A solution to the problem of finding lost submarines

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Abstract: *In this paper, based on the submersible's own dynamic characteristics, ocean currents, water resistance, seawater density and other factors, we construct the differential equation of submersible dynamics, solve and simulate the prediction of the submersible's real-time position by using the difference method, and find that the motion of the submersible presents a general trend of horizontal to the east and a small range of stable fluctuations. For the uncertainty, we analyzed both the influencing factors and the probability estimation, and obtained the uncertainty factors of the coupled influence of underwater environmental factors, sensor measurement errors, and seafloor topography.*

Keywords: *Network Structure Model; Search and Rescue Plan; Equations of the Dynamics of the Submersible*

1. Introduction

Deep sea expeditions are one of the most popular forms of extreme travel, offering adventurers the opportunity to witness the wreckage of a sunken ship. However, since the sinking of the Titanic, stricter demands have been placed on the safety of deep-sea travel. In this paper, we develop a model for predicting the position of a submersible at a given time and propose a scheme for searching for lost submersibles that is applicable to other waters and searching for multiple lost submersibles [1-2].

A submersible is a deep-diving device used for underwater exploration, seabed exploration, seabed exploitation and salvage, life-saving and other tasks [3]. With the growing interest in underwater exploration, it has become an adventurous activity that attracts tourists. However, companies have to prioritize safety and environmental impact while manufacturing submersibles to ensure a seamless and safe journey [4]. In case of any unforeseen mishap, the submersibles can still be located as soon as possible so that rescue operations can be carried out. Therefore, the study of submersibles is of significant research value as mankind continues to explore the resources of the deep sea [5].

Assume that the submersible is moving east and toward the sea floor. We created a Cartesian coordinate system with the position of the submersible as the origin, east as the positive direction of the x-axis, north as the positive direction of the y-axis, and vertically down as the positive direction of the z-axis.

Since the current direction in the Mediterranean Sea is mostly in the southeast and northeast directions, it is assumed that the submersible will be affected by the force of the currents on it, the resistance of the water, and the buoyancy of the ocean. The force of the currents on the submersible will be in the horizontal direction, and the resistance and buoyancy of the water will be in the vertical direction. We also have to consider the effect of seafloor topography, which can be approximated by looking at a map of the seafloor topography [6-7].

The seafloor topography affects, among other things, where the submersible descends and the direction of the currents. This kinematic model first considers the simple underwater motion of the submersible starting from the seafloor and only considers the effect of topographic factors on its motion after descending to the seafloor [8]. The topography is shown in Figure 1. Therefore, for the uncertainty, this paper analyzes both the influencing factors and the probability estimation, and obtains the coupled influence of underwater environmental factors, sensor measurement errors, and seafloor topography, which overcomes the difficulties of the traditional study about the search [9-10].

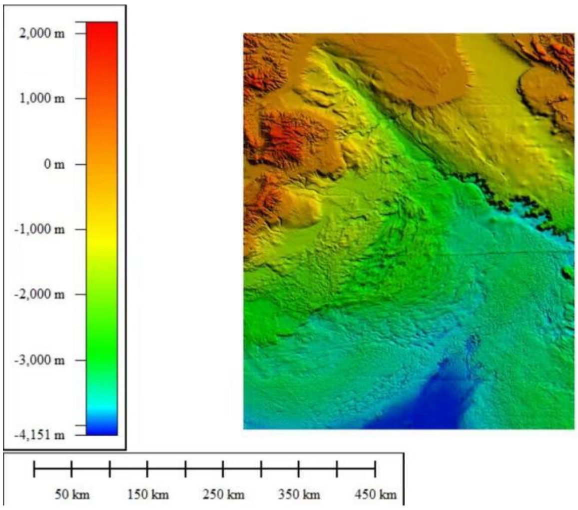


Figure 1: Topographic map of the seabed in Ionian Sea

2. The fundamental of data analysis

We denote the combined external force on the submersible as:

$$F = f\left(\frac{H}{R_e}\right)6\pi\eta UR_e \tag{1}$$

Where, h is dynamic viscosity. U is undisturbed fluid velocity on the centerline of the sphere. R is the radius of the sphere. R_e is the equivalent spherical radius of the object. H is the distance between the surface and the center of the equivalent sphere. By searching the literature, we find a continuous function that is valid for all distances from the distance surface, fitted as follows:

$$f\left(\frac{H}{R_e}\right) = 0.700\left(\frac{H}{R_e}\right)^{-1.082} + 1.001 \tag{2}$$

Based on the characteristics of density stratification of seawater, we establish a hyperbolic tangent density profile model by searching related literature, which is widely used to simulate the density stratification structure of fluids. The density of seawater is as follows:

$$\rho_c = \overline{\rho} - \frac{\Delta\rho}{2}\frac{\sinh\left(\frac{z - z_{pyc}}{d_{pyc}}\right)}{\cosh\left(\frac{z - z_{pyc}}{\lambda \cdot d_{pyc}}\right)} \tag{3}$$

Where, λ is the coefficient. The parameters in the model are set as follows: the density difference and the mean density are $\Delta\rho = 6.5\ kg/m^3$ and $\overline{\rho} = 1025.25\ kg/m^3$. Respectively, the depth at which the density jump layer is located, and the thickness of the density jump layer. 10m.The depth at which the density leap is located is $z_{pyc} = 10m$ and the density leap layer thickness is $d_{pyc} = 200m$. Through equation above, we derive the variation of density of seawater in relation to depth of the ocean, as follows Figure 2:

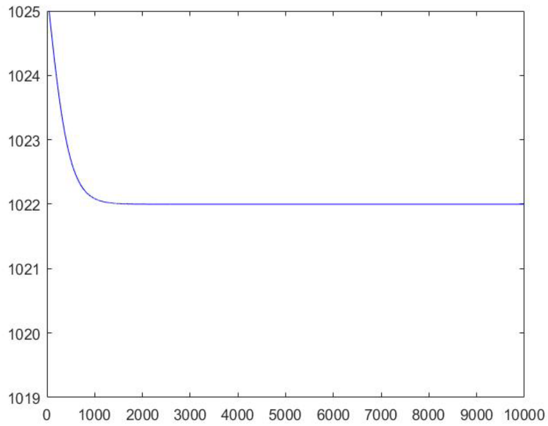


Figure 2: Relationship between density of seawater and depth of the ocean

When an object moves in liquid, there is a force acting on it in the opposite direction of the object's motion. Many previous experiments have shown that the main factors that constitute resistance are the shape of the object, its cross-section, the speed of relative motion, and the density of the seawater. For spherical objects, we can use a simplified formula to estimate the resistance to motion. As we consider the submersible as an elliptical gelatinous capsule-like body, its cross-section is similar to a circle. Therefore, we can express the resistance of water to the submersible.

$$f_r = 0.5 \cdot \rho_c \cdot k \cdot S \cdot v^2 \tag{4}$$

Where, k is a resistance coefficient and a dimensionless number. For spherical objects, k usually lies between 0.5 and 0.9, depending on the smoothness of the surface of the object. S is the area of contact between the water and the submersible, i.e., the cross-sectional area of the object, i.e., the area of projection of the object in the horizontal direction. v is the speed at which the submersible moves through the water. From the table, we can see that the resistance is proportional to the coefficient of resistance of the object and also includes the surface properties of the object.

As the submersible is in normal operating condition, it has a propulsion from the body to push it in the direction of the target. To make a prediction of the position of the submersible, we need to know the acceleration of the submersible and the combined external force applied. In a three-dimensional coordinate system, we take the decomposition of the total combined external force to the x-axis and y-axis, which can be seen in below equation. φ is the angle between F and the positive direction of the x-axis.

$$\begin{cases} F_x = F\cos\varphi \\ F_y = F\sin\varphi \end{cases} \tag{5}$$

Since we only consider the force of the current on the submersible horizontally, the current's component force on the x-axis on the submersible must be a driving force. However, since the submersible moves in an easterly direction, the force on the submersible in the y-axis may be either a driving force or a resistance force. We analyze the forces on the submersible in the x, y, and z directions. Figure 3 shows the force on the submersible in the x-axis.

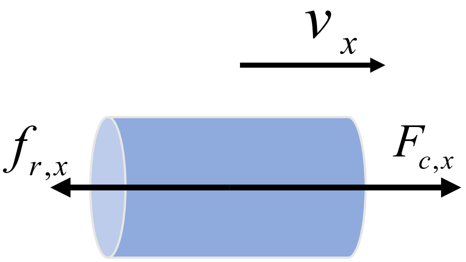


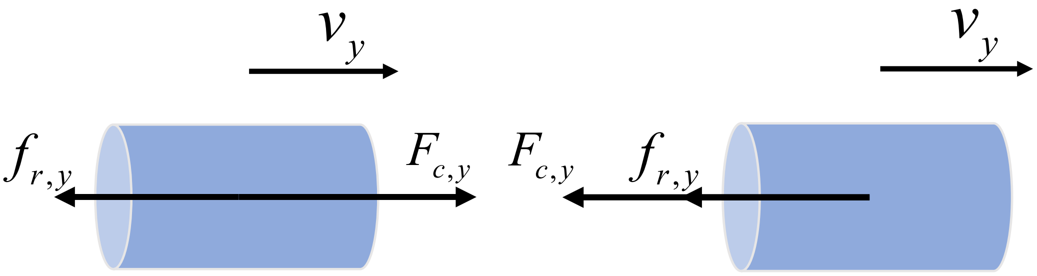
Figure 3: Force on the x-axis of the submersible

The combined external force on the submersible in the x-axis direction is expressed as shown in

Figure 3:

$$F_x = F_{c,x} - f_{r,x} \tag{6}$$

Figure 4 shows the two force cases of the submersible in the y-axis.



(a) (b)

Figure 4: Force on the y-axis of the submersible

We evaluate the direction of the force of the current on the submersible as the same as or opposite to the direction of the resistance of water using K, that is:

$$K = \frac{F_{c,y} \cdot v_y}{\left| F_{c,y} \cdot v_y \right|} = \begin{cases} 1, & F_{c,y} \text{ and } v_y \text{ are in the same direction} \\ -1, & F_{c,y} \text{ and } v_y \text{ are in the opposite direction} \end{cases} \tag{7}$$

Then the combined external force on the submersible in the y-axis direction can be expressed as:

$$F_y = F_{c,y} - K \cdot f_{r,y} \tag{8}$$

Figure 5 shows the force diagram of the submersible in the z-axis.

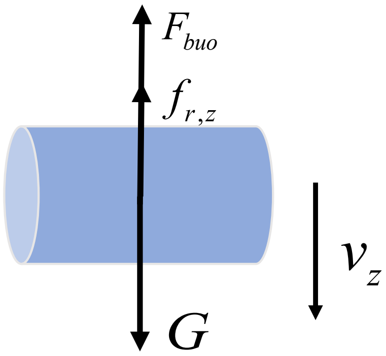


Figure 5: Force on the z-axis of the submersible

3. The establishment of data analysis

Based on the position prediction model, we considered the effects of ocean currents, water drag magnitude, and differences in seawater density on the motion of the submersible. Therefore, the following uncertainties are derived in this paper: underwater environmental factors: changes in ocean currents, fluctuations in seawater density and changes in seawater resistance are intertwined and may have an impact on the trajectory of the submersible. Changes in seawater temperature, pressure, current direction and speed will not only change the propulsion direction of the submersible, but will also affect the density and resistance of seawater. As the submersible dives deeper and deeper, changes in seawater density can cause changes in seawater buoyancy, which can lead to inaccurate predictions of the trajectory of the submersible. Sensor Measurements: Submersibles are often equipped with a variety of sensors, such as sonar, cameras, and seawater temperature sensors, which are used to sense the environment and monitor the state of the submersible itself. Measurement errors in these sensors may affect the prediction of the trajectory of the submersible. Geography of the seabed: The terrain of the seabed may include undulating terrain, rocks, dunes, seagrasses, etc. Due to the variability and complexity of the seabed terrain, the submersible may need to adjust its direction of travel at any time, and all these factors may affect the prediction of the motion trajectory of the submersible.

Based on the above uncertainties, we conclude that the submersible needs to carry appropriate equipment to send the following information to the host vessel on a regular basis. Thus, we can reduce these uncertainties before an accident occurs.

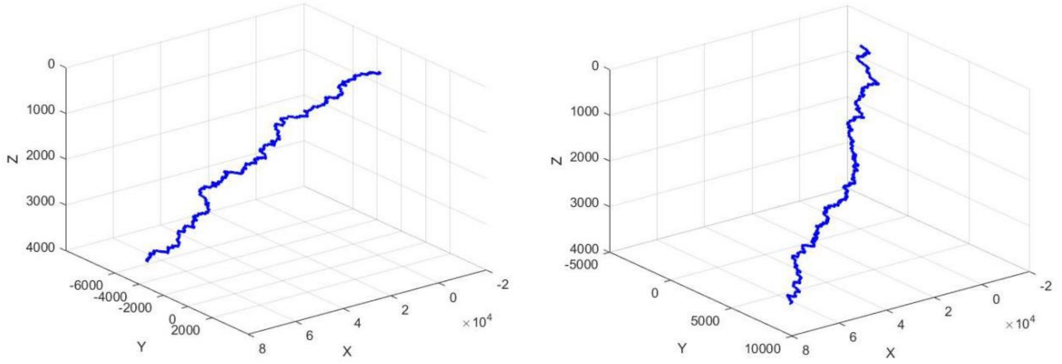
GPS can accurately obtain information about the position and trajectory of a submersible. This information can be sent to the host ship at regular intervals to help the host ship understand the current position and movement of the submersible.

Radios, satellite communications, etc. are used for real-time communication and data transfer with the host ship. They can regularly send information on the position, status and measurement data of the submersible to the host ship, helping the host ship to grasp the operation of the submersible, and make adjustments and interferences when necessary.

The sonar bathymetry system regularly sends information to the host ship about the undulation of the seafloor terrain, rocks, sand dunes, etc., which can help the host ship better understand the environment in which the submersible is located.

Sonar, seawater temperature sensors and cameras are used to sense the underwater environment and monitor the status of the submersible itself. By regularly acquiring the measurement data from these devices and sending it to the host, it can help the host understand the changes in the underwater environment and the working condition of the submersible itself.

In summary, the probability distribution for that ocean depth is calculated separately based on the step size of the ocean depth, and the accuracy of the prediction is estimated using the confidence interval values. First, we partition the 3000-meter ocean in 0.01-second time steps to form 5000 deep ocean levels based on the position prediction model. Second, we assume that the submersible moves vertically downward to the east. However, the direction of the ocean current may be southeast or northeast. Therefore, we simulate the prediction using differential equations based on the position of the submersible and sum to obtain the trajectories for the two cases in Figure 6.



(a) *(b)*

Figure 6: Three-dimensional trajectory of the submersible

As can be seen in figure 6 (a), the submersible moves from the origin towards the positive x, negative y and positive z axes, i.e. towards the sea floor in a southeasterly direction. From figure b, it can be seen that the submersible moves from the origin to the positive x-axis, positive y-axis, and positive z-axis, i.e., it moves to the sea floor in the northeast direction.

We projected the position coordinates of the submersible onto the xy plane and analyzed the xy coordinates. Then, we use the visualization image to plot the oscillatory relationship between xy

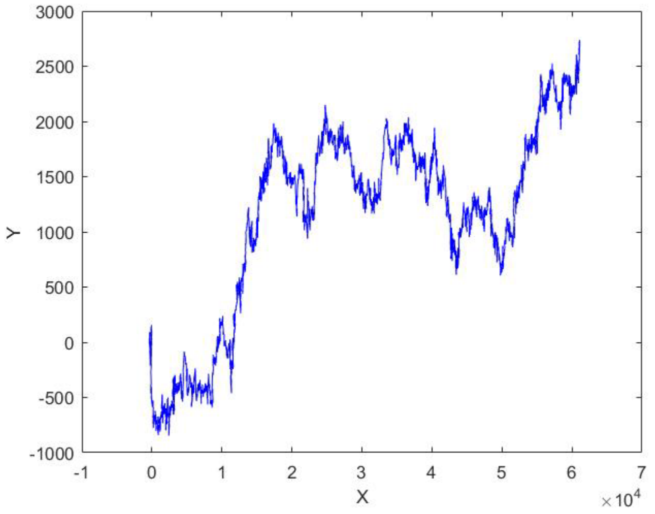


Figure 7: Oscillation curve in xy-plane

We make 400 stochastic prediction simulations for the same release location. Then we obtain two-dimensional confidence intervals for the 400 two-dimensional point sets in each layer of the statistic by simulating the prediction with multiple stochastic parameters. Next, we randomly selected points within the two-dimensional confidence intervals of each level for the overall characterization in the xy plane, as shown in figure 7.

In statistics, a confidence interval is a range of intervals for an estimate of a parameter. It indicates the level of confidence we have in that estimate. It is usually expressed as a probability value (e.g., 95%) indicating the probability that the interval contains the true parameter value over many repetitive samples. The formula for calculating the variance is shown as follows:

$$\sigma^2 = E\left[\left(X-\mu\right)^2\right] \tag{9}$$

Where, X are the coordinates of a randomly selected two-dimensional set of points. μ is the mathematical expectation of X, σ is the overall standard deviation of X. The confidence interval is shown in equation (10).

$$\left[\mu-1.96*\frac{\sigma}{\sqrt{n}}, \mu+1.96*\frac{\sigma}{\sqrt{n}}\right] \tag{10}$$

4. Conclusions

In order to ensure the safety of the submersible during its activities, a position prediction model was developed to predict the position of the submersible over a period of time. Considering the complexity of the seafloor environment, factors such as current speed and direction, ocean density variations and seafloor geography are taken into account in this paper to improve the accuracy of the prediction. Our submersibles are also equipped with the necessary equipment, including positioning sensors and communication equipment, to ensure real-time transmission of position information to the host computer and to reduce uncertainty. When the submersible is lost, the search device on the main vessel will be activated. Considering the cost-effectiveness and performance advantages, we screened the rescue devices in the market and finally chose the multibeam bathymetry system and scanning sonar as the additional devices on the rescue vessel to achieve timely rescue. Meanwhile, through the best path decision optimization model that can be obtained based on intelligent algorithms, we found the best search path with the least time to find the lost submersible. Based on the time and cumulative search results, we can determine the probability of finding the diver and adjust the search strategy as needed. Such a research process provides research ideas for the underwater exploration process.

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