ISSN 2616-5805 Vol. 7, Issue 1: 79-85, DOI: 10.25236/AJMS.2026.070111

Derive and prove a Completely New Maximum(minimum) Energy Formula of any Equilibrium System by the Variational Method

Anqi Lv*, Tingyu Rong

Southwest Jiaotong University, Jinniu District, Chengdu City, China
** Corresponding Author*

Abstract: *First we accurately derived the maximum (minimum) functional formula using the variational method. This is a brand-new maximum (minimum)functional formula in the world today. It solves how to construct the maximum (minimum) functional of the known problem and find the minimum functional required. In practice, we take two precise generalized variational principles of the great mathematicians Qian Weichang, Hu Haichang as examples. The calculation result is exactly the same with Qian Weichang, Hu Haichang variational principles. This indices that the functional maximum (minimum) formula is also a maximum (minimum) formula for calculating energy.Because the variational principle itself represents the minimum energy of a system. So we have found the calculation formula for the minimum energy of any system in the universe. The universe is so perfect, because every system in the universe is in a stable equilibrium of minimum energy. What important formula this is in the universe. now to cast a brick to attract jade. I hope everyone can research and apply it. Especially in the microscopic world. Because the derivation of the formula is entirely based on the formula of functional variation equals zero, and all tiny positive numbers are greater than zero. Of course, this article is limited to studying macro issues.*

Keywords: *The λ transformation of a functional; Independent variable function; maximum (minimum) energy formula*

1. Introduction

Due to the rapid development of finite element method,it is need of minimal functional of a equilibrium system. **See literature [4], [6].** "When known the field equation system and boundary condition equation system of known mathem" "atical and physical"problems, how to find corresponding minimal functional. Now, first time,we have exactly found "maximum (minimum) functional formula". Just need to substitute the field equation system and boundary condition equation system of a problem into the formula,the maximum (minimum) functional of the problem is obtained and the minimal functional can be obtained through the practical analysis method in this article, or by using mathematical methods. If the second-order variation of the functional>0 the resulting functional is a minimal functional. If the second-order variation,Multiply the functional by- 1 to obtain the minimum value of the functional.

2. Derive the Maximum (Minimum) Functional Formula

We start with Euler's research in 1696:

$$v=\int_{x_0}^{x_1} F[y(x),y'(x)]dx \tag{1}$$

This is a simple functional studied by Euler in that time, which is in an extreme state. Taking the variation of (a), we obtain:

$$\delta v=\int_{x_0}^{x_1} (F_y \delta y+F_{y'}\delta y')dx =0 \tag{2}$$

In the formula, y(x), y' (x) is a variable function, x_0, x_1 are the boundary points of the field. See literature [1] (p. 22). Introduced Definition 1: Independent variable function: Functional are functions of variable functions,when functional is varied with respect to a variable function, this variable function is called an independent variable function.

ISSN 2616-5805 Vol. 7, Issue 1: 79-85, DOI: 10.25236/AJMS.2026.070111

y is an independent variable function,when the boundary conditions are met, it can change in any function patter. y' is not, because y'=dy/dx, so y' can only change according to the relationship of dy/dx. According to the functional definition, y' is a functional of y(x). y', y''...y^n can be called sub functional. See literature [10] (p. 180). Sub functional are a special type of functional,their formation is just adding one or several differential symbols d /dx before the independent variable function y. Therefore, according to the definition of functional variation, the variation of sub functional can only be $\delta y'=\frac{\partial y'}{\partial y}\delta y$,...$\delta y^n=\frac{\partial y^n}{\partial y}\delta y$. It represents the linear principal part of the functional y ' difference.

Substituting $\delta y'=\frac{\partial y'}{\partial y}\delta y$ into (b)get: $\boldsymbol{\delta v}=\int_{x_0}^{x_1}[F_y+F_{y'}\frac{\partial y'}{\partial y})\delta y\mathbf{dx}=\int_{x_0}^{x_1}\frac{\partial F}{\partial Y}\boldsymbol{\delta} y dx=0$, According to the basic preparation theorem of Variational method,Since can vary arbitrarily, requires $\delta \boldsymbol{v}[y(x)]=0$,there must be:

$$\frac{\partial F}{\partial y}=F_y+F_{y'}\frac{\partial y'}{\partial y}=E=0 \tag{3}$$

The formula represents the condition that each variable function in the field should meet, that is, It is the field equation of the problem.It is not an Euler equation.Definition of functional variation according literature [1] (p. 13).

$$[\frac{\partial}{\partial\alpha}\mho[y(x)+\alpha\delta y]_{\alpha=0}=\frac{\partial\mho[y(x)+\alpha\delta y]}{\partial[y(x)+\alpha\delta y]}\frac{\partial[y(x)+\alpha\delta y}{\partial\alpha}\mid_{\alpha=0}=\frac{\partial\mho[y(x)+\alpha\delta y]}{\partial[y(x)+\alpha\delta y]}\delta y\mid_{\alpha=0}=\frac{\partial\mho[y(x)]}{\partial y(x)}\delta y$$

The form of and functions are the same, but the variables used for variation are different,but for variation,they are all as a real variable.Note that functional F only contains one **Independent variable function** y(x) in the formula (c). If it is a multivariate, then taking the derivative of each independent variable function and then adding them together. Let the functional be in a maximum (minimum) state:

$$\boldsymbol{\Pi}[\mathbf{y(x)}]=\int_v G[y(x)]dv+\int_\Sigma g[y(x)]d\boldsymbol{\Sigma}=\int_v G[y_0(x)]dv+\int_\Sigma g[y_0(x)]d\boldsymbol{\Sigma} \tag{4}$$

We only study functionals in this state. Π, G, and g have second-order variation.Functional G has i=1, 2,...n independent variable functions, and boundary g has j=1, 2,...k independent variable functions.V is the study field, and∑ is the surface of the study field V.Σ_σ is surface of surface forces act, and is the surface of displacement is limited.In this formula, there are integral of field V and integral of boundary surface,the object is in a extreme state under the combined action of boundary force, boundary displacement and physical force.Therefore, integral of study field and integral of boundary surface seem to be separate, but they are a whole.Introduce the parameter variable λ,λ is nothing to do with x and y, and let $\lambda y_0=y$.when the functional is in extreme state,y_0 form is fixed.For the variable λy_0,function form and properties of the functional are not changed.But for the variable. when $\lambda\neq1$, the functional is not in the extreme state, $\delta\Pi(\lambda y_0)\neq0$, only when $\lambda=1$, the functional is in extreme state,y_0 is an extreme curve in extreme state.λy_0 more truly reflects the situation of the functional is about to enter extreme state.When the functional is about to enter extreme state, the changing form of the independent variable function must change from completely arbitrary to enter the form of λy_0.After λy_0 substitution, formula (1) becomes:

$$\Pi[\lambda y_0(x)]=\int_v G(\lambda y_{0i})dv+\int_\Sigma g(\lambda y_{0j})d\Sigma \tag{5}$$

According to formula (c), calculate the derivatives of the functional independent variable functions in formula (5) one by one, and then add them together:

$$\delta\Pi[(\lambda y_0)]=\int_v\frac{\partial G(\lambda y_0)}{\partial(\lambda y_{0i})}\delta(\lambda y_{0i})dv+\int_\Sigma\frac{\partial g(\lambda y_0)}{\partial(\lambda y_{0j})}\delta(\lambda y_{0j})d\Sigma$$

This is tensor represent of sum of multiple terms. $\delta(\lambda y_{0i})<\varepsilon$, and parameter variabl λ is nothing to do with x and y,but λ is a real variable, so $\delta\lambda=d\lambda$,y_0 is the extreme curve, its curve shape is unchanged so $\delta Y_{0i}=0$.∴ $\delta(\lambda Y_{0i})=Y_{0i}\delta\lambda+\lambda\delta Y_{0i}=Y_{0i}d\lambda$.

$$\therefore\quad \delta\Pi[(\lambda y_0)]=v\frac{\partial G(\lambda y_0)}{\partial\lambda y_{0i}}\delta(\lambda Y_{0i})dv+\int_\Sigma\frac{\partial g(\lambda y_0)}{\partial\lambda y_{0j}}\delta(\lambda Y_{0i})d\Sigma=v\frac{\partial G(\lambda y_0)}{\partial\lambda y_{0i}}Y_{0i}d\lambda dv+\int_\Sigma\frac{\partial g(\lambda y_0)}{\partial\lambda y_{0j}}Y_{0j}d\lambda d\Sigma$$

$$=v\frac{\partial G(\lambda y_0)}{\partial\lambda y_{0i}}\frac{\partial\lambda y_{0i}}{\partial\lambda}d\lambda dv+\int_\Sigma\frac{\partial g(\lambda y_0)}{\partial\lambda y_{0j}}\frac{\partial\lambda y_{0j}}{\partial\lambda}d\lambda d\Sigma=\int_v dG(\lambda y_0)dv+\int_\Sigma dg(\lambda y_0)d\boldsymbol{\Sigma}=d\boldsymbol{\Pi}[\lambda y_0]$$

$$=\int_v E_i(\lambda y_0)\,y_{0i}d\lambda dv+\int_\Sigma\boldsymbol{\xi_j}(\lambda y_0)y_{0j}d\lambda d\Sigma \tag{6}$$

Because these differential dG, dg, and d**Π** in formula (3) are all for λ. Integrate the λ from 0→1 in

ISSN 2616-5805 Vol. 7, Issue 1: 79-85, DOI: 10.25236/AJMS.2026.070111

formula (6), for the integral concise, a definition is introduced:

Definition 2: The λ transformation formula of functional l(y,y') : The functional is integrated to λ from 0→1, where the formula of λ=1 is called the λ transformation formula l̃of functional ℓ(y,y').

y is an independent variable function, and its λ transformation formula is: $\frac{1}{2}$y.y' is not an independent variable function, but y'=$\frac{dy}{dx}$, then $\int_0^1 \frac{d\lambda y}{dx} d\lambda = \int_0^1 \lambda y' d\lambda$=.Therefore, the λ transformation formula of y' is $\frac{1}{2}$ y'.For equation (3) λ is integrated from 0→1: $\int_v \int_0^1 E_i(\lambda y_0) y_{0i} d\lambda dv = \int_v [\tilde{E}_i (y_0) y_{0i} - \tilde{E}_i(0) y_{0i}] dv$, $\int_\Sigma \int_0^1 \xi_j(\lambda y_0) y_{0j} d\lambda d\Sigma = \int_\Sigma [\tilde{\xi}_j(y_0) y_{0j} - \tilde{\xi}_j(0) y_{0j}] d\Sigma$, $= \Pi(y_0) - \Pi(0)$. Then

$$\Pi(y_0) - \Pi(0) = \int_v [\tilde{E}_i(y_0) y_{0i} - \tilde{E}_i(0) y_{0i}] dv + \int_\Sigma [\tilde{\xi}_j(y_0) y_{0j} - \tilde{\xi}_j(0) y_{0j}] d\Sigma$$

$$\therefore \quad \Pi[0] = \int_v \tilde{E}_i(0) y_{0i} dv + \int_\Sigma \tilde{\xi}_j(0) y_{0j} d\Sigma, \Pi[y_0] = \int_v [\tilde{E}_i(y_0) y_{0i}] dv + \int_\Sigma \tilde{\xi}_j(y_0) y_{0j} d\Sigma.$$

$$\text{Let} \quad \tilde{E}(y_0)^T = [\tilde{E}_1(y_0), \tilde{E}_2(y_0) \ldots \tilde{E}_n(y_0)], \quad \tilde{\xi}(y_0)^T = [\tilde{\xi}_1(y_0), \tilde{\xi}_2(y_0) \ldots \tilde{\xi}_k(y_0],$$

They are all row matrix.$y = [y_{01}, y_{02} \cdots y_{0Z}]^T$is a column matrix.In V,Z=n,On the border, Z=k.Then

$$\Pi[y_0(x)] = \int_v G[y_0(x)] dv + \int_\Sigma g[y_0(x)] d\Sigma = \int_v \tilde{E}_i(y_0) y_{0i} dv + \int_\Sigma \tilde{\xi}_j(y_0) y_{0j} d\Sigma = \int_v \tilde{E}(y_0)^T y dv + \int_\Sigma \tilde{\xi}(y_0)^T y d\Sigma.$$

Change to y in the formula. Therefore

$$\Pi[y(x)] = \int_v G[y(x)] dv + \int_\Sigma g[y(x)] d\Sigma$$

$$= \int_v \tilde{E}_i(y) y_i dv + \int_\Sigma \tilde{\xi}_j(y) y_j d\Sigma$$

$$= \int_v \tilde{E}(y)^T y dv + \int_\Sigma \tilde{\xi}(y)^T y d\Sigma \tag{7}$$

(A) IS functional Formula in maximum (minimum) State.that we exactly proved.

If you want to find the minimum functional of a problem,you must first find their E=0, ξ=0.And then find their λ transformation column matrix. substitute these equations into(7). The independent variable functions y are found from E=0,ξ=0, but how they correspond to these equations are unknown.That is,we don't know how to put in order y.We only need to replace them with the column matrix of Parameter variable,thus obtaining a functional with parameter variable expressed by.Take variation $\delta\overline{\Pi}$=0,according to $\delta\overline{\Pi}$=0,we got some equations,that can find these parameter variables.Therefore,from now on people knew how to construct the functional of the problem.But the functional found may be extremely large or small.But we want is an extremely small functional.Please see **literature** [7] "Chinese Teachers" 2026, No. 10, p. 135. And No. 11 (p. 144). There, we used the functional maximum (minimum) formula (7) to derive the HuHaichang Variational principle and the Qian Weichang variational principle. And the variational principle of a certain system represents the minimum energy of the system. That means that formula (7) is also a formula for calculating maximum (minimum) energy of the system.and is also a new method for establishing the variational principles.This is what an important formula in the world.

3. Analysis for the Euler equation

$$\text{Euler equation:} \quad F_y - \frac{d}{dx} F_{y'} = 0 \tag{8}$$

Equation (8) represents that a equation of the extreme functional F should satisfy. F can get it by integration. Obviously Euler didn't succeed. Euler didn't got maximum (minimum)functional formula.

See literature[1] (p. 19). But we got it right now.

4. Example

The two most famous and complex variational principles in the world come from China, both of which are exact solutions and with precise field equations and boundary condition equations.We will use precise existing variational principles as examples.

ISSN 2616-5805 Vol. 7, Issue 1: 79-85, DOI: 10.25236/AJMS.2026.070111

Example 1: Establish HuHaichang-Jiujin's variational principle for three types of variables of linear elasticity.

This was first proposed by Professor Hu Haichang of China in the mid-20th century.who obtained an exact solution using analytical methods, so there is only a result without a process.The debate was extremely heated, many people believed that his principle was only for two-class variables, not for Three class variables.Now, using our new formula, Check if it's two types or three types?

The field equations and boundary condition equations are as follows: see literature[4] (p. 408).

field equations:$E=\begin{bmatrix}\sigma_{ij,j}+\bar{F}_i\\ \sigma_{ij}-\alpha_{ijkl}e_{kl}\\ e_{ij}-\frac{1}{2}(u_{i,j}+u_{j,i})\end{bmatrix}=0$, boundary condition equations:$\xi=\begin{cases}u_i-\bar{u}_i=0 & \text{on } \Sigma_u.\\ \sigma_{ij}n_j-\bar{p}_i=0 & \text{on } \Sigma_\sigma.\end{cases}$

In the formula $\bar{F}_i$ represent body force, $\bar{u}_i$ already knows displacement, and $\bar{P}_i$ already knows boundary surface force,The elastic body is in a state of static force equilibrium, and the field equations and boundary condition equations are written in such an equilibrium state.In the formula, i=1,2,3, j=1,2,3, k=1,2,3, ℓ=1,2,3, $\alpha_{ijkl}$is the elastic constant at small deformation, and $n_j$is the cosine of the normal direction outside.Analyze the independent variable functions from these equations as,e_{ij}, u_i and obtain their λ transformation equations $\tilde{E}=\begin{bmatrix}\frac{1}{2}\sigma_{ij,j}+\bar{F}_i\\ \frac{1}{2}(\sigma_{ij}-\alpha_{ijkl}e_{kl})\\ \frac{1}{2}\left(e_{ij}-\frac{1}{2}u_{i,j}-\frac{1}{2}u_{j,i}\right)\end{bmatrix}$ inV, $\tilde{\xi}=\begin{cases}\frac{1}{2}u_i-\bar{u}_i & \text{On } \Sigma_u.\\ \frac{1}{2}\sigma_{ij}n_j-\bar{P}_i & \text{on } \Sigma_\sigma.\end{cases}$

Let the undetermined parametric function column matrix:$y=[\alpha_i, \mu_{ij}, \varepsilon_{ij}]^T$.On the boundary, then the functional with parameters is:

$$\bar{\Pi}[Y(X)]=\bar{\Pi}(Y)=\int_V\begin{bmatrix}\frac{1}{2}\sigma_{ij,j}+\bar{F}_i\\ \frac{1}{2}(\sigma_{ij}-\alpha_{ijkl}e_{kl})\\ \frac{1}{2}(e_{ij}-\frac{1}{2}u_{i,j}-\frac{1}{2}u_{j,i})\end{bmatrix}^T\begin{bmatrix}\alpha_i\\ \mu_{ij}\\ \varepsilon_{ij}\end{bmatrix}dv+\int_{\Sigma_\sigma}(\frac{1}{2}\sigma_{ij}n_j-\bar{P}_i)\beta_i d\boldsymbol{\Sigma}+\int_{\Sigma_u}(\frac{1}{2}u_i-\bar{u}_i)\gamma_i d\boldsymbol{\Sigma} \quad (9)$$

The variation of the functional with parameters is as follows:

$$\boldsymbol{\delta}\bar{\Pi}(Y)=\int_v[(\frac{1}{2}\sigma_{ij,j}+\bar{F}_i)\delta\alpha_i+\frac{1}{2}\alpha_i\delta\sigma_{ij,j}+\frac{1}{2}(\sigma_{ij}-\alpha_{ijkl}e_{kl})\delta\mu_{ij}+\frac{\mu_{ij}}{2}(\delta\sigma_{ij}-\alpha_{ijkl}\delta e_{kl})+\frac{1}{2}(e_{ij}-\frac{1}{2}u_{i,j}-\frac{1}{2}u_{j,i})\delta\varepsilon_{ij}+\frac{\varepsilon_{ij}}{2}(\delta e_{ij}-\frac{1}{2}\delta u_{i,j}-\frac{1}{2}\delta u_{j,i})]dv+\int_{\Sigma_\sigma}[(\frac{1}{2}\sigma_{ij}n_j-\bar{p}_i)\delta\beta_i+\frac{1}{2}\beta_i\delta\sigma_{ij}n_j]d\Sigma+\int_{\Sigma_u}[(\frac{1}{2}u_i-\bar{u}_i)\delta\gamma_i+\frac{\gamma_i}{2}\delta u_i]d\Sigma.$$

Simplified principles for the functional with parameters : a) Apply the partial integration method and Green's Formula to transform all variable function variations into independent variable function variations δu_i,$\delta\sigma_{ij}$,δe_{ij}.b) Cannot eliminate any one item, cannot eliminate equations equal to zero. c)The goal is to recover the field equations in body integration. Let $\alpha_i=u_i$ $\mu_{ij}=e_{ij}$,$\varepsilon_{ij}=\sigma_{ij.}$ And then:

$$\boldsymbol{\delta}\bar{\Pi}(Y)=\int_v[(\frac{1}{2}\sigma_{ij,j}+\bar{F}_i)\delta u_i+\frac{1}{2}u_i\delta\sigma_{ij,j}+\frac{1}{2}(\sigma_{ij}-\alpha_{ijkl}e_{kl})\delta e_{ij}+\frac{e_{ij}}{2}(\delta\sigma_{ij}-\alpha_{ijkl}\delta e_{kl})+\frac{1}{2}(e_{ij}-\frac{1}{2}u_{i,j}-\frac{1}{2}u_{j,i})\delta\sigma_{ij}+\frac{\sigma_{ij}}{2}(\delta e_{ij}-\frac{1}{2}\delta u_{i,j}-\frac{1}{2}\delta u_{j,i})]dv+\int_{\Sigma_\sigma}[(\frac{1}{2}\sigma_{ij}n_j-\bar{p}_i)\delta\beta_i+\frac{1}{2}\beta_i\delta\sigma_{ij}n_j]d\Sigma+\int_{\Sigma_u}[(\frac{1}{2}u_i-\bar{u}_i)\delta\gamma_i+\frac{\gamma_i}{2}\delta u_i]d\Sigma. \quad (10)$$

Note that : in the formula (10), $\frac{1}{2}u_i\delta\sigma_{ij,j}=(\frac{1}{2}u_i\delta\sigma_{ij})_{,j}-\frac{1}{2}u_{i,j}\delta\sigma_{ij}$.

In the same item, summation indices can be interchanged:

$\frac{1}{2}e_{ij}(-\alpha_{ijkl}\delta e_{kl})=-\frac{e_{ij}}{2}\alpha_{ijkl}\delta e_{kl}=-\frac{e_{kl}}{2}\alpha_{klij}\delta e_{ij}$,see literature[4] (p. 408) ∵ $\alpha_{ijkl}=\alpha_{klij}$.

∴ $-\frac{e_{kl}}{2}\alpha_{klij}\delta e_{ij}=-\frac{1}{2}\alpha_{ijkl}e_{kl}\delta e_{ij}=-\frac{1}{2}\sigma_{ij}\delta e_{ij}$.Without considering rigid rotation in small deformation $u_{i,j}=u_{j,}$see **literatur**e [5] p39. The last term of the body integral:

$$\frac{1}{2}\sigma_{ij}(-\frac{1}{2}\delta u_{i,j}-\frac{1}{2}\delta u_{j,i})=-\frac{1}{2}\sigma_{ij}\delta u_{i,j}=(-\frac{1}{2}\sigma_{ij}\delta u_i)_{,j}+\frac{1}{2}\sigma_{ij,j}\delta u_i.$$

Substituting these analyses into (A_{11}) gives: $\delta\bar{\Pi}(Y)=\int_v[(\frac{1}{2}\sigma_{ij,j}+\bar{F}_i)\delta u_i+(\frac{1}{2}u_i\delta\sigma_{ij})_{,j}-\frac{1}{2}u_{i,j}\delta\sigma_{ij}$

$+\frac{1}{2}(\sigma_{ij}-\alpha_{ijkl}e_{kl})\delta e_{ij}+\frac{1}{2}e_{ij}\delta\sigma_{ij}-\frac{1}{2}\sigma_{ij}\delta e_{ij}+\frac{1}{2}(e_{ij}-u_{i,j})\delta\sigma_{ij}+\frac{\sigma_{ij}}{2}\delta e_{ij}-(\frac{1}{2}\sigma_{ij}\delta u_i)_{,j}+\frac{1}{2}\sigma_{ij,j}\delta u_i]dv$

ISSN 2616-5805 Vol. 7, Issue 1: 79-85, DOI: 10.25236/AJMS.2026.070111

$+\int_{\Sigma_\sigma}[(\frac{1}{2}\sigma_{ij}n_j-\bar{p}_i)\delta\beta_i+\frac{1}{2}\beta_i\delta\sigma_{ij}n_j]d\Sigma+\int_{\Sigma_u}[(\frac{1}{2}u_i-\bar{u}_i)\delta\gamma_i+\frac{\gamma_i}{2}\delta u_i]d\Sigma.\because\sigma_{ij}=\alpha_{ijkl}e_{kl}$.combine like terms

$\therefore\ \delta\bar{\Pi}(Y)=\int_v[(\sigma_{ij,j}+\bar{F}_i)\delta u_i+(\sigma_{ij}-\alpha_{ijkl}e_{kl})\delta e_{ij}+[e_{ij}-\frac{1}{2}(u_{i,j}+u_{j,i})]\delta\sigma_{ij}]dv+\int_\Sigma\frac{1}{2}u_i\delta\sigma_{ij}n_jd\Sigma-\int_\Sigma\frac{1}{2}\sigma_{ij}n_j\delta u_id\Sigma$

$+\int_{\Sigma_\sigma}[(\frac{1}{2}\sigma_{ij}n_j-\bar{p}_i)\delta\beta_i+\frac{1}{2}\beta_i\delta\sigma_{ij}n_j]d\Sigma+\int_{\Sigma_u}[(\frac{1}{2}u_i-\bar{u}_i)\delta\gamma_i+\frac{\gamma_i}{2}\delta u_i]d\Sigma.$

Now considering the boundary conditions:let $\beta_i=-u_i,\gamma_i=\sigma_{ij}n_j$.

$$\therefore\ \delta\bar{\Pi}(Y)_{\Sigma_\sigma}=\int_\Sigma\frac{1}{2}u_i\delta\sigma_{ij}n_jd\Sigma-\int_\Sigma\frac{1}{2}\sigma_{ij}n_j\delta u_id\Sigma-\int_{\Sigma_\sigma}[(\frac{1}{2}\sigma_{ij}n_j-\bar{p}_i)\delta u_i+\frac{1}{2}u_i\delta\sigma_{ij}n_j]d\Sigma$$

Note that $\sum$ includes Σ_u, Σ_σ and the above three equations are added together to obtain:

$$-\int_{\Sigma_\sigma}(\sigma_{ij}n_j-\bar{p}_i)\delta u_id\Sigma.\text{Similarly:}+\int_{\Sigma_u}[(u_i-\bar{u}_i)\delta\sigma_{ij}n_j]d\Sigma.$$

Substitute into the above formula to obtain:

$$\delta\bar{\Pi}(Y)=\int_v[(\sigma_{ij,j}+\bar{F}_i)\delta u_i+(\sigma_{ij}-\alpha_{ijkl}e_{kl})\delta e_{ij}+[e_{ij}-(u_{i,j}+u_{j,i})]\delta\sigma_{ij}]dv$$
$$-\int_{\Sigma_\sigma}(\sigma_{ij}n_j-\bar{p}_i)\delta u_id\Sigma+\int_{\Sigma_u}[(u_i-\bar{u}_i)\delta\sigma_{ij}n_j]d\Sigma=0$$

According to the above equation, we can obtain: $\alpha_i=u_i$, $\mu_{ij}=e_{ij}$,$\varepsilon_{ij}=\sigma_{ij}$, $\beta_i=-u_i$, $\gamma_i=\sigma_{ij}n_j$,Substitute into equation (9) to obtain:

$$\Pi[Y(X)]=\int_v[(\frac{1}{2}\sigma_{ij,j}+\bar{F}_i)u_i+\frac{1}{2}(\sigma_{ij}-\alpha_{ijkl}e_{kl})e_{ij}+\frac{1}{2}(e_{ij}-\frac{1}{2}u_{i,j}-\frac{1}{2}u_{j,i})\sigma_{ij}]dv$$
$$-\int_{\Sigma_\sigma}(\frac{1}{2}\sigma_{ij}n_j-\bar{P}_i)u_id\Sigma+\int_{\Sigma_u}(\frac{1}{2}u_i-\bar{u}_i)\sigma_{ij}n_jd\Sigma \qquad (11)$$

(B_1) is the functional obtained from the maximum (minimum)functional formula, ~ The maximum (minimum) functional of the three types of variables in linear elasticity theory. Simplify: Because

$$\int_v\frac{1}{2}\sigma_{ij,j}u_idv=\int_v[(\frac{1}{2}\sigma_{ij}u_i)_j-\frac{1}{2}\sigma_{ij}u_{i,j}]dv.\ \because\ \int_v(\frac{1}{2}\sigma_{ij}u_i)_jdv=\int_\Sigma\frac{1}{2}\sigma_{ij}n_ju_id\Sigma.$$

Analyze these substitute into(B_1) obtain:

$\Pi[Y(X)]=\int_v-\frac{1}{2}\sigma_{ij}u_{i,j}+\bar{F}_iu_i+\frac{1}{2}(\sigma_{ij}-\alpha_{ijkl}e_{kl})e_{ij}+\frac{1}{2}(e_{ij}-\frac{1}{2}u_{i,j}-\frac{1}{2}u_{j,i})\sigma_{ij}]dv+\int_{\Sigma_\sigma}\bar{P}_iu_id\Sigma+\int_{\Sigma_u}(u_i-\bar{u}_i)\sigma_{ij}n_jd\Sigma,$

but $-\frac{1}{2}\alpha_{ijkl}e_{kl}\,e_{ij}=\frac{1}{2}\sigma_{ij}(-\frac{1}{2}u_{i,j}-\frac{1}{2}u_{j,i})$ .Substitute into the above formula

$\Pi[Y(X)]=\int_v-\frac{1}{2}\sigma_{ij}u_{i,j}+\bar{F}_iu_i+(e_{ij}-\frac{1}{2}u_{i,j}-\frac{1}{2}u_{j,i})\sigma_{ij}]dv+\int_{\Sigma_\sigma}\bar{P}_iu_id\Sigma+\int_{\Sigma_u}(u_i-\bar{u}_i)\sigma_{ij}n_jd\Sigma$

Multiplying the above equation by -1 gives:

$$\Pi[Y(X)]=\int_v[\frac{1}{2}\sigma_{ij}e_{ij}-\bar{F}_iu_i-(e_{ij}-\frac{1}{2}u_{i,j}-\frac{1}{2}u_{j,i})\sigma_{ij}]dv-\int_{\Sigma_\sigma}\bar{P}_iu_id\square-\int_{\Sigma_u}(u_i-\bar{u}_i)\sigma_{ij}n_jd\Sigma \qquad (12)$$

see literature[3] (p. 120), formula (12). **HuHaichang-Jiujin's variational principle**, which is completely consistent with (B_{11}).Theoretically proves that the HuHaichang variational principle is indeed a generalized variational principle with three types of variables u_i, σ_{ij}, and e_{ij}.The principle of minimum potential energy of the system is obtained by omitting the term equal to zero from the equation:

$$\Pi[Y(X)]=\int_v[\frac{1}{2}\sigma_{ij}e_{ij}-\bar{F}_iu_i]dv-\int_{\Sigma_\sigma}\bar{P}_iu_id\Sigma \qquad (13)$$

Only because multiplying -1 we obtain**HuHaichang-Jiujin's variational principle**, ~also is the principle of minimum energy. So in this time,functional from the maximum (minimum)functional formula is the maximum value of the functional.From we obtained the principle of minimum potential energy, we know the physical meaning of the maximum (minimum) functional formula is actually the maximum (minimum) energy of a system.What an important discovery. The functional minimum formula has been found. The calculation formula for the minimum energy of any system in the world has also been found.

Example 2: **Establish Qian Weichang variational principle for three types of variables in finite deformation elastic theory.**See **literature [3] (p. 257)**.

field equations and boundary condition equations are more complex as follows:

ISSN 2616-5805 Vol. 7, Issue 1: 79-85, DOI: 10.25236/AJMS.2026.070111

$$E=\begin{bmatrix} [(\delta_{ik}+u_{i,k})\sigma_{kj}]_{,j}+\bar{F}_i \\ e_{ij}-\frac{1}{2}(u_{i,j}+u_{j,i}+u_{k,j}u_{k,i}) \\ \sigma_{ij}-\frac{\partial A}{\partial e_{ij}} \end{bmatrix}=0,\ \xi=\begin{Bmatrix} u_i-\bar{u}_i=0, & (\delta_{ik}+u_{i,k})\sigma_{kj}n_j-P_i, & on\Sigma_u \\ & (\delta_{ik}+u_{i,k})\sigma_{kj}n_j-\bar{P}_i=0, & on\Sigma_\sigma \end{Bmatrix}$$

Note: On the displacement boundary, we calculate the unknown reaction force,so that the boundary conditions are comprehensive.

By applying the method shown in the functional maximum (minimum) formula step by step, as in Example 1. See **literature[3] (p. 271)**, formula (13). Finally found Qian Weichang's variational principle:

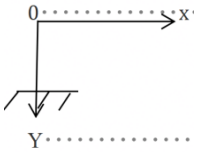
$$\Pi(X,y)=\int_v \{A-\bar{F}_i u_i-[e_{ij}-\frac{1}{2}(u_{i,j}+u_{j,i})-\frac{1}{2}u_{k,i}u_{k,j}]\sigma_{ij}\}dv-\int_{\Sigma_u}(u_i-\bar{u}_i)(\delta_{ik}+u_{i,k})\sigma_{kj}n_j\,d\Sigma-\int_{\Sigma_\sigma}\bar{p}_i u_i d\Sigma \quad (14).$$

(B_{13}) equal to formula (14). We got The principle of minimum potential energy by omitting the term equal to zero from the equation:

$$\Pi(X,Y)=\int_v [A-\bar{F}_i u_i]dv-\int_{\Sigma_\sigma}\bar{P}_i u_i d\Sigma.$$

where A represents the finite deformation strain energy.Practice is the sole criterion for testing truth!! The **Hu Haichang** variational principle is an exact solution obtained by analytical methods.The Chien Wei-zang variational principle is also an exact solution obtained by the Lagrange multiplier method, and the maximum (minimum)functional formula is a pure mathematical exact solution obtained by the variational method.Obtain two precise variational principles through the exact maximum (minimum)functional formula,so,it prove that they are all exact solutions. It further indicates that the maximum(minimum) functional formula actually is the formula for calculating the maximum minimum energy of any equilibrium system.This is such an important discovery. Those old scientists thought that strange constants could be calculated.

Example 3: S**oft landing problem of a spacecraft.**



When t = 0, v =v_0, the initial velocity of the raft.When t=T,v=0 the velocity of spacecraft lands.

Suppose the spacecraft moves with uniform deceleration downward along y, m represents the sum of the mass of the ship and the jet material. the m is a function of the time t, k is a constant. field equations of the problem:

$$\dot{m}+kf=0 \quad (15)$$

$$\dot{v}+f/m-g=0 \quad (16)$$

field equations (1),(2), See literature [9] (p. 2).

First, it must be checked whether these equations can ensure stability during the spacecraft's movement. ensure that the energy is always minimum during the spacecraft's motion. formulas (1) and (2) are yes or no the governing equations for motion stability? To achieve this goal, the maximum (minimum) functional formula (7) is applied.Two equations,two independent variable functions v, f.$\boldsymbol{\lambda}$ transformations of field equations (15), (16) are $\dot{m}+\frac{1}{2}kf$, $\frac{1}{2}\dot{v}+\frac{f}{2m}-g$.And then constitute the functional with parameters $\overline{\Pi}$:

$$\overline{\Pi}(t,f,v)=\int_0^T\left(\dot{m}+\frac{1}{2}kf\right)\alpha+\left(\frac{1}{2}\dot{v}+\frac{f}{2m}-g\right)\beta] \quad (17)$$

the variation of functional with parameters (3):

$$\delta\overline{\Pi}(t,f,v)=\int_0^T[\left(\dot{m}+\frac{1}{2}kf\right)\delta\alpha+\frac{\alpha}{2}\delta kf+(\frac{1}{2}\dot{v}+\frac{f}{2m}-g)\delta\beta+\frac{\beta}{2}\delta(\dot{v}+\frac{f}{m})]dt=0,$$

$$\text{Let } \alpha=kf,\ \beta=(\dot{v}+\frac{f}{m}).\delta\overline{\Pi}(t,f,v)=\int_0^T[(\dot{m}+kf)\delta kf+(\dot{v}+\frac{f}{m}-g)\delta(\dot{v}+\frac{f}{m})]dt=0$$

$$\delta^2\Pi(t,f,v)=\int_0^l[(\delta kf)^2+(\dot{m}+kf)\delta^2 kf+[\delta(\dot{v}+\frac{f}{m})]^2+(\dot{v}+\frac{f}{m}-g)\delta^2(\dot{v}+\frac{f}{m})]dt$$

ISSN 2616-5805 Vol. 7, Issue 1: 79-85, DOI: 10.25236/AJMS.2026.070111

$$=\int_0^T \{(\delta kf)^2+\left[\ \delta\left(\dot{v}+\frac{f}{m}\ \right)\right]^2\}\,d\,t\ >0$$

It can be seen that the functional obtained from field equations (1) and (2) is a minimum functional :

$$\Pi(t,f,v)=\int_0^T [\ \left(\dot{m}+\frac{1}{2}kf\right)kf+\left(\frac{1}{2}\dot{v}+\ \frac{f}{2m}-g\ \right)(\ \dot{v}+\frac{f}{m}\ )]dt \tag{18}$$

It is also the minimum value of the system energy. That the spacecraft flight will be stable. The field equations are the control equations for the stable flight of the spacecraft,so we can design according to field equations (1) and (2): The spacecraft is in uniform deceleration motion:

$\mathbf{V=v_0-\frac{v_0}{T}t,\ \dot{v}=-\frac{v_0}{T}}$.From (2) yields:$\mathbf{f=m(g-\dot{v})}=m\mathbf{(g\ -\frac{v_0}{T})}$,From (1) yields: $\mathbf{\dot{m}+km}(g-\frac{v_0}{T})\mathbf{=0,\ \dot{m}=\frac{dm}{dt},\ \frac{dm}{dt}=}$ $km(g-\frac{v_0}{T})$, $= k(g-\frac{v_0}{T})dt$.Integral from 0→T, $\int_M^{m_T}\frac{dm}{m}=\int_0^T\mathbf{-\ k}(g-\frac{v_0}{T})\mathbf{dt}$,$\mathbf{t=0,m=M}$

$$\mathbf{Ln\ \ m_T-Ln\ \ M=-\ k\left(g-\frac{v_0}{T}\right)T,\quad Ln\ \frac{m_T}{M}=-\ k(g-\frac{v_0}{T})T\ ,\qquad \frac{m_T}{M}=e^{-\ k(g-\frac{v_0}{T})T}}$$

$$\therefore\qquad m_T\ =\frac{M}{e^{(kgT-V_0)}} \tag{19}$$

Formula (19) represents the relationship that must be satisfied between and K,$\frac{f}{m}$,T.Only in this way the spacecraft can maintain flight stability and follow the predetermined flight path.To fly safely, equation (19) must be satisfied.When ensuring the safety of spacecraft flight,we can consider saving jet materials and other issues.

5. Summary

a) Maximum(minimum) functional formula (7) is a pure mathematics precise formula,so, it can be applied to linea,no linea,and incremental types,dynamics, statics problems.It is universal.

b) Maximum (minimum) functional formula (7) **is** also calculated minimum energy formula of any equilibrium system in the universe It is also the control equation that ensures the stability of the flying object during flight. That is also a new method for establishing variational principles.Note : when applying maximum (minimum) functional formula (7), the key is to have the true field equations and boundary conditions of the ploblems.

References

[1] Ailishigeerci, "variational method" (M). Beijing: Higher Education Press (1956).
[2] Lao Dazhong. Fundamentals of Variational Methods (3rd ed.). National Defense Industry Press(2015)
[3] Qinnwei Zhang. Multidisciplinary Academic Lecture Series (3) Generalized variational principle [M.. Shanghai Knowledge Publishing House (1985).
[4] Qinnwei Zhang.Variational Method and Finite Element [M] Volume 1,1980 Science Press
[5] Wang Longfu, Elasticity Theory [M] Science Press (1979)
[6] Ting-yu Rong, GENERALIZED MIXED VARIATIONAL PRINCIPLES AND NEW FEM MODELS IN SOLID MECHANICS. Int.J.Solids Structures Vol24.No.11.pp1131−1140. (1988) Printed in Great Britain.
[7] Teacher's Journal, 2026 Vol 10, pp. 135-136.
[8] Teacher's Journal,2026 Vol 11, pp. 144-146.
[9] "Optimal Control" - Theory and Applications [M]edited by Xie Xueshu, Tsinghua University Press, July 1986, First Edition
[10] Liang Lifu, Variational Principle and Its Applications[M],Harbin Engineering University Press, 2007