 : A_3x + B_3y + C_3 = 0 \\ m_2 : A_4x + B_4y + C_4 = 0 \end{cases} \tag{15}$$

Because the outgoing and incoming curves are centrally symmetric about the center of the curve:

$$\begin{cases} A_3 = A_4 \\ B_3 = B_4 \\ C_3 = -C_4 \end{cases} \tag{16}$$

The turn-around path is tangent to both the disk-in and disk-out curves:

$$\begin{cases} -\frac{A_3}{B_3} = \left. \frac{dy_1}{dx} \right|_{x=x_D} \\ -\frac{A_4}{B_4} = \left. \frac{dy_2}{dx} \right|_{x=x_A} \end{cases} \tag{17}$$

And there is a known radius relation $R_1 = 2R_2$, we can draw the image of the turnaround path based

on the geometric relation, and the turnaround path is a fixed value, as shown in Figure. 4.

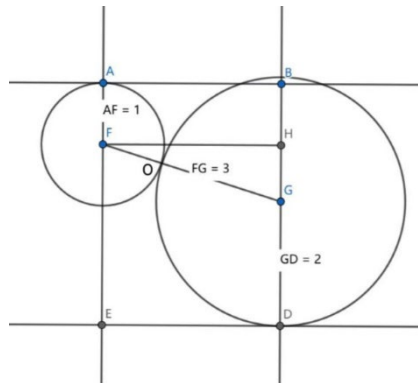


Figure 4: Geometric diagram of the turnaround path

Substituting into the formula for calculation, we get the radius of the latter arc $R_{AF} = 1.5027104m$, and the radius of the former arc $R_{GD} = 3.0054208m$.

5.2 Position modeling

Let the chord length between the front and rear handles of the i knuckle on the isometric curve be m_i , and the position of the front handle of the knuckle is $K(r_i, \theta_i)$ and the position of the rear handle is $L(r_{i+1}, \theta_{i+1})$. By the cosine theorem:

$$m_i^2 = r_i^2 + r_{i+1}^2 - 2r_i r_{i+1} \cos(\theta_i - \theta_{i+1}) \tag{18}$$

Using the chord length formula for each knuckle position iteration by iteration, it follows that the knuckle lengths are equal:

$$\begin{aligned} & r_i^2 + r_{i+1}^2 - 2r_i r_{i+1} \cos(\theta_i - \theta_{i+1}) \\ &= r_{i+1}^2 + r_{i+2}^2 - 2r_{i+1} r_{i+2} \cos(\theta_{i+1} - \theta_{i+2}) = \Delta d \\ & (i = 1, \dots, 221; i \in N^+) \end{aligned} \tag{19}$$

At the same time, because the disk out curve and the disk into the curve about the center of the curve is centrally symmetric, set the disk into the curve on a point for $M(\beta_m, \alpha_m)$, at this time the disk out of the curve with its corresponding point is $N(\beta_n, \alpha_n)$, then there are:

$$\begin{cases} \alpha_m - \alpha_n = \pi + 2k\pi & (k = 0, \pm 1, \dots) \\ \beta_m = -\beta_n \end{cases} \tag{20}$$

According to the known geometrical relationship, we can draw a curve architecture in the process of disk in, turnaround and disk out, the curve architecture of the head of the front handle of the travel path schematic diagram in Figure. 5, in which the outer circle of blue isometric curve on behalf of the disk in the route, the outer circle of red isometric curves on behalf of the disk out of the route, the red point and the blue point on behalf of the head of the path of the center of the large arc and the arc of the small circle, respectively.

After finding out the equation of the traveling route of the head front handle of the curve architecture of the “curve architecture”, we can deduce the position coordinates of each node and the tail of the curve architecture at each moment as follows: at a given moment, according to the position coordinates of the head front handle of the curve architecture, each node can be regarded as a chord of equal lengths of the isometric curve, and then according to the position coordinates of the head front handle of the curve architecture, each node can be regarded as a chord of equal lengths of the isometric curve. At a given moment, according to the position coordinates of the handle in front of the head of the curve, each segment can be regarded as a string of equal length on the equidistant curve, and then according to the formula for the length of the string on the equidistant curve, the position coordinates of the front and rear

handles of each segment as well as the rear part of the curve can be found out based on the position coordinates of the handle in front of the head of the curve structure, and the positions of the head of the curve structure, the rear part of the curve structure and the position of each segment of the structure can be solved at each moment of the traveling process.

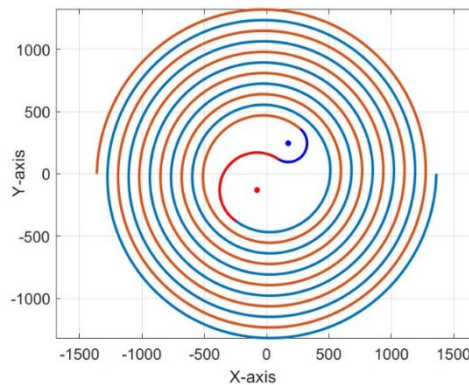


Figure 5: Curve structure of the head before the handle traveling route diagram

Figure. 6 below shows an example of an image of each section of the curved architecture at the 100th second:

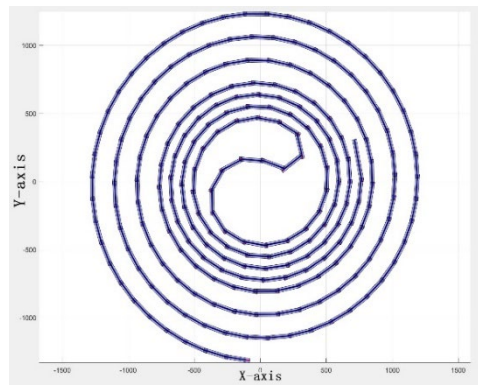


Figure 6: Image of each section of the curve architecture

5.3 Velocity modeling

When Δt is extremely small, the value of $\frac{\Delta s}{\Delta t}$ can be approximated as the velocity at that point, from which the position coordinate model can be used to calculate the velocities of the head of the curved architecture, the node bodies, and the tail of the curved architecture at a given moment.

$$v = \frac{ds}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} \tag{21}$$

Variable description: s is the polar path at a given moment in time.

5.4 Model solving

After establishing the model about the velocity, the above model combined with known data was solved using programming methods to obtain the positions of the front handles of the head of the curve architecture, the front handles of the 1st, 51st, 101st, 151st, 201st nodes of the body behind the head of the curve architecture and the rear handles of the tail of the curve architecture at the time of -100 s, -50 s, 0 s, 50 s, 100 s. architecture tail rear handle's position and velocity. Where the velocity results are shown in table 3 below.

Table 3: Velocity solution

| Case | -100s | -50s | 0s | 50s | 100s |
|-------------------|----------|----------|----------|----------|----------|
| Head (m/s) | 0.999953 | 0.999973 | 0.999980 | 0.999969 | 0.999979 |
| Section 1 (m/s) | 0.984095 | 0.974338 | 0.969521 | 1.002656 | 1.009475 |
| Section 51 (m/s) | 0.984551 | 1.031125 | 0.762933 | 0.961813 | 0.988460 |
| Section 101 (m/s) | 1.075293 | 0.980081 | 0.773674 | 0.934599 | 1.121085 |
| Section 151 (m/s) | 0.867394 | 0.751295 | 0.737654 | 0.945578 | 1.308064 |
| Section 201 (m/s) | 0.983502 | 0.558568 | 0.812215 | 1.036099 | 1.232246 |
| Tail (m/s) | 1.030471 | 0.582150 | 0.766311 | 1.093138 | 1.133243 |

6. Maximum travel speed analysis

6.1 Modeling

In order to solve the maximum traveling speed of the head of the curve architecture, this section adopts a step-by-step traversal method of the speed as follows: firstly, according to the known conditions, select the appropriate speed interval to be traversed one by one. The first traversal of the interval is.

$$[v_i, v_{i+1}] \quad (i = 1, 2, \dots; i \in N^+) \tag{22}$$

Perform n -equivalent divisions of the velocity interval with the step size of.

$$h = \frac{v_{i+1} - v_i}{n} \tag{23}$$

Take respectively:

$$v = v_i + kh \quad (k = 0, 1, \dots, n; k \in N^+) \tag{24}$$

Judge whether there is a collision between the sections one by one. If no collision occurs at $v = v_i + k_0h$ and a collision occurs at $v = v_i + (k_0 + 1)h$, then $(v_i + k_0h)$ is the maximum travel speed.

In judging whether collision occurs between knots, the following method is used: according to the principle of kinematics and the knowledge of calculus, it can be obtained:

$$v = \frac{ds}{dt} \tag{25}$$

Then by integrating the velocity in time, the magnitude of the change in displacement can be obtained. The magnitude of the change in displacement of the section P body from time j to time $j + \Delta t$ is:

$$s_{p,j} = \int_j^{j+\Delta t} v dt \quad (p = 1, \dots, 224; k \in N^+) \tag{26}$$

From this, we can iterate step by step to get the position of each section of the body, and finally judge the collision situation, and then solve the maximum traveling speed of the head of the curved architecture.

6.2 Model solving

By traversing the velocity interval one by one, the maximum traveling speed of the head of the fixed-curve architecture is finally programmed and calculated to be 1.096400 m/s.

7. Conclusions

In this paper, a motion model of curved architecture based on the separation axis theorem and iterative algorithm is proposed to systematically analyze the positions, velocities and collisions of the curved architecture in the dynamic environment. Firstly, by iteratively calculating the positions of the head, knuckle body and tail of the curved architecture, combined with the chord length and kinematics, the accurate description of the velocities of each part is successfully realized. Second, in the collision analysis,

this paper utilizes the separation axis theorem to effectively determine the collision state of the curved architecture and the knuckle body, ensuring that potential collision risks are avoided in the dynamic adjustment. The research results show that the maximum traveling speed of the head of the curved architecture is 1.096400 m/s under specific conditions, ensuring that the speed of each handle is within the safe range. This study not only provides a quantitative analysis method for the motion process of complex curved structures, but also lays the foundation for collision prediction and path planning in practical applications.

References

- [1] Yan Zhang, Qinglong He. Modified Landweber iterative algorithm based on momentum method[J/OL]. *Journal of Chongqing Technology and Business University (Natural Science Edition)*, 1-11[2024-11-20]. <http://kns.cnki.net/kcms/detail/50.1155.n.20241113.0932.004.html>.
- [2] WU Hao, WANG Qifeng, WANG Qi, et al. Collision detection method of robotic arm based on friction disturbance compensation[J]. *Mechanical Design*, 2024, 41(09):101-110. DOI: 10.13841/j.cnki.jxsj.2024.09.023.
- [3] Liu YC. Design of collision avoidance system for stockyard based on separation axis theorem[J]. *Science and Technology Communication*, 2015, 7(19):131-132. DOI:10.16607/j.cnki.1674-6708.2015.19.082.
- [4] F. Wang, T.N. Zhang, J. Zhu. Architecture design of plant and station side planned value curve receiving system[J]. *Electrotechnology*, 2022, (22):220-222. DOI: 10.19768/j.cnki.dgjs.2022.22.068.
- [5] J. H. Li, C. C. Li, L. Jiang, et al. Non-iterative simulation algorithm for non-Gaussian pulsating wind pressure[J]. *Vibration and Shock*, 2017, 36(23):216-222. DOI:10.13465/j.cnki.jvs.2017.23.032.
- [6] Pan Z, Zhu YH, Zhang CH, et al. Research on A*-based path optimization algorithm for mobile robots[J]. *Information Technology and Informatization*, 2024, (07):190-193.