

A Study on the Relationship between the Sex Ratio of Seven Leaf Trees and Ecosystems

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Abstract: The sex ratio of lampreys changes with the environment will regulate its population, and the paper will explore the relationship between the sex ratio of lampreys and the ecosystem. First, in order to get the effect of different sex ratios of lampreys on the ecosystem, a food chain model was established, combining the Lotka-Volterra model as well as the improved logistic model to simulate the changes in the number of individual populations. The fourth-order Lungkuta method was used to solve the model to obtain the population changes of lampreys as well as its natural enemies and food sources, reflecting the effects of sex ratio changes on the whole ecosystem. Second, the paper study the advantages and disadvantages of lampreys population size. Through genetic inheritance modeling, we analyze the effects of genetics on lampreys sex and species numbers. Changes in sex ratio can affect the adaptive capacity of lampreys and the stability of the ecosystem. Third, in order to explore the influence of lampreys sex ratio on ecosystem stability, we established an ecosystem stability evaluation model by adding three indicators such as Shannon Wiener index, constructed a food chain model, and obtained the number of various populations as well as the degree of ecosystem stability by substituting into the quantitative model and stability indicators, respectively. The simulated annealing algorithm was used to find out the sex ratio when the ecosystem stability degree was maximized.

Keywords: Sex Ratios, Ecosystems, Logistic Modeling, Longo Kuta Method

1. Introduction

The sex ratio of lampreys is influenced by environment and density. Prior to control of invasive sea lampreys, adult populations in the Great Lakes region were dominated by males (~65%). After control measures, the population was reduced by 90%, resulting in a more female-dominated adult population (~40%), with no significant difference in sex ratio vulnerability to control measures. Currently, the sea lampreys population is still being held at 10% of its historic high, and the sex ratio of adult sea lampreys eels in the Great Lakes region is estimated to be 55%^{[1][2]}. Sex ratios in creek lampreys populations tend to be more male-oriented in high-density populations, and sex-specificity in mortality rates is not thought to be responsible for changes in sex ratios, the paper simulate the change of lampreys population size under different sex ratios by building a food chain model and a modified logistic model^[3]. The fourth-order Lungkuta method was used to solve the differential equations, and finally the relationship between sex ratio and ecosystem was obtained, the paper analyzed the effect of sex-related genes on the sex ratio and population size of lampreys through the genetic model, and with the combination of get the ecosystem model based on the genetic model.

2. Research on Improved Logistics Modeling

2.1 Lotka-Volterra equations

The variation of the population size $U(t)$ of the prey and the population size $V(t)$ of the predator with time t is given by the following Lotka-Volterra equation.

$$\begin{cases} \frac{dU}{dt} = \alpha U - \lambda UV \\ \frac{dV}{dt} = -\beta V + e\lambda UV \end{cases} \tag{1}$$

Here α is seen as the natural growth rate of the prey and β as their natural mortality rate. λ can be seen as the proportion of prey caught by the predator per unit of time, so the total number of prey caught in interval dt is $\lambda dt UV$. Generally the predator always needs to catch a larger number of prey in order to have a chance to raise a new individual, so here there is a conversion relationship, and the factor that appears in the equation is such a conversion factor^[4].

2.2 Analysis of the fourth-order Lungkuta algorithm and comprehensive modeling

To describe the initial value problem of the first-order differential equation as in (2), this paper uses the fourth-order Runge-Kutta method with high accuracy and a truncation error of $O(h^5)$. The recursive formula for solving the fourth-order Runge-Kutta method is (3).

$$\begin{cases} \frac{dy}{dt} = f(t, y) \\ y(t_0) = y_0 \end{cases} \tag{2}$$

where y_0 is the initial state, t_0 is a known time, and $f(t, y)$ is a known function with respect to t, y .

$$\begin{cases} t_{n+1} = t_n + h \\ k_1 = f(y_n, t_n) \\ k_2 = f(y_n + \frac{h}{2}k_1, t_n + \frac{h}{2}) \\ k_3 = f(y_n + \frac{h}{2}k_2, t_n + \frac{h}{2}) \\ k_4 = f(y_n + hk_3, t_n + h) \\ y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \end{cases} \tag{3}$$

Any value $y(n)$ is determined by the sum of the current value $y(n-1)$ and the amount of change per unit time. The unit time change is the product of the time interval h and the approximate slope. For any time, the estimated slope is a weighted average of the following slopes. k_1 : Slope of time starting point; k_2 : Time midpoint slope, The slope k_1 is used to approximate the value of y at point $t_n + \frac{h}{2}$ according to Euler's method; k_3 : Slope at midpoint of time, using slope k_2 to approximate value y ; k_4 Time endpoint slopes, using slope k_3 to approximate y values.

(1) Determination of differential equations

$$\begin{aligned}\frac{\partial M}{\partial t} &= r_M \left(1 - \frac{M}{M_m}\right)M \\ \frac{\partial F}{\partial t} &= r_F \left(1 - \frac{F}{F_m}\right)F\end{aligned}\tag{4}$$

M and F are the number of male and female lampreys, respectively, t is the elapsed time, r_M and r_F are the natural growth rate of male lampreys, respectively, and M_m and F_m are the maximum environmental carrying capacity for male and female lampreys, respectively.

(2) Changes in the number of lampreys due to predation by the previous trophic level (hereinafter referred to as predator)

$$\begin{aligned}\frac{\partial M}{\partial t} &= -\lambda_M Q M \\ \frac{\partial F}{\partial t} &= -\lambda_F Q F\end{aligned}\tag{5}$$

λ_M and λ_F denote the proportion of male and female lampreys captured by predators per unit time, respectively, and Q denotes the number of predators.

(3) Changes in the number of lampreys due to the capture of prey by lampreys

$$\begin{aligned}\frac{\partial M}{\partial t} &= b_M e P M \\ \frac{\partial F}{\partial t} &= b_F e P F\end{aligned}\tag{6}$$

b_M and b_F denote the proportion of male and female lampreys successfully capturing prey per unit time, respectively, and P denotes the number of prey. e denotes the conversion relationship between the number of prey captured and the number of newborn lampreys.

(4) Changes in predator and prey numbers over time due to natural population growth

$$\begin{aligned}\frac{\partial Q}{\partial t} &= r_Q \left(1 - \frac{Q}{Q_m}\right)Q \\ \frac{\partial P}{\partial t} &= r_P \left(1 - \frac{P}{P_m}\right)P\end{aligned}\tag{7}$$

(5) Changes in predator population size over time due to predation on lampreys by predator

$$\frac{\partial Q}{\partial t} = b_Q e Q (M + F)\tag{8}$$

(6) Changes over time in prey numbers due to predation by lampreys

$$\frac{\partial P}{\partial t} = -\lambda_P P (M + F)\tag{9}$$

(7) According to the question, when food is more abundant, the proportion of males is larger; when food is more scarce, the proportion of males is relatively smaller. Define food abundance f , food abundance and male share x become negatively correlated, assuming that food abundance and male share

are linearly negatively correlated

$$f = -ax + d \tag{10}$$

Both a and d are constants. In order to establish a food abundance assessment system [2], ranging from 0 to 1, the closer the value is to 1, the richer the food is. When the food is abundant, the most suitable food abundance within the environmental carrying range is 0.75, and in the case of food shortage, it is 0.25. 78% have a lower food supply. When $f=0.75$, $x=0.56$. When $f=0.25$, $x=0.78$. (8) can be solved as:

$$f = -\frac{25}{11}x + \frac{2225}{1100} \tag{11}$$

(9) The relationship between the rate of natural increase $r_i (i = M, F, Q, P)$ and the birth rate (or sex selection rate for lampreys) C_i and the death rate S_i is:

$$r_i = C_i - S_i \tag{12}$$

(10) Since the mortality rate is independent of the sex ratio for the sex selection of lampreys, the mortality rate of lampreys is constant, and the sex selection rate $C_i (i = M, F)$ of lampreys is related to the sex ratio x

$$\begin{aligned} C_M &= (z - s) * 2 * x \\ C_F &= (z - s) * 2 * (1 - x) \end{aligned} \tag{13}$$

(11) Z is the average birth rate of lampreys (regardless of sex) and S is the mortality rate of lampreys. The relationship between food abundance and the natural growth rate of lampreys can be obtained by connecting (10), (11), and (12) (in males).

$$r_M = (z - s_M) * 2 * \frac{(-f + \frac{2225}{1100}) * 11}{25} - s_M \tag{14}$$

This model combines both the Logistic model and the predator-prey relationship with the Lotka-Volterra equation, and then improves on the natural growth rate in the Logistic model. The first big question model is organized as:

$$\left\{ \begin{aligned} \frac{dM}{dt} &= r_M \left(1 - \frac{M}{M_m}\right) M - \lambda_M QM + b_M ePM \\ \frac{dF}{dt} &= r_F \left(1 - \frac{F}{F_m}\right) F - \lambda_F QF + b_F ePF \\ \frac{dQ}{dt} &= r_Q \left(1 - \frac{Q}{Q_m}\right) Q + b_Q eQ(M + F) \\ \frac{dP}{dt} &= r_P \left(1 - \frac{P}{P_m}\right) P - \lambda_P P(M + F) \\ r_M &= (z - s_M) * 2 * \frac{(-f + \frac{2225}{1100}) * 11}{25} - s_M \\ r_F &= (z - s_F) * 2 * \left(1 - \frac{(-f + \frac{2225}{1100}) * 11}{25}\right) - s_F \\ f &= -\frac{25}{11}x + \frac{2225}{1100} \end{aligned} \right. \tag{15}$$

Application of algorithms and model solving

The paper modeled changes in the abundance of predator, female lampreys, male lampreys, and prey

of lampreys over a 10-year period under female lampreys and male lampreys sex ratios of 1:1, 1:4, 1:8, and 1:10, as shown in Figs 1, 2, 3, and 4, respectively.

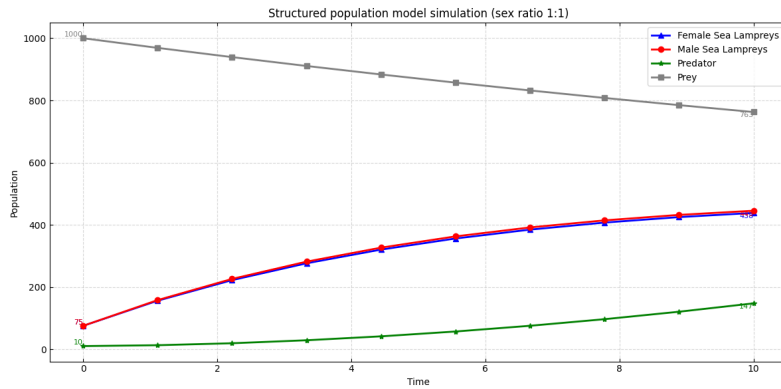


Figure 1: Changes in respective numbers at a sex ratio of 1:1

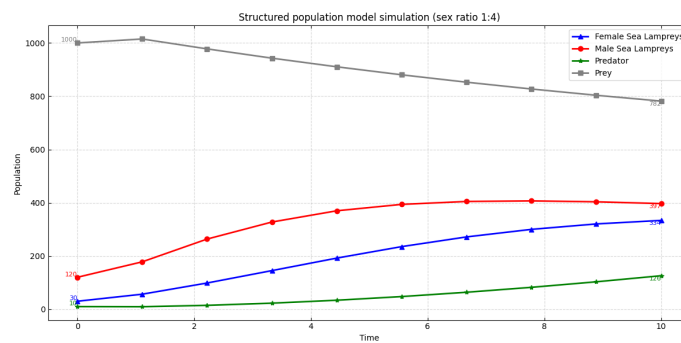


Figure 2: Changes in respective numbers at a sex ratio of 1:4

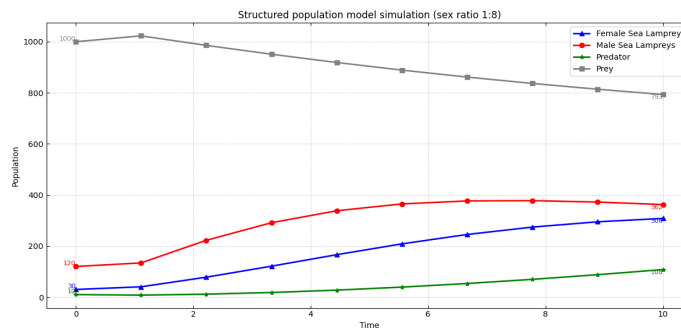


Figure 3: Changes in respective numbers at a sex ratio of 1:8

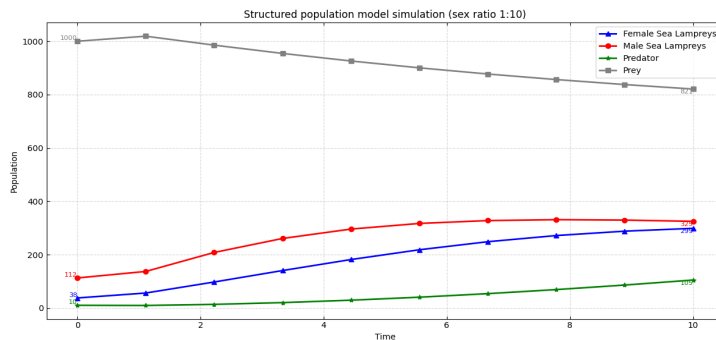


Figure 4: Changes in respective numbers at a sex ratio of 1:10

In our simulation of the structured population model, it was set according to Lindemann's theorem that the number of each tier is 1/10 of the number of the previous tier. based on the data obtained, it was observed that during this 10-year evolution, when the sex ratio (female lampreys: male lampreys) went

from 1:1 to 1:10, the data were as follows Table.1:

Table 1: Changes in Population Numbers

Female: Male Number (pcs)	Males lampreys	Male lampreys	Predator	Preys
1:1	445	438	147	763
1:4	397	334	126	782
1:8	362	308	108	793
1:10	325	299	105	821

Overall, the growth in the number of lampreys has been suppressed. This means that the demand for food by lampreys slowed down during the change in sex ratio, resulting in a suppressed trend of decreasing food numbers. Specifically, the reduction in the number of prey taken by lampreys per unit of time and the reduction in the number of lampreys as a food source for predators also affected the trend in the growth of their predator population. This finding emphasizes the impact of sex ratio changes on the entire ecosystem with respect to lampreys numbers, food chain stability, and predator-predator interactions.

2.3 Strengths and weaknesses of lampreys populations

Through scientific research^[5], it can be found that of the 186,945 genes in lampreys, 638 of them are genetically related, known as "germline-specific genes GSG", and 409 of them are related to sex, and it is also pointed out in the literature that these 409 genes are 36 times more likely to be expressed in the testes than in the ovaries, and therefore it can be found that the sex of newborns is most likely inclined to males, and the genetic inheritance model is as follows:

$$\left\{ \begin{array}{l} P_M = \frac{G_{gsum} * p}{G_{sum} * (1+p)} + \frac{G_{sum} - G_{gsum}}{2 * G_{sum}} \\ P_F = \frac{G_{gsum} * 1}{G_{sum} * (1+P)} + \frac{G_{sum} - G_{gsum}}{2 * G_{sum}} \\ O_n = \frac{O_M}{O_F} \\ N_M = \frac{N_F * P_M}{O_n * P_F} \\ N_n = \frac{N_F + P_M}{O_M + N_M} \\ O_n = N_n \end{array} \right. \quad (16)$$

P_M denotes the probability of total male genes, P_F denotes the probability of total female genes, G_{gsum} denotes the number of sex-linked genes, G_{sum} denotes the number of genes, p denotes the multiplicity of the odds that the genes are expressed in the testes and the ovaries, O_n represents the multiplicity of the current number of males with respect to the number of females, O_M denotes the number of original males, O_F denotes the number of original females, N_M denotes the number of new males, N_F denotes the number of new females, and N_n denotes the sex ratio of the female-to-male lampreys obtained from the update^{[6][7]}.

Based on the model developed above, we can get the following relationship, when the sex ratio is 1:1, the newborn individuals are male-biased, but due to the constraints of O_n in relationship, it shows that when the sex ratio changes from 1:1 to 1:n ($n > 1$), the rate of growth of the number of newborn males is suppressed, and this negative feedback suggests that the sex ratio will not always remain 1:1, and at the same time, the value of n will not always increase linearly. The results of the model implementation are shown in Fig 5.

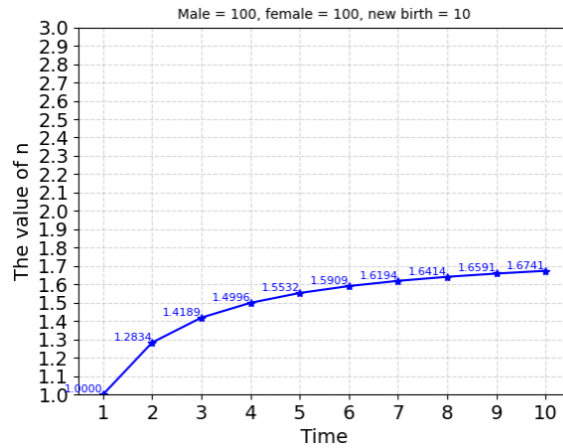


Figure 5: Map of changes in the number of species

Advantage: Increased adaptive capacity, the adjustment of sex ratio helps the seven gills to adapt better in changing environments^[8]. For example, in harsh environments or when resources are scarce, changes in sex ratio can increase reproductive competition between individuals and improve the probability of offspring survival.

Disadvantage: Reproduction is limited, and changes in gender ratio may limit the reproduction of the seven gilled fish population. If there is a significant deviation in gender ratio, some individuals may find it difficult to find suitable mates for reproduction, which can affect the increase in the number of offspring. In addition, changes in gender ratio may reduce genetic diversity, and having too many genders may lead to a narrow gene pool, increasing the risk of inbreeding and genetic defects^[9].

In the base model we discussed earlier, we considered the effect of the lampreys gene on population size and sex ratio^[10]. Now we want to go a step further and analyse how the sex ratio will change dynamically over the next 10 years under different initial sex ratio conditions. Previously we have examined the situation when the sex ratio is 1:1, and now we have modelled the respective changes in population size at 1:4, as shown in Fig 6.

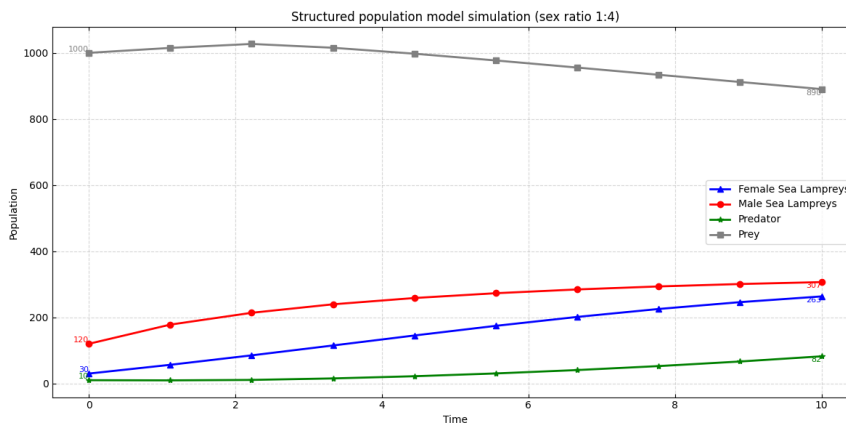


Figure 6: Changes in respective numbers over 10 years

Table 2: Population Numbers with Genetic Factors

Consideration of genes Number (pcs)	Males lampreys	Male lampreys	Predator	Preys
no	397	334	126	782
yes	307	263	82	890

According to Table 2, it can be seen that, considering genetic factors within the population, an increase in n with a sex ratio of 1: n leads to a decrease in the total population of seven leaf trees. This reduces the number of seven leaf trees that feed on prey and are preyed upon as predators, resulting in an increase in the number of remaining prey and a decrease in the number of predators after 10 years. The change in the number of seven leaf trees has both beneficial and adverse effects on the ecosystem.

Excessive consumption may reduce food sources and affect the stability of the food chain and biodiversity; Too little may increase food sources, once again affecting the stability of the food chain and biodiversity.

3. Conclusions

This article constructs a dynamic model of the change in the number of seven leaf trees, taking into account the predation and competition of species in the food chain. The relationship between the number of changes is obtained, and indicators related to ecosystem stability are established. The impact of changes in the number of each species on the ecosystem is analyzed, and model innovation is carried out. Finally, sensitivity analysis is conducted to analyze the impact of the sex ratio of seven leaf trees on the ecosystem under certain external environmental disturbances, combining the Lotka-Volterra model and establishing the improved logistic model is not only innovative, but also fully considered the finiteness of resources and other factors leading to changes in population size, making the model universal and strongly applicable. The fourth-order Runge-Kutta method is used to solve the differential equations, which improves the accuracy and stability compared with the Eulerian method. This helps to reflect more accurately the effect of lampreys sex ratio on the ecosystem and avoids oscillations or instability in the numerical solution.

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