

Design of hybrid catastrophe bonds based on the public-private partnership model

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Abstract: *From the perspective of insufficient supply and demand of catastrophe bonds, combining the occurrence of natural catastrophes with the performance of the capital market, a hybrid catastrophe bond based on the public-private cooperation model is designed innovatively. Firstly, the paper deconstructs the operating framework of the public-private cooperative model of catastrophe bonds, and shows that it can improve the supply of catastrophe bonds to a greater extent. Secondly, it proves that the bond combined with the stock market state and the catastrophe state can increase investors' holding demand. Finally, the future income cash flow including principal, interest, and option is designed, and a hybrid catastrophe bond based on the public-private partnership model is obtained. The validity of bond issuance is proved from the perspective of investors' psychological expectations, and it is proved that the hybrid catastrophe bond design based on the public-private partnership can alleviate the difficulty that catastrophe bond meets cold in both supply and demand.*

Keywords: *public-private partnership, hybrid catastrophe bonds, the supply of bonds, the demand of bonds, market downside risk, option*

1. Introduction and literature review

At present, the catastrophe insurance market is not mature yet. Because of the particularity of natural catastrophes, it is very difficult to establish a catastrophe insurance system through pure commercial insurance. As the most widely used catastrophe risk securitization product in the world, catastrophe bonds were first issued in 1997 by USAA Insurance^[1]. And a UK specialist insurance and reinsurance company issued the first catastrophe bond specifically for cyber hackers in 2023^[2]. However, few catastrophe bond products have been designed and successfully issued according to the characteristics of catastrophes. Therefore, designing an optimized catastrophe bond issuance mechanism is of great significance for catastrophe risk management. This paper summarizes the existing representative research results from the basic elements of bonds, including risk type, trigger mechanism, principal and interest repayment design, financing term, and so on. Among the types of risk coverage, catastrophe includes natural catastrophes and man-made catastrophes, and the risks covered by catastrophe bonds can be single catastrophe risks or multiple catastrophe risks^[3].

As a clearing condition designed in the bond contract, generally speaking, the trigger mechanism of a catastrophe bond includes loss compensation type, physical parameter type, industry index type, and model loss type^[4].

In the repayment design of principal and interest, the principal and interest repayment design of catastrophe bonds includes the principle can be guaranteed but interest cannot be guaranteed, the principal cannot be guaranteed but interest can be guaranteed, neither principal nor interest can be guaranteed^[4].

Regarding the financing period of bonds, the issuing period of catastrophe bonds is related to the characteristics of natural catastrophes. For example, for seasonal disasters, the financing term of flood and drought catastrophe bonds is generally 1 year^[5], and that of typhoon and hurricane bonds is generally 2 years^[6]. Earthquake risks are more harmful, so the financing term of earthquake bonds is generally 3 to 5 years^[7].

In the study of catastrophe bond pricing, the process of catastrophe bond pricing includes catastrophe modeling, determining the term structure of interest rate, and bond pricing. The classic catastrophe loss

intensity fitting methods are Log-normal distribution, Gamma distribution, Weibull distribution, and generalized Pareto distribution^[8]. The generalized Pareto distribution is segmented to the economic loss distribution function, which has obvious advantages in describing the distribution with a thick tail. The probability distribution of catastrophe occurrence times mainly includes geometric distribution, binomial distribution, and Poisson distribution^[9]. In the study of the term structure model of interest rate, according to to determine whether the specific form of risk market price is explicit, the interest rate term structure model includes an equilibrium model and a no-arbitrage model. At present, the most widely used arbitrage-free term structure models mainly include Ho and Lee models and BDT models^[10]. In the common pricing models, the financial derivatives model and actuarial model without arbitrage pricing are used to determine bond prices^[11].

Catastrophe risk is characterized by difficulty in forecasting and a huge amount of loss. According to the international issuance experience, both the commercial catastrophe insurance and reinsurance markets are faced with the dilemma of cold supply and demand, which can be alleviated to some extent by designing the issuance and operation mode of hybrid catastrophe bonds based on the public-private partnership (PPP) model.

2. Analysis of supply and demand of catastrophe bonds and design of factors

2.1 Analysis of the shortage of supply of traditional catastrophe Bonds

The basis of issuing catastrophe bonds is that the policyholder buys catastrophe insurance, and then the insurance company transfers the risk to the capital market by issuing catastrophe bonds. Therefore, the uninsurability of catastrophe risk directly restricts the issuance of catastrophe bonds and is also the reason for the shortage of catastrophe bonds^[12].

2.2 Expand the supply of catastrophe bonds: The improvement of the catastrophe bond issuance model of public and private cooperation

The problem of insufficient supply of catastrophe bonds can be alleviated by constructing the issuance mode of public and private cooperation. This paper improves the framework of catastrophe risk financing based on the public-private partnership model (PPP), as shown in Figure 1. Cash flows marked by solid lines in the framework of Figure 1 represent cash flow disbursements determined by the parties in the PPP financing framework through actuarial modeling or commercial negotiations. And the payment of this part of cash flows has nothing to do with the occurrence of catastrophe events. The cash flow marked by a dotted line indicates that the payment of the cash flow depends on the occurrence of the catastrophe. In figure 1, $\alpha, \beta \in (0, 1)$.

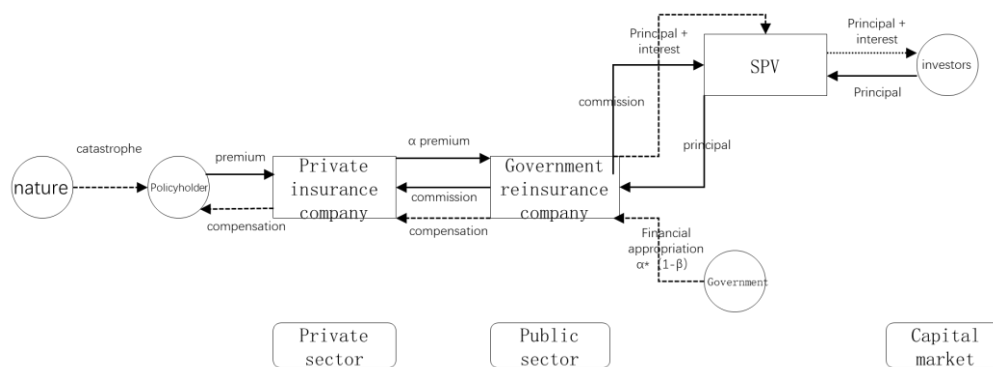


Figure 1: Catastrophe bond financing Framework in PPP

In the PPP catastrophe bond financing framework, the public sector includes government institutions and government reinsurance companies. Private institutions refer to insurance companies providing professional insurance services. The part of the capital market includes securities issuing intermediary SPVs and investors. Insurance holders are residents in areas prone to catastrophe events and other legal entities.

Based on the catastrophe risk financing framework of the PPP model, the government departments, with the technical support of the professional actuarial field, work out reasonable catastrophe insurance rates together with private insurance companies and require people in catastrophe-prone areas to purchase

catastrophe insurance from private insurance companies. Private insurers receive premiums and transfer a proportion of alpha to reinsurers with a government background according to their risk tolerance. As well as the proportion of the catastrophe risk transfer to the other party. The government reinsurance companies encourage the participation of private insurance companies to pay a certain commission.

Government reinsurance companies from private insurance companies charge after the proportion of premium also bear the corresponding catastrophe risk at the same time. In the insurance period, if any catastrophic event occurs, the percentage of benefits will be transferred to the insured by the government reinsurer through private insurers. Therefore, in the cooperation mode shown in Figure 1, the government reinsurance company will pay a certain commission to the private insurance company to encourage and support the private insurance company to provide professional insurance services in the capacity of the government. The PPP model will lay a foundation for the improvement of catastrophe risk participation and the promotion of the scope.

The government reinsurer finances its share of α catastrophe risk in two ways. By issuing catastrophe bonds to finance the catastrophe risk in proportion β of them, and the remaining $(1 - \beta)$ proportion of catastrophe risk is borne by financial allocation. In the operation channel of the government reinsurance company financing catastrophe events by issuing catastrophe bonds, that is, in the cash flow of the capital market section in the upper right corner of Figure 1, the government reinsurance company pays a certain commission to the financial intermediary that issues catastrophe bonds, and the intermediary SPV issues catastrophe bonds for the government reinsurance company in the capital market. In this financing framework, intermediaries do not bear any risks. They are employed by the government reinsurance company and are only responsible for the issuance of bonds and provide services of undertaking and underwriting of bonds. Their profits come from a certain percentage of commission paid by the government reinsurance company. Investors buy bonds through SPVs and pay the principal of bonds to the government reinsurance company through SPVs. In turn, the reinsurance company pays the principal and interest of bonds to investors through SPVs, so intermediaries are risk neutral. The interest on catastrophe bonds paid by the government reinsurance company will be covered by the reinsurance company's business operating profits, and the shortfall will be financed by financial grants.

The financing mechanism in Figure 1 involves private institutions, public sectors, and capital markets, thus constituting a catastrophe risk management system based on the PPP model. In the game between the public sector and the capital market, the public sector needs to strike a balance between the interest cost of issuing catastrophe bonds and the post-disaster financial backstop cost. Then investors in the capital market decide the purchase proportion of catastrophe bonds according to the bond interest given by the government and their expectations of catastrophe. Through the game between private institutions, public sectors, and capital markets, the reasonable sharing of catastrophe risks among them can be completed, thus laying the operational foundation for guaranteeing the issuance scale of catastrophe bonds, in other words, improving the operating mechanism and operation process for expanding the supply of catastrophe bonds.

3. Lack of demand for traditional catastrophe bonds and innovative hybrid catastrophe bonds

3.1 Analysis of the problem of lack of demand for catastrophe bonds

After being put in the capital market, catastrophe bonds are still not attractive enough to investors, mainly because investors have the wrong downside risk aversion, as shown in Figure 2. Because disasters convey a sense of fear, and there is a potential causal effect between one major disaster and another, markets may collapse, undoubtedly causing portfolio losses for bond investors. It is the aversion to the uncertainty of catastrophe occurrence and stock market crash, namely investors' downside risk aversion. That has partly caused the lack of demand for catastrophe bonds in the capital market.

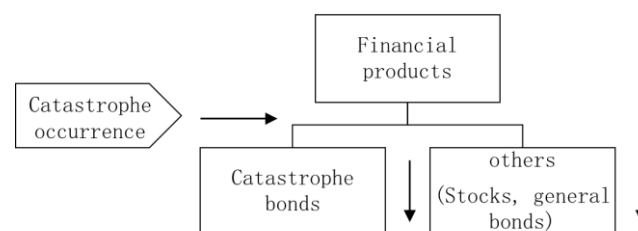


Figure 2: Investors' downside risk aversion

This article references the research ^[13], the theoretical design of hybrid catastrophe bonds combines the boom and recession of the stock market in the cash flow of catastrophe bonds, as to reduce the double risk exposure of investors. The following is an analysis of the operating mechanism and cash flow of simple catastrophe bonds and hybrid catastrophe bonds.

3.2 The operation of the traditional simple catastrophe bond analysis

In a traditional simple catastrophe bond transaction process, a financial institution issues a catastrophe bond with face value N for a government reinsurer (*Figure 3*). Government reinsurer A first determines the price μ_A for its transaction with intermediary B. Investor C determines the price μ_C for its transaction with intermediary B. $\mu_A < \mu_C$. At maturity, the government reinsurer gives the face value of the bond to the financial intermediary, who then gives it to the investor. In effect, μ_A is the level of financing obtained by catastrophe bonds in the capital market. μ_C is the amount of money invested by investors in the catastrophe bond market. The difference $\mu_C - \mu_A$ is the issuance fee paid to intermediary B.

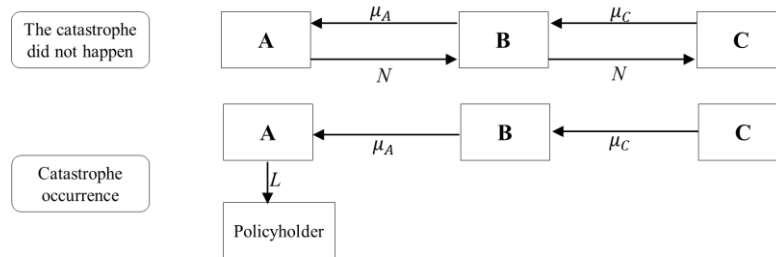


Figure 3: Cash flow of traditional simple catastrophe bonds

3.3 Expand demand for catastrophe bonds: introduce innovative hybrid catastrophe bonds

Now consider hybrid catastrophe bonds, which incorporate stock market booms and busts in their design. To reduce the investor's financial market investment loss caused by the market downturn caused by the catastrophe, in the hybrid catastrophe bond, it is designed that investor C buys a put option from intermediary B and investor C sells a call option to intermediary B. To maintain the intermediary's risk neutrality, the option price received or paid is transferred to government reinsurer A through Intermediary B.

A certain stock index is used to represent the trend of the whole stock market. The purchase and sale of hybrid catastrophe bonds are the same as that of traditional catastrophe bonds. A financial institution issues catastrophe bonds with face value N^h on behalf of a government reinsurer. Government reinsurer A first determines its trading price μ_A^h with intermediary B. Investor C determines its transaction price μ_C^h with financial Institution B. $\mu_A^h < \mu_C^h$. If no catastrophe occurs during the life of the bond, Intermediary B pays investor C the face value of the bond N^h . If the stock market is booming, as evidenced by an upward movement of the stock index, the stock index I is above a set level h , and a catastrophe does not occur, then C needs to pay B an additional consideration H . That is, the financial institution simply pays the investor $N^h - H$ and then transfers the received H to A. If the stock market declines, as indicated by a downward movement of the stock index, with the stock index I below a set level d and accompanied by a catastrophe, then B pays H to C. This H is then paid by A to B. The probability of catastrophe is denoted by p . The q_h and q_d denote the probability of stock market boom and bust events, respectively. The natural catastrophe state and the stock market state are shown in Figure 4.

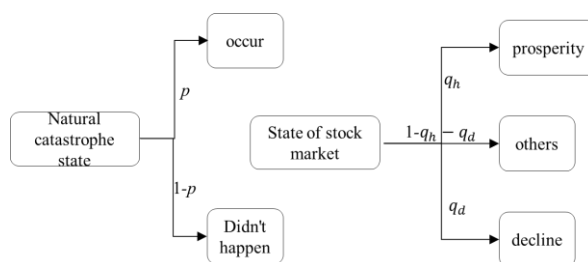


Figure 4: Graph of natural catastrophe state and stock market state

The following denotes the waiting time for the occurrence of two stock market boom events by $\bar{\tau}$. The $\underline{\tau}$ denotes the waiting time for the occurrence of two stock market recession events. $IA(\bar{\tau} \leq 1)$ and $IA(\underline{\tau} \leq 1)$ denote the indicative functions of stock market boom and recession events occurring during bond survival, respectively. q_h and q_d denote the probability of occurrence of stock market boom and recession events, respectively. We have $q_h = P_r(I > h)$ and $q_d = P_r(I < d)$. Figure 5 illustrates the cash flows of the three institutions in the hybrid catastrophe bond design.

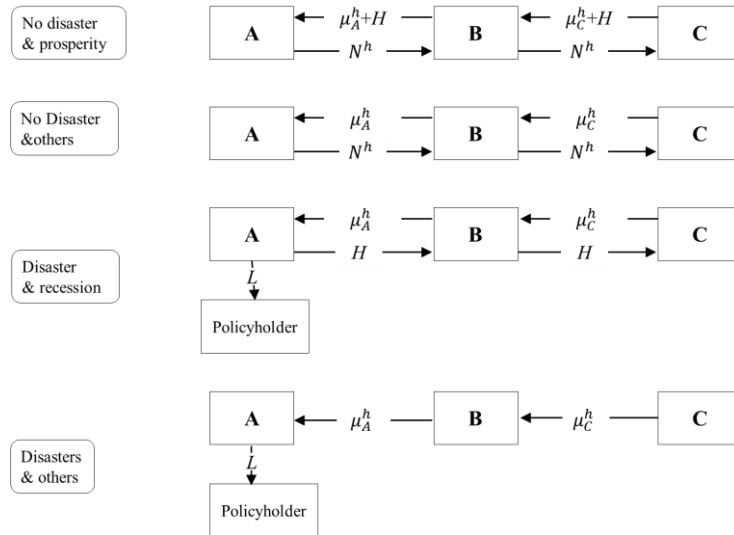


Figure 5: Graph of cash flow payments for hybrid catastrophe bonds

4. Design of hybrid catastrophe bonds based on a public-private partnership model

The pricing theory and bond elements involved in the issuance of catastrophe bonds are described above, and the design of hybrid catastrophe bonds based on the public-private partnership model in this paper is given next.

4.1 Determination of catastrophe bond elements

Based on the analysis of the financing period, this paper decides to design a catastrophe bond with a maturity of three years. This will give the issuer enough time to lock in the risk and thus protect the financing capacity while leaving enough time for the risk manager to monitor the changes in the portfolio.

In terms of bond triggers, industry-indexed and model-loss triggers can be designed without the participation of large catastrophe modeling firms. For the physical parameter-based trigger mechanism, the insured area needs to be subdivided as much as possible when determining the parameters of the catastrophe trigger event, but the available catastrophe sample data is limited, so the design of the sub-regional catastrophe trigger mechanism cannot be achieved. Therefore, this paper adopts the loss compensation type trigger mechanism.

In the design of the principal and interest repayment mechanism, to guarantee the scale of catastrophe financing and take into account the acceptance ability of investors, the payment of the bond principal still needs to be guaranteed. After comprehensive consideration, this paper decides to design the principal and interest repayment mechanism of earthquake bonds as follows. If a trigger event occurs during the duration of the bond, the principal will not be repaid temporarily after the maturity of the bond, and the repayment of the principal will be deferred for 5 years without interest.

4.2 Determination of the cash flow of catastrophe bonds

Firstly, catastrophe modeling was carried out for natural catastrophes to obtain the distribution of the occurrence frequency N of catastrophe risk loss and the distribution of the single loss limit X . Then, the approximate value of the one-year catastrophe loss distribution was obtained through Monte Carlo simulation [5], to estimate the catastrophe risk exposure. Next, the risk-sharing ratio among the public sector, private sector, and capital market is obtained by establishing a PPP model, and the scale of bond

issuance is determined according to the estimated catastrophe risk exposure.

In terms of the payment of bond interest, the loss trigger level is determined according to the fitting of the distribution of catastrophe losses. If the annual loss is higher than the trigger level, the event is regarded as the occurrence of catastrophe, then the annual interest CU will not be paid and the bond face value will be guaranteed to be paid at maturity. During the duration of the bond, the annual catastrophe event is fitted and the probability of annual interest payment is estimated. In the options of consideration in terms of payment, the stock market index will be used for empirical measurement, select representative points of the capital market prosperity events h , and calculate representative points of the capital market events recession d . After comprehensively considering the catastrophe that occurs and the prosperity and decline of the market, to determine the payment terms of the option price H .

Finally, in the pricing of catastrophe bonds, a random interest rate distribution model is constructed. Then, based on assuming bond issuance at par, the future cash flow of catastrophe bonds is discounted according to the pricing theory of financial derivatives^[11]. Then the coupon of catastrophe bonds is calculated, that is, the financial burden of catastrophe bonds issued by the government is calculated.

5. Conclusions

At present, the catastrophe bond is still in the exploration stage, there is much work to be done in popularizing the catastrophe bond to improve the catastrophe risk-sharing mechanism. The innovation of this paper is that by improving the public-private partnership model of catastrophe risk financing to adapt to catastrophe risk exposure, combining the natural state with the performance of the capital market to design hybrid catastrophe bonds, and exploring the allocation of risks and returns between the public sector and the market.

Under the framework of hybrid catastrophe bond issuance based on the public-private partnership model, the project design of earthquake catastrophe bond issuance by government departments is completed, which provides a new theoretical idea for further promoting catastrophe bonds as a catastrophe risk management method.

6. Recommendations

If catastrophe bonds are to be widely used as a financial product, the developed insurance market and securities market must provide effective technical support, such as catastrophe loss measurement by professional teams, timely loss determination, claim settlement after disasters, professional rating agencies, and evaluation standards.

Therefore, we should actively develop the capital market, focus on the basic work of issuing catastrophe bonds, and create technical conditions for issuing hybrid catastrophe bonds. We should also try our best to maintain a healthy and stable stock market so that the trend of the stock index can reflect the condition of the capital market to a greater extent, to provide an effective financial environment for the issuance of hybrid catastrophe bonds.

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References

- [1] Edesess M. *Catastrophe bonds: An important new financial instrument*[J]. EDHEC-Risk Institute: Nice, France, 2014.
- [2] Evans S. *Cat bond market could double to \$70bn within five years: John Seo at SIFMA*. [N] <https://www.artemis.bm/2023.03.06>
- [3] Polacek A. *Catastrophe bonds: A primer and retrospective*[J]. *Chicago Fed Letter*, 2018, 405: 1-7.
- [4] Johnson L. *Catastrophe bonds and financial risk: Securing capital and rule through contingency*[J].

Geoforum, 2013, 45: 30-40.

[5] Juahir H, Ibrahim R A, Saputra M P A, et al. Application of compound Poisson process in pricing catastrophe bonds: A systematic literature review[J]. *Mathematics*, 2022, 10(15): 2668.

[6] Herrmann M, Hibbeln M. Seasonality in catastrophe bonds and market-implied catastrophe arrival frequencies[J]. *Journal of Risk and Insurance*, 2021, 88(3): 785-818.

[7] Calvet L, Lopeman M, De Armas J, et al. Statistical and machine learning approaches for the minimization of trigger errors in parametric earthquake catastrophe bonds[J]. *SORT-Statistics and Operations Research Transactions*, 2017, 41(2): 373-392.

[8] Anggraeni W, Supian S, Halim N B A. Earthquake Catastrophe Bond Pricing Using Extreme Value Theory: A Mini-Review Approach[J]. *Mathematics*, 2022, 10(22): 4196.

[9] Nowak P, Romaniuk M. Pricing and simulations of catastrophe bonds[J]. *Insurance: Mathematics and Economics*, 2013, 52(1): 18-28.

[10] Chen X, Gao J. Uncertain term structure model of interest rate[J]. *Soft Computing*, 2013, 17: 597-604.

[11] Bretscher L, Schmid L, Sen I, et al. Institutional corporate bond pricing[J]. *Swiss Finance Institute Research Paper*, 2022 (21-07).

[12] Tavanaie M M, Linders D. Decomposition of natural catastrophe risks: Insurability using parametric CAT bonds[J]. *Risks*, 2021, 9(12): 215.

[13] Barrieu P, Louberge H. Hybrid CAT-Bonds[J]. *Journal of Risk and Insurance*, 2009, 76(3): 547-578.