Research of Cycling Race Strategy Based on Segment Optimization Technique

Honglin Liu
Sichuan University Software Engineering, Chengdu, Sichuan, 610207, China

Abstract: This paper examines race strategies in cycling to help athletes win. The three-parameter CP model, which is regarded to critical power CP, anaerobic work capacity AWC and the maximum duration T, and a total energy consumption model are establishment in this paper. We present a segmented optimum technique to investigate riders' energy distribution, whose target is to obtain relative optimality in each divided tiny time period. To build a total energy consumption model, the player's functional threshold power FTP, available anaerobic work W', and fatigue effect α are all taken into account throughout each time period t. Then we investigate two typical stages of courses: straight sections and slopes. The model is then applied to real track. The results reveal that the theoretical final time has little difference when compared to the champion's time.

Keywords: Three-parameter CP Model; Segmented Optimum Technique; Energy Distribution

1. Introduction

A bicycle road race, which includes a criterium, a team time trial, and an individual time trial, has traditionally been one of the most popular sports events. In the individual time trial of the cycling road race [1], each player must complete a fixed track alone, and their ranking is determined by the amount of time it takes to complete the course.

The player with the shortest amount of time will be crowned champion. Athletes' power consumption and endurance performance are validated with various sorts of courses [2]. As a result, we must investigate an individual rider's power curve and energy consumption distribution under various road conditions and environmental circumstances in order to make recommendations for driver training and competition.

2. Establishment of model

To aid in the analysis of the riders' power curve, we introduce the concept of critical power (CP). Scherrer and Monod [3] proposed the critical power, which refers to the linear relationship between the work done by the human and the maximum time where an athlete performs a maximum-capacity workout with a fixed load.

\[ W = W' + CP \times T \]  

(1)

On the basis of Scherrer and Monod's research, Hopkins [3] calculated CP by increasing the maximum cis-transient power. The equation proposed by Hopkins can be written as:

\[ T = \frac{AWC}{P - CP} + K \]  

(2)

where AWC refers to the anaerobic work capacity completed before reaching the maximum oxygen consumption, and T refers to the longest time an athlete can maintain at this power. When T=0, P achieves the maximum value \( P=P_{\text{max}} \), which represents the maximum exercise power when a person reaches his limited load. At this point there is:

\[ K = -\frac{AWC}{P_{\text{max}} - CP} \]  

(3)

Finally get the final P-T equation:
\[
T = \frac{AWC}{P - CP} - \frac{AWC}{P_{\text{max}} - CP}
\]

Divide the rider’s total riding process into \( k \) segments, and each segment has a time length of \( t_k \). Within each segment, the rider will run at a higher power for a part of the time (consumption stage) and ride at a lower power (recovery stage) for most of the rest time according to the characteristics of the road conditions so as to give full play to the strength of the rider without excessive use of energy, and the rider will repeat this strategy for each time period \(^4\).

Assume that the anaerobic work \( W' \) consumed by the athlete in each stage is equal to the energy he recovered in the previous stage, that is: \( t_k \times V_w = W' \).

From this, we can write the driver’s functional threshold power in the first stage:

\[
FTP_1 = CP + \frac{W'}{t_k}
\]

The formula is extended to the next \( i \)-th stage, and the influence of fatigue factors on athlete’s energy recovery is also considered:

\[
FTP_i = CP + (1 - \alpha_i) \frac{W'}{t_k}
\]

Sum all segments to get the total energy expended during the race:

\[
W_k = t_k \cdot \sum_{i=1}^{k} \beta_i \cdot FTP_i
\]

The average power of the rider during the race is:

\[
P = \frac{W_k}{k \cdot t_k}
\]

Where \( \beta_i \) is the correction factor of the \( i \)-th segment. We can change the values of \( k \) and \( t_k \) to find the best strategy.

Consider that it is difficult for athletes to change their speed frequently during the race, we combine the eq. (8) with the following eq. (9) to solve the speed \( v_k \):

\[
P = P_f + (m + m_b) \cdot g \cdot v_k \cdot \sin \theta + (m + m_b) \cdot g \cdot v_k \cdot \mu \cdot \cos \theta + 2 f \cdot v_k^2 + \frac{1}{2} C_d \cdot S \cdot \rho \cdot v_k^3
\]

Where \( \mu \) and \( f \) are the static friction coefficient and dynamic friction coefficient of the road section respectively, \( \theta \) is the inclination angle of the course. For straight sections, \( b=0 \). When riders are going downhill, \( \theta \) takes a negative value. \( g \) is the acceleration of gravity. \( C_d \) is the coefficient of air resistance. \( S \) is the projected area of the frontal plane of the cyclist and the athlete. \( \rho \) is the density of air.

The speed solving process is comparable to the speed on the straight segment when the slope is smaller than 2%. When the slope is greater than 2%, without loss of generality, the athlete must use more power to maintain a fair speed in order to enhance the results. To ascend the speed of climbing slope \( v_c \) and the speed on straight segment \( v_k \), we employ iterative law.

Firstly, we press \( \theta=0 \) into eq. (9) to acquire the average speed \( v_k \) and treat it as the first climbing speed \( v_c \). Next, we put the \( v_c \) into eq. (9) to solve the climbing power \( P_v \). The climbing section consumes energy \( W_c = (P_c - CP) \times T_c \). \( T_c \) and \( L_c \) are the time and the length of climbing the slope, \( T_c = L_c / V_c \).

Flat-segment consumption energy is \( W = (1 - \alpha_i) \times \gamma \times W' \) and the power \( P = W / (T_k \cdot T_c) \).

Put the value of \( P \) into eq. (9) to find out the first flat speed \( v^{(1)} \). Consider \( v^{(1)} \) as the first \( v_c \) and continue the above process, iterating continuously until \( |V_n^{(n+1)} - V_n^{(n)}| / V_n^{(n)} < 0.01 \). We accept \( v_c \) in the \( n \)-th iterations as the best strategy for athletes.
3. Solution of model

Male and female competitors compete on the same track in this competition. Male runners must cycle twice around the course whereas female runners cycle once. The map of 2021 Olympic Time Trial Course in Tokyo, Japan as shown in Figure 1 and list the relevant information of the track in Table 1:

![Map of 2021 Olympic Time Trial Course](image1.png)

Table 1: Course information of 2021 Olympic Time Trial in Tokyo, Japan

<table>
<thead>
<tr>
<th></th>
<th>Length of straight course(km)</th>
<th>Length of uphill section(km)</th>
<th>Length of downhill section(km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>male</td>
<td>15.4</td>
<td>14.8</td>
<td>14</td>
</tr>
<tr>
<td>female</td>
<td>7.4</td>
<td>7.4</td>
<td>7.7</td>
</tr>
</tbody>
</table>

According to our model, we can predict required time for time trial experts and sprinters to complete the race respectively and find the minimum time to complete the race. We list our prediction results in Table 2.

Table 2: Prediction results according to our model for 2021 Olympic Time Trial

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time trial specialist</td>
<td>Sprinter</td>
<td>Championship</td>
<td>Time trial specialist</td>
<td>Sprinter</td>
</tr>
<tr>
<td>$v_c$ (km/h)</td>
<td>42.01</td>
<td>36.44</td>
<td></td>
<td>41.29</td>
<td>35.84</td>
</tr>
<tr>
<td>$v_d$ (km/h)</td>
<td>57.01</td>
<td>52.93</td>
<td></td>
<td>56.99</td>
<td>52.12</td>
</tr>
<tr>
<td>$t$ (min)</td>
<td>54:10.43</td>
<td>61:01.18</td>
<td>55:04</td>
<td>26:13.47</td>
<td>31:00.24</td>
</tr>
</tbody>
</table>

Where $v_c$ and $v_d$ refers to the speed of the rider when he is uphill and downhill respectively.

Given that it is difficult for riders to strictly implement detailed strategies, the deviation coefficient $\beta_i$ is added into the model to indicate the difference between the athletes' actual and ideal conditions in the $i$-th period. To be more accurate, we change the deviation coefficient $\beta_i$ into a random number between 0.9 and 1.1 and increases the number of segments to be numerous enough. Based on this, we utilize random numbers to replicate 50 variances in real circumstances of athletes and calculate the corresponding speed of uniform riding. The outcomes are shown in Figure 2.

The variance between these 50 simulation outcomes is 0.026084 compared to the previous result $v_k=49.85$ km/h. The error range is:

$$\left[ \frac{(v_{\text{min}} - 49.85)}{49.85}, \frac{(v_{\text{max}} - 49.85)}{49.85} \right] = [-0.84\%, 0.50\%]$$

(10)

Because $\beta_i$ reflects the difference between the ideal and the actual circumstances in the $i$-th period, its divergence should be between 0.9 and 1.1. As a result, the athlete’s power distribution could have a fluctuation within 10%, and the final result fluctuation is between [-0.84%, 0.50%]. That is, the corresponding average speed of the entire race changes no more than 1% compared to identical circumstance.

From above statements, we can conclude that the athletes' minor deviation from the plan has little effect on the final result. Our model's robustness can be guaranteed.
4. Conclusion

In the establishment of the energy distribution model in our paper, the riding process is divided into multiple periods so that the athlete can reasonably allocate their own stored anaerobic energy in each segment, which not only ensures the fully utilization of athletes' strength but also avoid excessive physical strength consumption leading to poor performance.

This study fully addresses the athlete's fatigue component when determining the overall energy limit, which makes results more accurate. The addition of the energy distribution deviation coefficient fully incorporates the theoretical viability of the athlete in the real application of the model operation, increasing the model's applicability.

The model is then applied to real track. Track is adequately simplified and we obtain the speed and final time of various sorts of athletes in different stages. The results reveal that the theoretical final time has little difference when compared to the champion's time, indicating the model's effectiveness.

References