ISSN 2616-5805 Vol. 3, Issue 1: 8-14, DOI: 10.25236/AJMS.2022.030102

Rolle Theorem and Radon-Nikodym Theorem in Bochner Cost

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Abstract: *This paper are divided into two mains parts with different affairs to discuss. The first part are some history of how the modern strict mathmaticala analysis built up. And there's a simple counterexample for Rolle theorem. As we all known Rolle theorem has three requirement to work out. So, there's some counterexamples for which there's only two requirements exist, and what is going to happen. The second part gives two counterexamples on Radon-Nikodym theorem and Riesz theorem in the meaning of vector measure. Then there's a proof for the equivalent of the Radon-Nikodym theorem and Riesz theorem in the meaning of vector measure. Some recent research are given at the last, which means it's still a vigor field nowadays.*

Keywords: *Rolle Theorem, Radon-Nikodym Theorem, Riezs Theorem, Bochner integral*

1. Introduction

Analysis, this type of mathematics that uses unknown amounts to calculate, so this has a close relationship between the analysis and algebra. In the 18th century, the word became related to the calculus, and the subprinted is divided into the main use of analytical skills. In 1820, the first clear definition of Western Wesi was infinite. When Kexi defined the fixed integral, the continuity was assumed. He first considers the incrementum number $x_1, x_2, \ldots, x_n$ in the integral range $[a, b]$, and he defines the definition of the definition as the following limit::

$$S = (x_1 - a)f(a) + (x_2 - x_1)f(x_1) + \ldots + (b - x_n)f(x_n) \tag{1}$$

When n becomes large, use the fact that the median theorem and continuity is used to give a detailed argument for the limit. But this configuration is not always possible, and Kexi also claims that a convergence function must have continuous and. In 1826, Abel gave a famous antique to refute Kexi's fallacy:

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{\sin kx}{k} \tag{2}$$

This function is not continuous in the odd number of π. This antique will later got the development of mathematicians and finally distinguish between convergence and unanimous convergence.

The inspiration definition of Liman's definition depends on whether Digei is still converged in the original function when the function is in the Fourier class. So he further promoted the concept of fixed points: the increasing numbe$x_1, x_2, \ldots, x_n$, and ∂_n, in the corresponding interval between a and b, in which $0 < \partial_n < 1$, then Further promoted to the following form:

$$S = (x_1 - a)f[a + \partial_1(x_1 - a)] + (x_2 - x_1)f[x_1 + \partial_2(x_2 - x_1)] + \ldots + (b - x_n)f[x_n + \partial_n(b - x_n)]$$

The value of S is dependent on the selection of interval lengths and ∂_n, if it has the following properties: When $n \to \infty$, which means $x_i - x_{i-1} \to 0$, there is an endless approach to the fixed value A, So this value is called points $\int_a^b f(x)dx$

It is easy to construct an example in which each interval is different, but still can be integrated, which makes Liman's definitions more effective, and later this has become the definition of general modern integration. Due to the limited space, only several of the most important resentment in mathematical analysis are listed here. From the process of overall analysis, the structure of the antique is inspired.

In recent years, many scholars have made research in this regard. Wei Y H, Wang Y was published in

ISSN 2616-5805 Vol. 3, Issue 1: 8-14, DOI: 10.25236/AJMS.2022.030102

the article on the Journal of Chongqing University of Arts and Sciences (Natural Science Edition) in April 2014, studying the relationship between Radon - Nikodym theorem and condition[1]; Shi Y W, Chen W L, Feng J J published in February 2011 Journal of Hengshui University Journal in non-standard multi saturation models, discussing the Radon-Nikodym theorem in the complex Loeb measurement space[2]; Hu B J Published in October 2005, the article discussed by the Journal of Chongqing Jiaotong University discussion application Riesz-Frechet theorem proves the Radon - Nikodym theorem, the proposed proof improvement has improved its original brief proof [3]; Yu M Posted in the article in Journal of Yunnan Nationalities University (Natural Sciences Edition) introduced the concept of generalized Fuzzy measurement, proved the generalized Fuzzy measured Radon-Nikodym theorem. As soon as the Radon-Nikodym theorem has a certain research heat with the Rolle theorem. This paper is very valuable and the meaning of the times[4].

2. Rolle theorem and its opposing

2.1. Rolle theorem

If the function $f(x)$ satisfies the following three conditions:

(1) $f(x)$ is continuous in closed interval $[a, b]$;

(2) $f(x)$ can be guided on the opening section (a, b);

(3) $f(a) = f(b)$;

At least there is at least a point $\delta \in (a, b)$ such that $f'(\delta) = 0$.

2.2. Analysis of each condition in the Rolle theorem

(1) In Rolle theorem, $f(a) = f(b)$ is only sufficient condition, not a necessary condition. If a function $f(x)$ can be found, which continuous in the closed interval $[a, b]$, can be guided on the opening section (a, b). But in closed interval $[a, b]$, there isn't x_1, x_2, so that $f(x_1) = f(x_2)$. This way can prove conclusions.

Example 1 $f(x) = x + \sin(x)$, $-\pi \leq x \leq \pi$

Analysis: Function $f(x)$ is continuous on the closed interval $[-\pi, \pi]$ and can be guided on the opening section $(-\pi, \pi)$, $f'(0) = 0$, but in $[-\pi, \pi]$ on the function value is not equal. In fact, $f(x) = x + \sin(x)$ has an unlimited $f'(x) = 0$ in the real estimation, but $f(x)$ is a monotonic increment function.

$$\text{Example 2 } f(x) = \begin{cases} \frac{1}{\left[\frac{1}{x}\right]} \sin\frac{\pi}{x}, & x \neq 0, -1 \leq x \leq 1 \\ 0, x = 0 \end{cases} \tag{3}$$

Analysis: The function $f(x)$ is continuous in $[-1, 1]$; when $x_0 = \pm\frac{1}{k}$, $k = 2,3,4 \ldots$, $f(x)$ is not guided in x_0; when $x_0 = \frac{2}{2k+1}$, $k = 1,2,3 \ldots$, $f'(x_0) = 0$; and $-1 \leq x_1 \leq x_2 \leq 1$, make $f(x_1) = f(x_2)$. The description $f(x)$ can be guided in the opening section (a, b), which is not a sufficient condition, not a necessary condition [5].

2.3. Rolle Theorem is only satisfied with the two premises

Any one of the three conditions of the Rolle theorem is indispensable. If any three conditions only satisfy the two generally there will be repeated examples, making the guidance function in any point is not zero[6].

Details are as follows:

(1) When the function $f(x)$ is not met to be guided on the opening section (a, b), i.e., $f(x)$ is continuous in closed interval $[a, b]$, $f(a) = f(b)$, In addition to the opening section (a, b), $f(x)$ is possible to be guided, but there is no point $\delta \in (a, b)$ such that $f'(\delta) = 0$[7].

$$\text{Example 3 } f(x) = 1 - |x|, -1 \leq x \leq 1 \tag{4}$$

Analysis: $f(x)$ continuous in the closed interval $[-1,1]$, and $f(-1) = f(1) = 0$, except all $x = 0$

ISSN 2616-5805 Vol. 3, Issue 1: 8-14, DOI: 10.25236/AJMS.2022.030102

in $(-1,1)$, existing $f'(x)$ and $f'(x)$ has a boundary:

$$f'(x) = \begin{cases} -1, -1 < x < 0 \\ 1, 0 < x < 1 \end{cases} \quad (5)$$

However, there is no δ in $(-1,1)$ to make$f'(\delta) = 0$.

(2) When the function $f(x)$ is not satisfied in the closed interval $[a,b]$, that is, $f(x)$ can be guided on the opening section (a,b), $f(a) = f(b)$, at this time, at any $\delta \in (a,b)$, $f'(\delta) \neq 0$.

$$\text{Example 4 } f(x) = x - [x], 0 \leq x \leq 1 \quad (6)$$

where $[x]$ represents the maximum integer of not exceeding x

Analysis: $f(0) = f(1) = 0$, by definition direct verification f satisfying $[0,1]$, $f'(x) = 1, -1 < x < 1$, there is no δ make $f'(\delta) = 0$.

(3) When $f(x)$ does not satisfy $f(a) = f(b)$, that is, $f(x)$ can be guided on the opening section (a,b) and in closed interval $[a,b]$, any $a \leq x_1 \leq x_2 \leq b$, $f(x_1) \neq f(x_2)$, there is no point $\delta \in (a,b)$ such that $f'(\delta) = 0$.

$$\text{Example 5 } f(x) = x, -1 \leq x \leq 1 \quad (7)$$

Analysis: The nature of the primary function, $f(x)$ is continuous and can be connected to [0, 1], but in [0, 1], $f'(x) = 1$ constant, so there is no such δ can make $f'(\delta) = 0$[8].

3. Vector value measurement function

3.1. Knowledge preparation and sign notes

Concept of measurement space: The set line E of a given space X is satisfied with a collected non-negative function group, ie: a collection of two or two unhaneous collection $A_1, A_2, \dots, A_n \in E$, if $\bigcup_{n=1}^{\infty} A_n \in E$, there must be $\mu(\bigcup_{n=1}^{\infty} A_n) = \sum_{n=1}^{\infty} \mu(A_n)$[9]. And claiming that the space X satisfies can be satisfied and the operationally closed ring E is a σ ring, and an E, an E, X, E, μ, integrally formed of X, E, μ, is referred to as a measuring space.

Concept of linear space: Set X is linear space on digital F, if there is a mapping $T: X \to R^1, x \to ||x||$

If they are satisfied:

(1) Positiveness: $||x|| \geq 0, ||x|| = 0$ When and only when $x = 0$

(2) Tight: $||ax|| = |a| \, ||x||, a \in F$

(3) Triangular inequality: $x, y \in X, ||x + y|| \leq ||x|| + ||y||$

Then $||x||$ is the norm of element x. If the norms are defined in the linear space X as $|| \bullet ||$, this space X is referred to as an excellent linear space, recorded $(X, || \bullet ||)$.

Concept of **Banach space**: X is an excellent linear space, $\{x_n\}$ is the point of X, if[10]

$$d(x, x_0) = ||x - x_0|| \to 0, x - x_0 \to 0 \quad (8)$$

The $\{x_n\}$ is called $\{x_n\}$ in accordance with the norms to x (or $\{x_n\}$ strong converging reply to x); if $x \in X$, X is called complete excellent linear space, that is, the Banach space, referred to as (B) space[11].

Concept of modeling vector space: A function mapped to non-negative solids, if it meets the nature described in the introduction, is called a half; if only the zero vector function value is zero, then it is called the norm[12]. Vector space with a norm is called the vector space, and has a half-ray called semi-cultural vector space.

Concept of boundary variation function: Defines the real value function on the interval $[a,b]$, and any split P in $[a,b]$, which satisfies $a \leq b$. Work[13]

$$V_f(x_0, \dots, x_n) = \sum_{i=1}^{n} |f(x_i) - f(x_{i-1})| \quad (9)$$

Then the $V_f(x_0, \dots, x_n)$ is f about the division of $P = \{x_i\}_{i=0}^{n}$, making $V_a^b(f) = \sup\{V_f(x_0, \dots, x_n)\}$. Then $V_a^b(f)$ is called f in $[a,b]$ full variation[14]. If $V_a^b(f) < \infty$, then f is a boundary variant function

ISSN 2616-5805 Vol. 3, Issue 1: 8-14, DOI: 10.25236/AJMS.2022.030102

on $[a, b]$, and all the boundary variant functions on $[a, b]$ is $V(a, b)$.

Concept of boundary variance functions on the vector value function space: From the interval $[a, b]$ to the vector value function $x(t)$ of the Banach space $(a_i, b_i), i = 1,2, \dots,$ $\sup(\sum_{i=1}^{n}(a_i - b_i)) < \infty$. Then $x(t) \in X$ is called a boundary deterioration[15].

Concept of strongly determined: In (X, E, μ), there is a valuable step function column $\{x_n(s)\}$ such that $x(s) = \lim_{n\to\infty} x_n(s)$ establishes almost everywhere in X, the $x(s)$ is strongly determined in X.

Concept of Bochner integration: (Ω, E, μ) is a complete σ limited measurement space, $x(t)$ is a vector value function defined on the Ω to take the value of the Banach space X:

(1) If $x(t)$ is a value-value function on Ω, $\{A_k\}$ is a measurable set that is not intersecting each other in Ω, $\Omega = \bigcup_{k=1}^{\infty} A_k$, $x(t) = x_k(t \in A_k, k = 1,2, \dots)$. Besides

$$\sum_{k=1}^{\infty} ||x_k||\mu(A_k) < +\infty \tag{10}$$

It is said that $x(t)$ is Bochner accumulated on Ω, and $(B)\int_{\Omega} x(t)d\mu$ is the Bochner integration of $x(t)$, ie

$$(B)\int_{\Omega} x(t)d\mu = \sum_{k=1}^{\infty} x_k\mu(A_k) \tag{11}$$

(2) For a general strong measuring function $x(t)$, if it is a valuable function column $\{x_n(t)\}$, which about μ is almost a strong convergence limit, besides

$$\lim_{n\to\infty} \int_{\Omega} ||x(t) - x_n(t)||d\mu = 0 \tag{12}$$

Then say that $x(t)$ is on Ω is Bochner accumulated, and the Bochner score of $x(t)$ is

$$(B)\int_{\Omega} x(t)d\mu = \lim_{n\to\infty}(B)\int_{\Omega} x_n(t)d\mu \tag{13}$$

For the Bochner accumulated function $x(t)$, its integral value (vector) does not depend on $\{x_n(t)\}$, Bochner points is the direct promotion of the Lebesgue points in vector value function, in vector value measurement theory, Sub-theory, probability, random procedure, and Banach space geometric theory have extensive applications[16].

Concept of absolute continuity of vector value measurement: $\varphi: S \to X$ is vector value measurement, if there is any $\varepsilon > 0$, there is $\sigma > 0$ so that $D \in E$, $\mu(E) < \sigma$, $\|\varphi(D)\| < \varepsilon$, it is called absolute continuous.

3.2. Radon-Nikodym Theorem about a finite voltage of a space absolute continuous vector function

Radon-Nikodym Theorem at numeric measuring: Set (X, E, μ) is a full range limited measurement space, φ is a limited measure on E, where φ is absolutely continuous. Then there is limited tonable function g on S, make each $D \in E$,

$$\varphi(D) = \int_{D} fd\mu \tag{14}$$

This theorem can see the promotion of the Newton-Leibniz formula[17].

It is natural to think that whether the Radon-Nikodym theorem is still established for the vector value function, and which conditions need to meet. Discussion in the sense of Bochner can be discussed:

The introduction of the marker C_0 is an real value consecutive function on E with a tight support set[18].

Set: $X = [0,1]$, E is [0, 1] all of Leberg can be measured, and M is defined as Leberg measurement.

Specify mapping $F: R \to C_0$ is: $F(E) = \int_E \sin 2^n \pi t dm(t)$

$F(E) \in C_0$can be seen from the Newton-Lebneiz theorem on Leberg Points.

Definition f Norm: $\|F(E)\| = \sup\left\|\int_E \sin 2^n \pi t dm(t)\right\|$

So $\|F(E)\| = \sup\left\|\int_E \sin 2^n \pi t dm(t)\right\| \le m(E)$, under this definition, F is a boundary error and about m absolute continuous measure[19].

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If there is a Bochner volume function $f: E \to C_0$, make $F(D) = \int_E f_n(t)dm(t)$, $f(t) = \{f_n(t)\}$, $D \in E$, $F(D)$ for the nth coordinate $(F(D))_n = \int_E f_n(t)dm(t)$.

So for almost all t, $f_n(t) = \sin 2^n \pi t$; but if you record $E_n = \{t| \sin 2^n \pi t > t\}$, it is everything $n, m(E_n) = \frac{1}{2}$.

So $m(\lim E_n) \geq \lim m(E_n) = \frac{1}{2}$, but according to this measure, $m(\{t: f(t) \in C_0\}) \leq \frac{1}{2}$, ie f is not almost everywhere C_0 value, this contradicts the nature of F. So the Radon-Nikodym theorem has not been established on F.

3.3. A Riesz Representation T theorem is not established with a bounded linear operator

Riesz Representation T theorem of the real value function: If X is Banach space, $T: L^1(E, \mu) \to X$ is a bounded linear operator, whether there is $f \in L^\infty(X, E, \mu)$, make:

$$Tg = \int_D fgd\mu, \forall g \in L^1(X, \mu) \tag{15}$$

Then T is represented by Riesz.

Here $L^\infty(X, E, \mu)$, indicating f: f is B accumulated, $\operatorname{ess\,sup}\|f(t)\|_x < +\infty$, marked $\|f\| = \operatorname{ess\,sup}\|f(t)\|_x$.

Take the measurement space on the interval $[0, 1]$, $([0, 1], E, m)$, where m is Leberg measurement. Defining linear operator:

$$T: L[0,1] \to C_0,\ Tg(t) = \{\int_0^1 g(t) \sin 2^n \pi dm(t)\} \tag{16}$$

Obviously T is linear and there is:

$$\|Tg(t)\| = \sup\left|\int_0^1 g(t) \sin 2^n \pi dm(t)\right| \leq \int_0^1 |g(t)| dm(t) = \|g\| \tag{17}$$

That is T has a boundary, if there is $f \in L^\infty([0,1], C_0, m)$, make $Tg(t) = \int_0^1 f(t)g(t)dm(t), \forall g \in L[0,1]$

Using the previous vector value measured $F(E) = \{\int_E \sin(2^n\pi)dm(t)\}$, you can see $F(E) = T(\chi_E) = \int_E f(t)dm(t)$, again utilizing the methods and conclusions in the above example, and it is easy to know that there is no such T. You can get Riesz to indicate that theorem is not established for T.

4. Riesz about a race operator can represent an equivalent to the Radon-Nikodym theorem in Banach Space

From the above two inveractions, we can see that the Radon-Nikodym theorem and Riesz can represent contradictions in the same way. So naturally, it is equivalent to whether there is such riesz representative operators to the Radon-Nikodym theorem in the Banach space is equivalent[20].

Theorem: X About the full limited measurement space (Ω, R, μ) is charged with Radon-Nikodym properties

$$T \in B(L^1(\Omega, R, \mu) \longrightarrow X) \tag{18}$$

both can be represented.

Necessity: Set $T \in B(L^1(\Omega, R, \mu) \longrightarrow X)$, define $F: R \longrightarrow X$ is $F(E) = T(\chi_E)$. The χ_E is a feature function. Since $F(E) \leq |T|\mu(E)$, F is a vector value function having a boundary variation, and F is absolutely continuous about μ. Based on assumption, F has the nature of Radon-Nikodym, so there is $f \in B(\Omega, X, \mu)$, so that $F(E) = \int_E fd\mu$, at this time, π is a division of $E \in R$. So

$$\sum_{A\in\pi}\|F(A)\| = \sum_{A\in\pi}\left\|\int_A f(t)d\mu(t)\right\| \leq \sum_{A\in\pi}\left\|\int_A f(t)\right\| d\mu(t) = \int_A \|f(t)\| d\mu(t) \tag{19}$$

Therefore, $|F|(E) \leq \int_E \|f(t)\| d\mu(t)$.

Take a list of finite values $\{f_n\}$ at this time, so that $\lim \int_\Omega \|f(t) - f_n(t)\| d\mu(t) = 0$, set n_0 to make any $\varepsilon >\ 0$, always have $\int_\Omega \left\|f(t) - f_{n_0}(t)\right\| d\mu(t) < \varepsilon$.

Setting is a certain value Δ that allows $|F|(E) - \sum_{A\in\pi}\|f_{n_0}(t)d\mu(t)\| < \varepsilon$ and take $f_{n_0}(t)$ to get a common value of the measurable, it is not allowed to make π is added Δ, satisfying $|F|(E) - \sum_{A\in\pi}\int_E\|f(t)d\mu(t)\| < \varepsilon$. The following triangular inequality is established:

$$\left||F|(E) - \int_E\|f_{n_0}(t)\|d\mu(t)\right| \le \left||F|(E) - \sum_{A\in E}\int_A\|f(t)d\mu(t)\|\right| + \sum_{A\in E}\left|\int_A\|f(t)d\mu(t)\| - \left\|\int_A f_{n_0}(t)d\mu(t)\right\|\right| \le 2\varepsilon \quad (20)$$

So there is $|F|(E) = \lim\int_\Omega\|f_n(t)\|d\mu(t) = \int_E\|f(t)\|d\mu(t)$.

For any $E \in R$, $\|F(E)\| = \|T(x_E)\| \le \|T\|\|x_E\| = \|T\|\mu(E)$, therefore

$$|F|(E) \le \|T\|\mu(E) \quad (21)$$

So there is $\int_E\|f(t)\|\,d\mu(t) \le \|T\|\mu(E), \forall E \in R$

Therefore, $\|f(t)\| \le \|T\|$, because of all $E \in R$, $T(x_E) = \int_\Omega x_E f d\mu$, $Tg = \int_\Omega fgd\mu$ is set to all ladder functions. The ladder function is dense in $L^1(\Omega,\mu)$, that is, all $g \in L^1(\Omega,\mu)$ is established.

So, because $\|Tg\| = \left\|\int_\Omega fgd\mu\right\| \le \|f\|\|g\|$, $\|T\| \le \|f\|$, combined with the above formula we can conclude $\|T\| = \|f\|$, so there is $Tg = \int_\Omega fgd\mu$, so T is expressed[21].

Sufficiency: Set $F: R \to X$ is a vector value measurement with bound deterioration, and F is absolutely continuous about μ. $|F|$ is absolutely continuous about μ. Because of the Radon-Nikodym theorem of numerical measuring, there is function $h \in L^1(\Omega,\mu)$, $|F|(E) = \int_E hd\mu$ established to all $E \in R$. Let $E_n = \{t|n-1 \le h(t) \le n\}$, $n = 1,2,\ldots$, E_n does not intersect each other, $\Omega = \bigcup_{n=1}^\infty E_n$, $E \subset E_n$, has $(n-1)\mu(E) \le |F|(E) \le n\mu(E)$, g is a step function:

$$g = \sum_{i=1}^P a_i F(E\cap A_i) \quad (22)$$

$A_i \in R$, $A_i \cap A_j = \emptyset$. Definition, $T_n g = \sum_{i=1}^m a_i F(E_n \cap A_i)$, so there is

$$\left||T_n g|\right| \le \sum_{i=1}^P |a_i||F|(E\cap A_i) \le \sum_{i=1}^P |a_i| n\mu(E\cap A_i) \le n||g||_1 \quad (23)$$

Therefore, according to the hahn-Banach theorem, T_n can be continuously extended to a boundless linear operator of $L^1(\Omega,\mu) \to X$.

Set $t\_T_n \in B(L^1(\Omega,\mu) \to X)$. According to the hypothesis T_n has Riesz representation, there is existed $f_n \in L^\infty(\Omega, X, \mu)$ makes $T_n g = \int_\Omega f_n g d\mu, \forall g \in L^1(\Omega,\mu)$. For $E \in R$, $F(E\cap E_n) = T_n(\chi_E) = \int_E f_n d\mu$. Definition, $f(t) = f_n(t)$, $t \in E_n$

$$F(E) = \lim F\left(E\cap\left(\bigcup_{n=1}^\infty E_n\right)\right) = \lim\int_{E\cap(\bigcup_1^m E_n)} f(t)d\mu(t) \quad (24)$$

Since F has a boundary deterioration, for any $E \subset \bigcup_{n=1}^m E_n$, $F(E) = \int_E f(t)d\mu(t)$, there is

$$\int_{\bigcup_{n=1}^m E_m}\|f(t)\|\,d\mu(t) = |F|\left(\bigcup_{n=1}^m E_m\right) \le |F|(\Omega) \quad (25)$$

You can get $\|f(t)\| \in L^1(\Omega,\mu)$ according to Levi Lemma. Equation

$$F(E) = \lim F\left(E\cap\left(\bigcup_{n=1}^\infty E_n\right)\right) = \lim\int_{E\cap(\bigcup_1^m E_n)} f(t)d\mu(t) \quad (26)$$

Got: $F(E) = \int_E f(t)d\,\mu(t)$. Therefore, X is on (Ω, R, μ) has RNP properties.

5. Conclusion

In the 1930s, Bochner proposed Bochner points of vector value function, naturally, and he tried to promote the Radon-Nikodym theorem in real analysis to a vector value measurement. However, soon he found that this is not established in the L^∞ space. Later, Birkhoff and Gelfand have proven that the Radon-Nikodym theorem is established for Hilbert space and the anti-Banach space. So more mathematicians put into development of this theory. These conclusions promote the research of Banach space geometric theory, Diestel et al. Demonstrate that the Banach space X has RNP properties (space to meet the Radon-Nikodym theorem), and when each consistent bound X value is converged according to the B norm. The weakness of the tight convex space also has the nature of Radon-Nikodym. The research of Radon-Nikodym in Banach space has an important role in the Banach space. In terms of

ISSN 2616-5805 Vol. 3, Issue 1: 8-14, DOI: 10.25236/AJMS.2022.030102

probability, differential equation, integration of aggregation, operator theory, etc. have extensive applications. The study of RNP nature is still continuing.

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