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Research on Landing Control Scheme of Mars Probe Based on Differential Equation

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Abstract: *In this paper, a mathematical model is established to study the movement of the detector under different conditions. Firstly, in order to determine the control scheme with the shortest landing time, this paper analyzes the motion changes of the probe in each stage, and establishes the differential equations of velocity and angle changes in each stage, that is, the motion differential model of the landing process of the Mars probe. Then, before entering the Martian atmosphere, simplify the decision variables in the first stage as incident angle and incident velocity. After entering the Martian atmosphere, take the end time of pneumatic deceleration section and the end time of the second part of parachute deceleration section as decision variables, and take the minimum value of the end time of dynamic deceleration section as the objective function. A multivariate optimization model based on the motion differential model of the landing process of Mars probe is established. Finally, the optimal value of the probe after entering the Martian atmosphere is obtained by ergodic solution.*

Keywords: *ordinary differential equation, optimization model, planning model, ergodic solution*

1. Introduction

At 7:18 a.m. on May 15, China's first Mars probe tianwen-1 landed steadily at the preferred landing point - the utopian plain in the northern hemisphere of Mars, which means that China's first Mars exploration mission landed on Mars successfully, taking an important step in China's interstellar exploration journey and another milestone of great significance in the development of China's aerospace industry. The process of tianwen-1 probe from Mars synchronous orbit to hovering over the Martian surface is divided into several parts, such as the orbiter, the opening of the damping umbrella of the Mars probe, and the ignition of the engine system. By collecting the relevant audio-visual and text data of tianwen-1 probe, according to the motion law of the probe's landing process, it is divided into five stages: separation taxiing stage (before entering the Martian atmosphere), pneumatic deceleration stage, parachute system deceleration stage, power deceleration stage and landing buffer stage.

2. Differential model of detector motion variation

The motion of the detector in each stage is taken as the research object, starting with the law of its motion change. Because the constant change of the incident angle of the detector will greatly affect the parachute opening and the resistance coefficient of each stage, and the speed change is extremely important to the time, we establish the differential equations of speed change and angle change, calculate the landing time of each stage, and obtain the total landing time.

After being separated from the orbiter, the Mars probe slides into the Martian atmosphere in a flat throwing motion. At this time, the altitude is $H = 125km$ and the speed is $v_0 = 4.8km/s$.

Before entering the atmosphere:

Phase 1: separation of sliding section

In the motion process of this stage, because there are many variables in this stage, the process is simplified: the orbiter is separated from the lander, and the lander moves near the center of gravity towards Mars. It is assumed that the total amount of kinetic energy components in the direction of the connecting line between the lander and the center of mass of Mars remains constant, and the total kinetic energy changes with the incident angle (10 °~ 12 °).

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After entering the atmosphere:

Phase 2: pneumatic deceleration section

Set the total time of pneumatic deceleration section as t_1. Note that the initial time is 0, let the gravitational acceleration of Mars be g, and the resistance coefficient at this stage is k_1. The initial velocity of the Mars probe is v_0, the total mass is $M = 1634.9kg$, and the included angle between speed and horizontal direction is θ. The instantaneous angle between the probe and the horizontal direction after entering the Martian atmosphere is $\theta_0(10°\sim12°)$, when entering the atmosphere, the vertical partial velocity is $v_\perp = 833.5\text{m/s}$, tangential acceleration is a_τ. The normal acceleration is a_n. The radius of curvature is ρ. The instantaneous radius of curvature entering the Martian atmosphere is ρ_0, angular acceleration is α. Angular velocity is ω. The instantaneous initial angular velocity after separation is ω_0, the descending height is H_1.

For the force analysis and initial value state of the detector, the motion change model of the detector at this stage is:

$$\begin{cases} \dfrac{dv}{dt} = \dfrac{k_1 v^2}{M} - gsin\theta \\ \dfrac{d^2\theta}{dt^2} = \dfrac{\left(\dfrac{k_1 v^2}{M} - gsin\theta\right) gcos\theta}{v^2} \\ \rho_0 = \dfrac{v_0^2}{a_n} \\ \omega_0 = \dfrac{v_0}{\rho_0} \\ H - H_1 = \displaystyle\int_0^{t_1} vsin\theta dt \end{cases} \tag{1}$$

Phase 3: parachute deceleration section

In the movement process of this stage, it is divided into four parts: opening the umbrella, bouncing the umbrella, throwing the bottom and throwing the back cover. By collecting a lot of information about the motion changes in this stage [1], we know that the detector mainly uses parachute deceleration in this stage, so the detector is still only subject to gravity and resistance, but the resistance coefficient is different.

The part 1 of the phase 3: the process from opening the umbrella to throwing the bottom.

Let the time of throwing the bottom be $t_2 = t_1 + 20s$, let the resistance coefficient at this stage be $k_2 = \frac{1}{2} C\rho' S \cdot e^{-\frac{h}{h_s}}$, the parachute opening speed is v_1. The radius of curvature when opening the umbrella is ρ_1. The included angle between the speed and the horizontal direction when opening the parachute is θ_1. The descending height is H_2.

Similarly, the stress analysis of the detector can be obtained:

The motion change model of the detector at this stage is:

$$\begin{cases} \dfrac{dv}{dt} = \dfrac{k_2 v^2}{M} - gsin\theta \\ \dfrac{d^2\theta}{dt^2} = \dfrac{\left(\dfrac{k_2 v^2}{M} - gsin\theta\right) gcos\theta}{v^2} \\ \rho_1 = \dfrac{v_1^2}{a_n} \\ \omega_1 = \dfrac{v_1}{\rho_1} \\ H_1 - H_2 = \displaystyle\int_{t_1}^{t_2} vsin\theta dt \end{cases} \tag{2}$$

The part 2 of the phase 3: the process from throwing the bottom to throwing the back cover.

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Set the time for throwing the back cover as t_3. Let the resistance coefficient at this stage be $k_3 = \frac{1}{2}C\rho'S \cdot e^{-\frac{h}{h_S}}$, the speed of throwing the bottom is v_2. The mass of the outsole is $m_1 = 89.6kg$, the radius of curvature when opening the umbrella is ρ_2. The included angle between the speed and the horizontal direction when opening the parachute is θ_2. The descending height is H_3.

Similarly, the stress analysis of the detector can be obtained:

The motion change model of the detector at this stage is:

$$\begin{cases} \frac{dv}{dt} = \frac{k_3 v^2}{(M - m_1)} - gsin\theta \\ \frac{d^2\theta}{dt^2} = \frac{\left(\frac{k_3 v^2}{(M - m_1)} - gsin\theta\right) gcos\theta}{v^2} \\ \rho_2 = \frac{v_2^2}{a_n} \\ \omega_2 = \frac{v_2}{\rho_1} \\ H_2 - H_3 = \int_{t_2}^{t_3} vsin\theta dt \end{cases} \tag{3}$$

Phase 4: power deceleration section

The movement process in this stage is divided into two parts: dynamic deceleration and hovering obstacle avoidance. By collecting a lot of information about the motion changes in this stage, we know that the detector mainly uses 7500 n variable thrust engine to provide main power to decelerate in this stage. In addition, eight 250 n engines have the same direction as the main engine and cooperate with the main engine to decelerate [1], and the other attitude control engines are used for accurate attitude control [1]. At this stage, the detector is only subject to gravity and the reverse thrust brought by the engine. (ignore the delay time of hovering obstacle avoidance)

Note that the total reverse thrust received by the detector is F_n. Set the end time of this stage as t_4. The initial speed is v_3. The final speed is 0 and the mass of the back cover is $m_2 = 245.3kg$, the mass of landing patrol is $m_3 = M - m_1 - m_2 = 1300kg$, the acceleration is a, and the included angle between the speed a and the horizontal direction when throwing the back cover is θ_3. The descending height is H_4.

The force analysis of the detector shows that the motion change model of the detector at this stage is:

$$\begin{cases} \frac{dv}{dt} = -\frac{F_n}{m_3} + g \\ v_3 sin\theta = a(t_4 - t_3) \\ H_3 - H_4 = \int_{t_3}^{t_4} vsin\theta dt \end{cases} \tag{4}$$

Phase 5: landing buffer section

In the movement process of this stage, the probe adopts the reverse landing leg type to make a soft landing on the surface of Mars. At this stage, the landing patrol must find the optimal landing point, select a more flat terrain, confirm the final landing position and implement slow descent, and the height of $H_4 = 100m$ can accurately judge the land shape. Therefore, at this stage, the detector uses reverse thrust to slow down until it is 2-4m above the ground, turns off the engine and starts free fall, and finally realizes soft landing.

3. Planning model of theoretical motion

3.1. Establish planning model

It is required to analyze the shortest time required for the separation of the taxiing stage and the four stages after entering the atmosphere, as well as the scheme for the operation of the engine system. The time before entering the atmosphere is simplified and recorded as 3h. After entering the atmosphere, it is recorded that the time when the Mars Lander enters the Martian atmosphere is 0 and the end time of the

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pneumatic deceleration phase is t_1. The end time of the first part of the parachute deceleration section is t_2. The end time of the second part is t_3. The end time of power deceleration section is t_4. (since there are few controllable factors for landing buffer, this question simplifies the fourth stage - landing buffer section, only t_5 = 30s).

So let the decision variable be t_1 and t_3.

The objective function is:

$$minZ = t_4 \tag{5}$$

Constraints are:

1)The model satisfies the equations (1)-(4) of the four stage basic motion model.

2) The model satisfies that the sum of descent height values in four stages is H = 125km.

$$\begin{cases} 0 < t_1 < t_2 < t_3 < t_4 \\ v_0 = \dfrac{v_\perp}{sin\theta_0} \\ v_1 = \displaystyle\int_0^{t_1} \left(\frac{k_1 v^2}{M} - gsin\theta\right) dt + v_0 \\ v_2 = \displaystyle\int_{t_1}^{t_2} \left(\frac{k_2 v^2}{M} - gsin\theta\right) dt + v_1 \\ v_3 = \displaystyle\int_{t_2}^{t_3} \left(\frac{k_3 v^2}{(M - m_1)} - gsin\theta\right) dt + v_2 \\ v_3 sin\theta = a(t_4 - t_3) \\ v_4 = 0 \\ H = 125km \end{cases} \tag{6}$$

That is, when Z minimum is satisfied, t_1, t_2, t_3 and t_4 is obtained from which the above model is obtained.

3.2. Solving programming model

Because the model is difficult to solve from the perspective of obtaining analytical solutions, that is, equations (1)-(4) are difficult to write explicit analytical solutions. Here, the difference form of equations (1)-(4) is given, and a column of different t_1, t_2, t_3 is obtained by numerical calculation. Corresponding v, H, θ, ω under the value. Then they are fitted with polynomials, that is, the actual relationship is described in the form of polynomials, so the model is transformed into an unconstrained optimization problem, and the approximate solution of the problem can be obtained by solving it.

Firstly, set the time step as dt and traverse the end time t_1 of pneumatic deceleration section. End time t_3 of the second part of the parachute deceleration section. (t_2 = t_1 + 20s) Assume at time t_n, the speed is v_n, height h_n, angle is θ_n, angular velocity is ω_n. Therefore, the motion change model of each stage is changed to the difference form, that is, the equations (1)-(4) are changed to the difference form as follows:

Pneumatic deceleration section:

$$\begin{cases} v_{n+1} = \displaystyle\int_0^{t_{n+1}} \left(\frac{k_1 v^2}{M} - gsin\theta\right) dt + v_n \\ \theta_{n+1} = \displaystyle\int_0^{t_{n+1}} (\int_0^{t_{n+1}} \frac{\left(\frac{k_1 v^2}{M} - gsin\theta\right) gcos\theta}{v^2} dt + \omega_n) dt + \theta_n \\ \rho_{n+1} = \dfrac{v_{n+1}^2}{a_n} \\ \omega_{n+1} = \dfrac{v_{n+1}}{\rho_{n+1}} \\ H_{n+1} = \displaystyle\int_0^{t_{n+1}} vsin\theta dt \end{cases} \tag{7}$$

Parachute deceleration section part 1:

$$\begin{cases} v_{n+1} = \int_0^{t_{n+1}} \left(\frac{k_2 v^2}{M} - gsin\theta\right) dt + v_n \\ \theta_{n+1} = \int_{t_1}^{t_{n+1}} (\int_{t_1}^{t_{n+1}} \frac{\left(\frac{k_2 v^2}{M} - gsin\theta\right) gcos\theta}{v^2} dt + \omega_n) dt + \theta_n \\ \rho_{n+1} = \frac{v_{n+1}^2}{a_n} \\ \omega_{n+1} = \frac{v_{n+1}}{\rho_{n+1}} \\ H_{n+1} = \int_{t_1}^{t_{n+1}} vsin\theta dt \end{cases} \tag{8}$$

Parachute deceleration section part 2:

$$\begin{cases} \boldsymbol{v_{n+1}} = \int_{t_2}^{t_{n+1}} \left(\frac{\boldsymbol{k_3 v^2}}{\boldsymbol{(M - m_1)}} - \boldsymbol{gsin\theta}\right) \boldsymbol{dt} + \boldsymbol{v_n} \\ \boldsymbol{\theta_{n+1}} = \int_{t_2}^{t_{n+1}} (\int_{t_2}^{t_{n+1}} \frac{\left(\frac{\boldsymbol{k_3 v^2}}{\boldsymbol{(M - m)}} - \boldsymbol{gsin\theta}\right) \boldsymbol{gcos\theta}}{\boldsymbol{v^2}} \boldsymbol{dt} + \boldsymbol{\omega_n}) \boldsymbol{dt} + \boldsymbol{\theta_n} \\ \boldsymbol{\rho_{n+1}} = \frac{\boldsymbol{v_{n+1}^2}}{\boldsymbol{a_n}} \\ \boldsymbol{\omega_{n+1}} = \frac{\boldsymbol{v_{n+1}}}{\boldsymbol{\rho_{n+1}}} \\ \boldsymbol{H_{n+1}} = \int_{t_2}^{t_{n+1}} \boldsymbol{vsin\theta dt} \end{cases} \tag{9}$$

Power deceleration section:

$$\begin{cases} v_{n+1} = \int_0^{t_{n+1}} \left(-\frac{F_n}{m_3} + g\right) dt + v_n \\ v_{n+1} sin\theta = a(t_4 - t_n) \\ H_{n+1} = \int_{t_3}^{t_{n+1}} vsin\theta dt \end{cases} \tag{10}$$

3.3. Solving programming model

Based on the differential equation of motion change in the first four stages, through traversing t_1, t_3. On the premise of safe landing, the optimal solution t_4 is obtained. Get t_1, t_2, t_3 at the same time.

The best control scheme is - when entering the Martian atmosphere, the best incident angle is $\theta_0 = 10.6°$, the optimum incident velocity is $v_0 = 4531m/s$, after entering the Martian atmosphere, open the umbrella in 282.00s, throw the bottom in 302.00s, throw the quilt cover in 401.00s, and the engine power system is at t_3 = 401.00s fire and maintain the maximum thrust at t_4 = 427.65s, decelerate to 0 and hover, at this time, the thrust becomes 4836N. To sum up, the shortest time from the separation of the orbiter and the lander to the hovering of the lander is (427.65 + 3 * 3600)s.

References

[1] Zhao Cong. The thrilling 9 minutes of tianwen-1 Mars landing [J]. Space exploration, 2021 (06): 7-11.
[2] Feng Wei, Yi Wangmin, Yang Wang, Li Qunzhi, Hou Senhao, Zheng Shengyu, Meng fanwei. Design and verification of launch test system for "tianwen-1" detector [J]. Aerospace return and remote sensing, 2021, 42 (03): 23-31.
[3] Peng Yuming. Research on new Mars EDL navigation, guidance and control technology [D]. Nanjing University of Aeronautics and Astronautics, 2011.