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How Can Random Walk Be Applied to Analysis Stock

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Abstract: *This paper discuss how the model of random walk applies in analyzing the stock price. Firstly, the graph of symmetric random walk and the graph of the stock price of the company AMD are shown to observe the similarity of them directly. Secondly, we give the stock price model driven by random walk theoretically. The result of our research helps us to understand the application of random walk on analyzing the stock price by deriving the equations.*

Keywords: *Random walk, Stock price, mathematics*

1. Introduction and background

As the economic develop rapidly, the stock industry has become increasingly popular. People try to find a method to analyze the changes of stock and to conduct risk management that finally gets a high return. Random walk is one of the methods that can be used to analyze the stock price changes. Random walk refers to the inability to predict future development steps and direction based on past performance, and it is the ideal mathematical state of Brownian motion. In 1953, when the British statistician Kendall applied time series analysis to study stock price fluctuations, he found out that there is no pattern in stock prices, which means the changes in stock price match with the Random walk. As a result, people started to use Random walk to analyze the stock market.

Random walk-the stochastic process formed by successive summation of independent, identically distributed random variables which is one of the most basic and well-studied topics in probability theory. More details and derivative processes can be found in the book [1], and here we will briefly introduce the formal expression of random walk.

Assume that $Zn = X1 + ... + X_n$, and $\{X1,..., X_n\}$ are independent random variables with distribution

$$P(X_1 = 1) = p,\ P(X_1 = 0) = 1 - p$$

Where $p \in (0,1)$. We call $\{Z_1,..., Z_n\}$ as random walk.

Lemma 1.1: The distribution of random walk is shown below:

$$P(Zn = k) = C_n^{\frac{n+k}{2}} p^{\frac{n+k}{2}} (p-1)^{\frac{n-k}{2}}$$

Proof*:* We assume that a is the amount of movements that the object move to the right, and b is the amount of movements that the object move to the left. Also, a and b satisfy the following equations

$a + b = n$
$a - b = k$

After simple calculation, we derivate

$$a = \frac{n+k}{2}$$
$$b = \frac{n-k}{2}$$

Then we have

$$P(Zn = k) = C_n^a p^a (p-1)^b = C_n^{\frac{n+k}{2}} p^{\frac{n+k}{2}} (p-1)^{\frac{n-k}{2}}$$

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2. Main observations

In this section, we will show our observations on symmetric random walk of the price in the stock market. Also, we will use python to draw symmetric random walk. In addition, we take the stock price of AMD as an example, giving the graph of it.

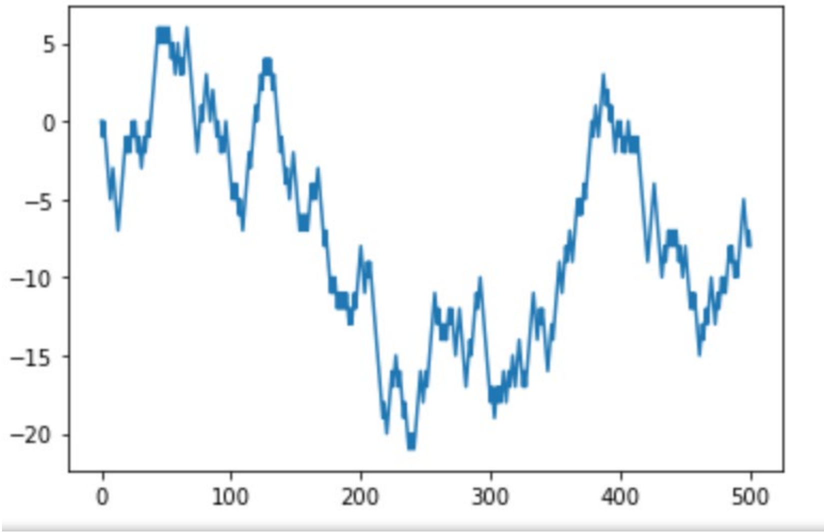

Figure 1: A graph of symmetric random walk

We use Python to show a graph of the symmetric random walk in figure1, and the detailed code is showing in Section Appendix.

Now we have a graph of Advanced Micro Devices (AMD)'s real stock price, we find its data and graph in address:https://www.futunn.com/stock/AMD-US?seo_redirect=1&channel= 1244&subchaanel =2&from=BaiduAladdin&utm_source=alading_ user&utm_medium=website_growth

Figure 2: A graph of the stock price of AMD

The figure 2 above shows the real stock price of AMD from September to the September 20th, 2021.

We observe that the graph of the symmetric random walk is really similar to the graph of the stock price of AMD. Thus, we deduce that random walk is an ideal model for our researching about stock price.

3. Theory derivation

In this section, we analyze two different models for stock price. The first one is based on a binomial

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tree model, having two parameters, *u* and *d (u* represents the upward movements, and *d* represents the downward movements), and we call it STOCK PRICE MODEL 1. This model is given by Cox [2] in 1978. They give a simple approach for option pricing and clarify its links between this model and Black-Scholes model. Here we introduce it again. The second one is also based on a binomial tree model but with other two parameters μ and σ (μrepresents its rate of return, and σ represents its volatility), and we call it STOCK PRICE MODEL 2. Realistically, μ and σ can be observed from historical data, therefore having a higher practical value. In this paper, we want to use elementary mathematics with a simple and clear way to introduce the relationship between these two stock price models.

3.1. Stock price model 1

We suppose that the first step price of the stock is S_0, and the next step of the stock S_1 is driving by the single-period binomial model which is described by two factor *u* and *d*. Also, *u* and *d* satisfy the following equation

$$S_1 = S_0 u$$
$$S_1 = S_0 d$$

So we have *d < 1 < u*. We denote the binomial tree as *(u, d,* p), suppose S_n be the stock price at time *n,* then it satisfies the following equation

$$p(\frac{S_n}{S_{n-1}} = u) = p$$
$$p\left(\frac{S_n}{S_{n-1}} = d\right) = 1 - p$$

Let

$$\Upsilon_\mathrm{n} = \ln \frac{S_n}{S_{n-1}}$$

Then we derivate $\Upsilon_1, \Upsilon_2, ...$ have the same distribution

$$p(\Upsilon_\mathrm{n} = lnu) = p$$

$$p(\Upsilon_\mathrm{n} = lnd) = 1 - p$$

Notice that

$$ln\frac{S_n}{S_0} = ln\frac{S_n}{S_{n-1}}\frac{S_{n-1}}{S_0} = ln\frac{S_n}{S_{n-1}} + ln\frac{S_{n-1}}{S_0}$$
$$= \Upsilon_\mathrm{n} + ln\frac{S_{n-1}}{S_0} = \Upsilon_\mathrm{n} + ln\frac{S_{n-1}}{S_{n-2}}\frac{S_{n-2}}{S_{n-3}}\dots\frac{S_1}{S_0}$$
$$= \Upsilon_\mathrm{n} + \Upsilon_\mathrm{n-1} + \cdots \Upsilon_1$$

Thus, we have an observation

$$ln\frac{S_n}{S_0} = \Upsilon_1 + \Upsilon_2 + \cdots + \Upsilon_\mathrm{n}$$

From

$$P(\Upsilon_1 = lnu, \Upsilon_2 = lnd) = \mathrm{P}(\Upsilon_1 = lnu)\mathrm{P}(\Upsilon_2 = lnd)$$

Deviating $\{\Upsilon_1, \Upsilon_2, \dots\dots\}$ is identical independent variables.

Setting

$$\mathrm{X_n} = 2\frac{\Upsilon_\mathrm{n} - \frac{\mathrm{lnu} + \mathrm{lnd}}{2}}{\mathrm{lnu} - \mathrm{lnd}}$$

We have

$$\mathrm{P}(\mathrm{X_n} = 1) = \mathrm{P}(\Upsilon_\mathrm{n} = \mathrm{lnu}) = \mathrm{p}$$
$$\mathrm{P}(\mathrm{X_n} = -1) = \mathrm{P}(\Upsilon_\mathrm{n} = \mathrm{lnd}) = 1 - \mathrm{p}$$

By calculations, we have

$$\Upsilon_\mathrm{n} = \frac{\mathrm{lnu} - \mathrm{lnd}}{2}\mathrm{X_n} + \frac{\mathrm{lnu} + \mathrm{lnd}}{2}$$
$$ln\frac{S_n}{S_0} = \sum_{i=1}^{n} \Upsilon_\mathrm{i} = \frac{\mathrm{lnu} - \mathrm{lnd}}{2}\sum_{i=1}^{n} \mathrm{X_i} + \frac{\mathrm{lnu} + \mathrm{lnd}}{2}$$

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Finally, we derivate that

$$S_n = S_0 EXP(\frac{\text{lnu} - \text{lnd}}{2}\sum_{i=1}^{n} X_i + \frac{\text{lnu} + \text{lnd}}{2})$$

The stock price can be expressed by Random walk.

$$S_n = S_0 EXP(\frac{\text{lnu}-\text{lnd}}{2} Z_n + \frac{\text{lnu}+\text{lnd}}{2} n) \tag{3.1}$$

3.2. Stock price model 2

The practice realistic characterization of the stock is its rate of return *μ,* volatility *σ.*

$$Variance = E\,(X — EX)^2$$
$$Standard\ variance = \sqrt{VAR(x)}$$

For each single step Δt in the interval [0, Δt], $S_0$and S_{Δ_t} are regarded as random variables. Suppose the expectation of some function of S_{Δ_t} are equal to the the rate of returnμ, variance of some function of S_{Δ_t} equal to volatility σ. We represent our suppose as follows:

$$E\left(f\left(\frac{S_{\Delta_t}}{S_0}\right)\right) = \mu\Delta t$$
$$\sqrt{\text{Var}\left(f\left(\frac{S_{\Delta_t}}{S_0}\right)\right)} = \sigma\Delta t$$

Let

$$X_n = ln\frac{S_{n\Delta t}}{S_{(n-1)\Delta t}}$$

Let special $\Delta t = \frac{1}{n}$

$$X_1 + X_2 + \cdots + X_n = ln\frac{S_{\frac{1}{n}}}{S_0} + ln\frac{S_{\frac{2}{n}}}{S_{\frac{1}{n}}} + \cdots + ln\frac{S_1}{S_{\frac{n-1}{n}}} = ln\frac{S_{\frac{1}{n}}}{S_0}\frac{S_{\frac{2}{n}}}{S_{\frac{1}{n}}}\cdots\frac{S_1}{S_{\frac{n-1}{n}}} = ln\frac{S_1}{S_0}$$

Calculation

$$\mu = E\left(ln\frac{S_1}{S_0}\right) = nE(X_1)$$
$$\sigma^2 = Var\left(ln\frac{S_1}{S_0}\right) = Var(X_1 + X_2 + \cdots + X_n) = nVar(X_1)$$

The above implies that

$$E(X_1) = \mu\frac{1}{n}$$
$$Var(X_1) = \sigma^2\frac{1}{n}$$

Recalling: $\frac{S_{\Delta t}}{S_0}$ is a random variable raking values u and d with probability p.

$$\mu\Delta_t = E\left(ln\left(\frac{S_{\Delta_t}}{S_0}\right)\right) = plnu + (1-p)lnd$$

We have

$$p = \frac{\mu\Delta t - \text{lnd}}{\text{lnu} - \text{lnd}}$$

Note: Var(x) = $E\,(X - EX)^2 = E\,(X^2) - (EX^2)$

Thus:

$$\left(Var\frac{S_{\Delta t}}{S_0}\right) = E\left(ln\left(\frac{S_{\Delta t}}{S_0}\right)\right)^2 - \left(E\left(ln\left(\frac{S_{\Delta t}}{S_0}\right)\right)\right)^2$$

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$$= p(\ln u)^2 + (1-p)(\ln d^2) - (p\ln u + (1-p)\ln d)^2$$
$$= (\ln u + \ln d)^2 p(1-p)$$

Recall that

$$\sigma^2\Delta t = \mathrm{Var}(ln\left(\frac{S_{\Delta t}}{S_0}\right)) = (\ln u + \ln d)^2 p(1-p)$$

Substituting of p, we have

$$\sigma^2\Delta t = (\mu\Delta t - \ln d)(\ln u - \mu\Delta t)$$

Summary: we need to choose *u,d,p* such that

$$\mu\Delta t = p\ln u + (1-p)\ln d$$
and
$$\sigma^2\Delta t = (\mu\Delta t - \ln d)(\ln u - \mu\Delta t)$$

We derivate stock price can be expressed by Random walk.

$$S_n = S_0 EXP(\frac{\ln u - \ln d}{2} Z_n + \frac{\ln u + \ln d}{2} n)$$

Model Cox-Rosee-Bubinstein: ud=1:

$$S_n = S_0 EXP(\ln u Z_n)$$

We need to choose *u, d, p* such that $\ln d = -\ln u$

$$\mu\Delta t = p\ln u + (1-p)\ln d = 2p\ln u - \ln u$$

And

$$\sigma^2\Delta t = (\mu\Delta t - \ln d)(\ln u - \mu\Delta t) = (\mu\Delta t + \ln u)(\ln u - \mu\Delta t) = (\ln u)^2 - (\mu\Delta t)^2$$

From the formula we get above

$$\sigma^2\Delta t = (\ln u)^2$$

We have $\ln u = \sigma\sqrt{\frac{T}{n}}$

$$Z_n \ln u = X_1 \ln u + X_2 \ln u + \cdots + X_n \ln u$$

Now we introduce p* and λ, assume that

$$p^* = \frac{1}{2} + \lambda\sqrt{\Delta t}$$
$$\lambda = \frac{1}{2\sigma}(\mu - \frac{\sigma^2}{2})$$

And we get

$$p(X_1 = 1) = p^* P(X_1 = -1) = 1 - p^*$$

By some calculation, we have$E(Z_n \ln u) = (\mu - \frac{\sigma^2}{2})T$

$$\mathrm{Var}(Z_n \ln u) = n\sigma^2\Delta t(1 - \frac{1}{\sigma^2}\left(\mu - \frac{\sigma^2}{2}\right)^2 \Delta t)$$

Again, we can ignore any term going to zero faster than Δt, then we have

$$\mathrm{Var}(Z_n \ln u) = n\sigma^2\Delta t = \sigma^2 T \tag{3.2}$$

For the following derivation, we apply the central limit theorem. The complete process of its derivation can be found in the book [3], and here we will only mention and apply this theorem.

Theorem 3.2: Central Limit Theorem: For {X1, X2, ...Xn}, when n is big enough, we have

$$\frac{X_1 + X_2 + \cdots + X_n - E(X_1 + X_2 + \cdots + X_n)}{\sqrt{\mathrm{Var}(X_1 + X_2 + \cdots + X_n)}} \to N(0,1)$$

Where N (0,1) donates standard normal distribution. Apply the central limit theorem to $Z_n \ln u$, we have

$$\frac{Z_n \ln u - E(Z_n \ln u)}{\sqrt{\mathrm{Var}(Z_n \ln u)}} \to N(0,1) = W_1$$

Here Wt donates one dimensional Brownian motion. Then

$$Z_n \ln u \to \sqrt{\mathrm{Var}(Z_n \ln u)} W_1 + W(Z_n \ln u) + E(Z_n \ln u)$$

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$$\to \sigma\sqrt{T}W_1 + \left(\mu - \frac{\sigma^2}{2}\right)T$$

The last equation is based on 3.2. Therefore:

$$S_n = S_0 EXP(Z_n \mathrm{lnu}) \to S_0 EXP(\sigma\sqrt{T}W_1 + \left(\mu - \frac{\sigma^2}{2}\right)T)$$

As n goes to infinity, from

$$ln\frac{S_n}{S_0} = ln\frac{S_{\frac{T}{n}}}{S_0} + ln\frac{S_{\frac{T}{n}2}}{S_0} + \cdots + ln\frac{S_{\frac{T}{n}n}}{S_{\frac{T}{n}(n-1)}} = ln\frac{S_{\Delta t}}{S_0} + ln\frac{S_{2\Delta t}}{S_{\Delta t}} + \cdots + ln\frac{S_{n\Delta t}}{S_{\Delta t(n-1)}}$$
$$= L_1 + L_2 + \cdots + L_N$$

Then, we have in

$$S_T = S_0 EXP(L_1 + L_2 + \cdots + L_N) = S_0 EXP(Z_n \mathrm{lnu})$$

In fact, we know and

$$S_T = S_{n\Delta t} = S_0 EXP(Z_{n\Delta t}\mathrm{lnu})$$

And

$$Z_{n\Delta t}\mathrm{lnu} = X_{\Delta t}\mathrm{lnu} + X_{2\Delta t}\mathrm{lnu} + \cdots + X_{n\Delta t}\mathrm{lnu}$$
$$= \widehat{X_1}\mathrm{lnu} + \widehat{X_2}\mathrm{lnu} + \cdots + \widehat{X_n}\mathrm{lnu}$$

Let $\widehat{X}_i = \widehat{X}_{i\Delta t}$ we have

$$p(\widehat{X_\imath} = 1) = p^* = \frac{1}{2} + \lambda\sqrt{\Delta t}$$

For convenience, we still donate $\widehat{X}_i$ as X_i

We derivate stock price can be expressed by one dimensional Brownian motion.

$$S_T = S_0 EXP(\sigma\sqrt{T}W_1 + \left(\mu - \frac{\sigma^2}{2}\right)T) \tag{3.3}$$

From 3.1 we know that the stock price is driven by random walk, thus we can explain the similarity between the graph of random walk and the graph of stock price.

References

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[3] Fischer, Hans. A history of the central limit theorem: from classical to modern probability theory. New York: Springer, 2011.

Appendix

This appendix is the python script to show image 1 in Chapter Two.

```
import matplotlib.pyplot as plt
import random
position = 0
walk = [ position ]
 steps = 500
for i in range ( steps ):
        step = 1 if random. randint (0, 1) else -1
        position = position+step
        walk.append( position )
fig = plt.figure ()
 ax= fig.add_subplot(111)
 ax. plot (walk)
p l t . show ( )
```