

Research on bearings-only passive positioning based on unmanned aerial vehicle formation

Genhua Zhang^{1,*,#}, Qingyao Liu^{2,#}, Siyan Xie^{1,#}, Yijing Chen^{1,#}

¹Maynooth International Engineering College, Fuzhou University, Fuzhou, China

²College of Physics and Information Engineering, Fuzhou University, Fuzhou, China

*Corresponding author: ricky0803@foxmail.com

#These authors contributed equally.

Abstract: This article focuses on the method of adjusting the position of unmanned aerial vehicles using bearings-only passive positioning. Based on the plane geometric relationship, a unmanned aerial vehicle positioning model, a circular queue adjustment scheme, and a conical queue adjustment scheme are established, and MATLAB is used to solve the known data. Firstly, the line connecting FY00 and FY01 is used as the polar axis of the polar coordinate system, and the sine theorem in the triangle is used to obtain the functional relationship between the polar diameter offset and polar angle offset with the angle received by the receiving drone. Based on this relationship, a positioning model for passive signal receiving drones can be established. The relative position relationship between the passive receiver and the two transmitters on the circumference, as well as the positive or negative offset, can lead to differences in the model. Expressions for the angle and offset in polar coordinates in four different situations can be obtained.

Keywords: Sine theorem; Greedy strategy; UAV positioning model; Bearings-only passive localization

1. Introduction

Unmanned aerial vehicle formation flying can more efficiently complete tasks in military reconnaissance, target strike, battlefield evaluation, and other scenarios. When attacking hostile targets, multiple combat drones flying in formation can simultaneously strike the same target from different angles, increasing their lethality. The drones in the queue need to collaborate and share location information by emitting electromagnetic waves from each other, so that the drones are in the appropriate position in the queue. To maintain the formation, it is planned to use a bearings-only passive positioning method to adjust the position of unmanned aerial vehicles. This involves transmitting signals from a few drones in the formation and passively receiving signals from other drones to extract directional information for positioning, in order to adjust the position of the drones. Each drone in the formation has a fixed number, and the relative position relationship with other drones in the formation remains unchanged. The direction information received by the drone receiving the signal is agreed as the angle between the drone and any two drones transmitting the signal. In order to reduce external electromagnetic interference, drones in the queue should emit as few electromagnetic wave signals as possible. Therefore, it is necessary to establish a mathematical model to search for more efficient collaborative positioning strategies for drones in the formation.

2. Model establishment and analysis

2.1 Model analysis

The direction information received by the drone receiving the signal is only the angle between the drone and any two drones transmitting the signal. Therefore, the form of positioning achieved by the drone receiving the signal is based on the received angle;^[1] The angle is crucial for solving this problem, so a polar coordinate system that meets the requirements of the problem can be established for solving. Using the line connecting FY00 and FY01 as the positive half axis of the polar coordinate system, the mathematical geometric relationship is used to obtain the functional relationship between the polar diameter offset and polar angle offset, as well as the angle received from the three transmitters. When the transmission point number is known, the polar diameter deviation and polar angle deviation of the receiver relative to its standard position in the formation can be obtained from the received data $(\alpha_1, \alpha_2, \alpha_3)$.^[2]

2.2 Model establishment

Firstly, with the drone FY00 as the pole and FY01 as the endpoint, connect FY00 and FY01, and use this ray as the positive half axis of the polar coordinate to establish a polar coordinate system,^[3] as shown in Figure 1. Assuming the polar diameter of the circle is R, one of the transmitting signal drones on the circle is FY01, and after converting it into polar coordinates, its corresponding polar coordinates are (R, 0).^[4]

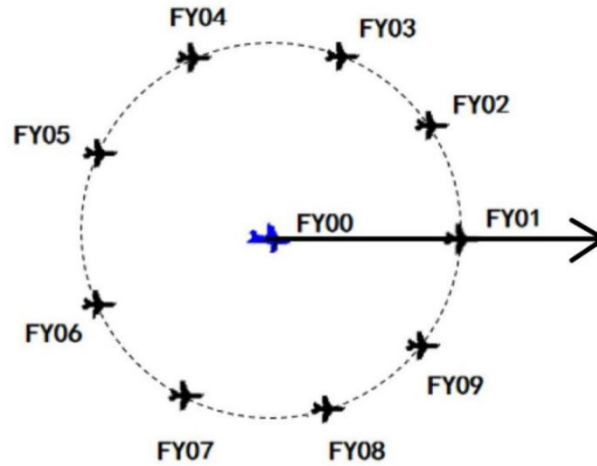


Figure 1: UAV Signal Emission Diagram

According to the question, the drone located at the center of the circle (FY00) and two other drones in the formation emit signals, while the drones with slight deviations in other positions passively receive signals. In this question, we assume that the drones transmitting signals are FY00 and FY01, and the other drone transmitting signals is set as FY0T. Due to the possibility that two launch drones on the circumference may be located on the same or different sides of the receiver, and the positive or negative pole angle deviation of the receiver to be located relative to its standard position in the formation may cause slight differences in the model, the four possible combinations are discussed below.

For the convenience of discussion, the position of the unmanned aerial vehicle FY0T transmitting signals on the circumference is set as point B, the position of the unmanned aerial vehicle FY00 transmitting signals on the center of the circle is set as point O, and the deviation position of the unmanned aerial vehicle receiving signals is set as E.^[5] Let the angle between the signals transmitted by the two unmanned aerial vehicles (FY00 and FY0T) transmitting signals be $\alpha_1 = \angle BEO$, and the angle between the signals transmitted by the two unmanned aerial vehicles (FY00 and FY01) transmitting signals be $\alpha_2 = \angle AEO$. Let the angle between the signals transmitted by the two unmanned aerial vehicles (FY00 and FY0T) transmitting signals to the receivers to be located at their respective positions in the formation be α_1 , and the angle between the signals transmitted by the two unmanned aerial vehicles (FY00 and FY01) transmitting signals to the receivers to be located at their respective positions in the formation be α_2 . In addition, the distance between transmitter 1 and receiver target position (j) is (frame):

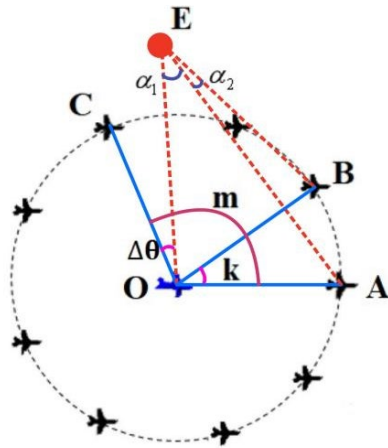
$$m = \begin{cases} (j-1), & j-1 < 5 \\ 9-(j-1), & \text{else} \end{cases} \quad (1)$$

The interval between transmitter 1 and FY0T (i) is (frame):

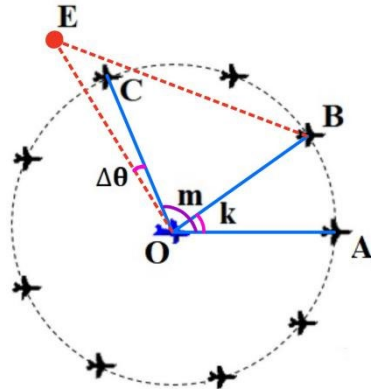
$$k = \begin{cases} (i-1), & i-1 < 5 \\ 9-(i-1), & \text{else} \end{cases} \quad (2)$$

After analysis, it can be concluded that at that time, the two transmitters on the circumference were located on the same side of the receiver to be located (assuming that there was only a slight deviation

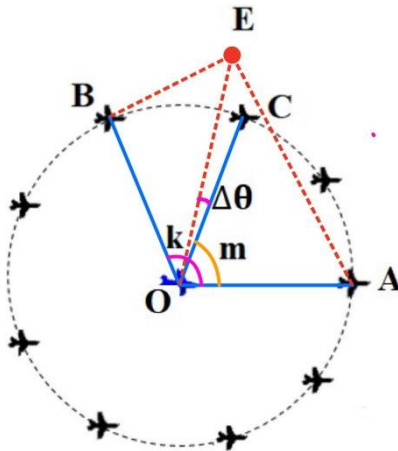
between the receiver and its target position).^[6] At that time, the two transmitters on the circumference were located on the opposite side of the receiver to be located.



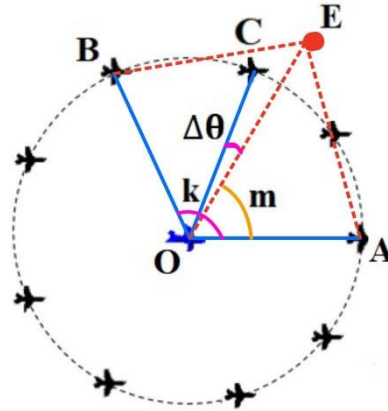
$$\begin{cases} m > k \\ \angle R_x < m * 40^\circ \\ \angle EOB = (m - k) * 40^\circ - \Delta\theta \\ \angle AOE = m * 40^\circ - \Delta\theta \end{cases} \quad (3)$$



$$\begin{cases} m > k \\ \angle R_x > m * 40^\circ \\ \angle EOB = (m - k) * 40^\circ + \Delta\theta \\ \angle AOE = m * 40^\circ + \Delta\theta \end{cases} \quad (4)$$



$$\begin{cases} m < k \\ \angle R_x > m * 40^\circ \\ \angle EOB = (k - m) * 40^\circ - \Delta\theta \\ \angle AOE = m * 40^\circ + \Delta\theta \end{cases} \quad (5)$$



$$\begin{cases} m < k \\ \angle R_x < m * 40^\circ \\ \angle EOB = (k - m) * 40^\circ + \Delta\theta \\ \angle AOE = m * 40^\circ - \Delta\theta \end{cases} \quad (6)$$

Observing the above four figures, the position of the drone FY0T transmitting signals on the circumference can be set as point A, the position of the drone FY00 transmitting signals on the center of the circle can be set as point O, and the deviation position of the drone receiving signals can be set as point E. Set the angle between the signals transmitted by the two unmanned aerial vehicles FY00 and FY0T as, and the angle between the signals transmitted by the two unmanned aerial vehicles FY00 and FY01 as. Set the angle between the signals transmitted by the two unmanned aerial vehicles FY00 and FY0T as The angle between the transmitting signals is, and the angle between the two unmanned aerial vehicles FY00 and FY0T transmitting signals to the deviated unmanned aerial vehicle is As shown in Fig 2; From this, the following values can be calculated:

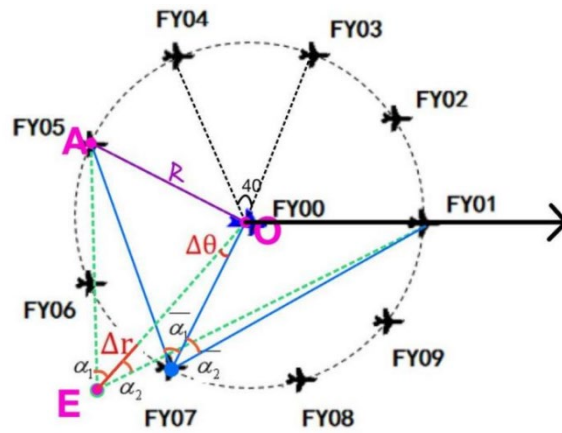


Figure 2: Drones transmitting signals to deviating drones

Deviation polar diameter in polar coordinate system

$$\Delta r = OE - R \quad (7)$$

Deviation angle in polar coordinate system $\Delta\theta$

$$\Delta\theta = m * 40^\circ - \angle AOE \quad (8)$$

Wherein

$$m = \begin{cases} |i - j|, & |i - j| \leq 4 \\ 9 - |i - j|, & |i - j| > 4 \end{cases} \quad (9)$$

(i, j are the numbers of the two transmitters on the circumference)

Given the mathematical expression of Δr and $\Delta\theta$, in ΔAOE , the mathematical expression of AE can be obtained from the cosine theorem:

$$AE = \sqrt{AO^2 + OE^2 - 2 * AO * OE * \cos \angle AOE} \quad (10)$$

Bring each value into the above equation:

$$AE = R^2 + (R + \Delta r)^2 - 2R(R + \Delta r) \cos[(i - j) * 40 - \Delta\theta] \quad (11)$$

Furthermore, the cosine theorem can be used to find α_1 :

$$\cos \angle \alpha_1 = \frac{OE^2 + AE^2 - AO^2}{2 * OE * AE} \quad (12)$$

$$\alpha_1 = \cos^{-1} \frac{OE^2 + AE^2 - AO^2}{2 * OE * AE} \quad (13)$$

Finally, obtain the numerical expression for α_1 :

Scenario 1:

$$\alpha_1 = \frac{2(\Delta r + R)^2 - 2R(R + \Delta r) \cos[(k) * 40 + \Delta\theta]}{2(R + \Delta r) * \sqrt{R^2 + (R + \Delta r)^2} - 2R(R + \Delta r) \cos[(k) * 40 + \Delta\theta]} \quad (14)$$

$$\alpha_2 = \frac{2(\Delta r + R)^2 - 2R(R + \Delta r) \cos[(m - k) * 40 - \Delta\theta]}{2(R + \Delta r) * \sqrt{R^2 + (R + \Delta r)^2} - 2R(R + \Delta r) \cos[(m - k) * 40 - \Delta\theta]} \quad (15)$$

Scenario 2:

$$\alpha_1 = \frac{2(\Delta r + R)^2 - 2R(R + \Delta r) \cos[(k) * 40 - \Delta\theta]}{2(R + \Delta r) * \sqrt{R^2 + (R + \Delta r)^2} - 2R(R + \Delta r) \cos[(k) * 40 - \Delta\theta]} \quad (16)$$

$$\alpha_2 = \frac{2(\Delta r + R)^2 - 2R(R + \Delta r) \cos[(m - k) * 40 + \Delta\theta]}{2(R + \Delta r) * \sqrt{R^2 + (R + \Delta r)^2} - 2R(R + \Delta r) \cos[(m - k) * 40 + \Delta\theta]} \quad (17)$$

Scenario 3:

$$\alpha_1 = \frac{2(\Delta r + R)^2 - 2R(R + \Delta r) \cos[(k - m) * 40 - \Delta\theta]}{2(R + \Delta r) * \sqrt{R^2 + (R + \Delta r)^2} - 2R(R + \Delta r) \cos[(k - m) * 40 - \Delta\theta]} \quad (18)$$

$$\alpha_2 = \frac{2(\Delta r + R)^2 - 2R(R + \Delta r) \cos[(k) * 40 - \Delta\theta]}{2(R + \Delta r) * \sqrt{R^2 + (R + \Delta r)^2} - 2R(R + \Delta r) \cos[(k) * 40 - \Delta\theta]} \quad (19)$$

Scenario 4:

$$\alpha_1 = \frac{2(\Delta r + R)^2 - 2R(R + \Delta r) \cos[(k - m) * 40 + \Delta\theta]}{2(R + \Delta r) * \sqrt{R^2 + (R + \Delta r)^2} - 2R(R + \Delta r) \cos[(k - m) * 40 + \Delta\theta]} \quad (20)$$

$$\alpha_2 = \frac{2(\Delta r + R)^2 - 2R(R + \Delta r) \cos[(k) * 40 + \Delta\theta]}{2(R + \Delta r) * \sqrt{R^2 + (R + \Delta r)^2} - 2R(R + \Delta r) \cos[(k) * 40 + \Delta\theta]} \quad (21)$$

3. Research on Position Deviation of Unmanned Aerial Vehicles

Assuming that the drone only deviates slightly from the target position, the closest target position can be selected by comparing the received angle information with the angle between the transmitter and all standard positions in the formation. After determining the target position again, it is necessary to determine the polar diameter and inclination deviation of the receiver from the target position. The number of transmitters on the required circumference needs to be determined by comparing the number of unknowns with the number of equations.^[7]

3.1 Biased receiving drone

Assuming that the receiving drone with slight deviation knows its own number, it is necessary to first determine the numbers of other transmitters except for FY00 and FY01. Assuming that there is only one transmitter with an unknown number on the circumference except for FY01, denoted as FY0T, the transmitter may be in all 7 standard positions on the circumference except for this receiver and FY01. The set of line angles ($\alpha_1, \alpha_2, \alpha_3$) from unknown signal sources (all possible transmitter positions) can be compared with the actual angle information received by the receiver ($\alpha_1, \alpha_2, \alpha_3$). Since the receiver position deviates slightly from its target position, the transmitter position corresponding to the element with the smallest deviation from ($\alpha_1, \alpha_2, \alpha_3$) in ($\alpha_1, \alpha_2, \alpha_3$) is the actual position of the transmitting drone.

The set of angles $\overline{\alpha_1}$ between the receiver (R) and the origin, as well as the line connecting FY01 (T1):

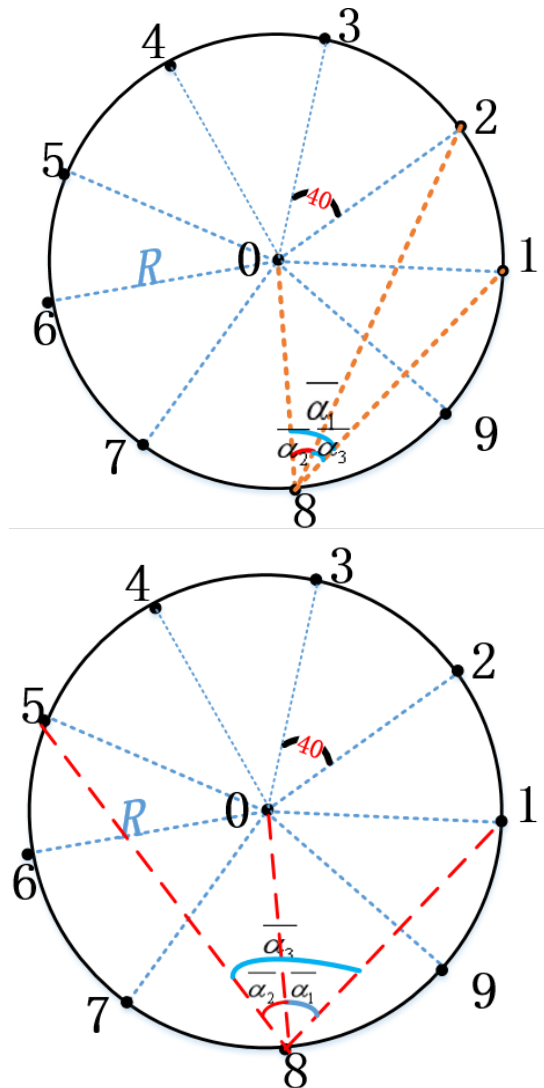
$$\overline{\alpha_1} = \{\overline{\alpha_{T_1RO}}\} \quad (22)$$

The set of angles $\overline{\alpha_2}$ between the receiver (R) and the origin, as well as the unknown number of transmitter FY0T(T_i) connections:

$$\overline{\alpha_2} = \{\overline{\alpha_{T_iRO}} \mid (1 < i < 10) \cap (i \neq R), i \in Z^+\} \quad (23)$$

The set of angles $\overline{\alpha_3}$ between the receiver and the connection between FY01(T_1) 0 and FY0T(T_i):

$$\alpha_3 = \{\overline{\alpha_{T_1RT_i}} \mid (1 < i < 10) \cap (i \neq R), i \in Z^+\} \quad (24)$$



$$\overline{\alpha_3} = \overline{\alpha_1} - \overline{\alpha_2}$$

$$\overline{\alpha_3} = \overline{\alpha_1} + \overline{\alpha_2}$$

From the figure, it can be seen that the angle between FY01 and the two connecting lines between the origin and receiver is $\overline{\alpha_1}$ for each transmission point. Due to the uniform distribution of the formation drone on the circumference,^[8] there are two transmitters with different standard positions that can allow the receiver to receive the same $\overline{\alpha_2}$. However, due to the different $\overline{\alpha_3}$ corresponding to the positions of these two transmitters, the source of the transmission signal can be determined through the three angle information.^[9]

4. Conclusion

A circular formation consisting of 10 drones is formed, with one drone located at the center of the circle and the other 9 drones evenly distributed around the circumference.^[10] Based on their own perceived altitude information, the drones maintain the same altitude for flight. The drone located at the center of the circle and two other drones in the formation transmit signals, while the other drones with slight deviations in position passively receive signals. When the drone transmitting the signal has no deviation in position and its number is known, it is required to establish a model to enable the drone passively receiving the signal to locate. A slightly deviated drone in a certain position receives signals from drones numbered FY00 and FY01, as well as signals from several numbered drones in the formation. If the drone's position is not deviated, in addition to FY00 and FY01, calculate how many more drones need to emit signals to achieve effective positioning of the drone. According to the formation requirements, one drone is located at the center of the circle, and the other nine drones are evenly distributed on a circle with a polar diameter of 100m. The initial position of the drone's polar coordinates has been given. When the position of the drone deviates slightly and the deviation of each drone is known, the drone with the number FY00 and up to three drones on the circle are selected each time to transmit signals, while the other drones receive signals and make adjustments. After multiple adjustments, finally, 9 drones were evenly distributed on a certain circumference.

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