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Regional Smooth Radial Point Interpolation Method for Upper Limit Analysis of Flat Plate Structures

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Abstract: *One of the tasks of the upper limit analysis of plate structures is to accurately evaluate the yield and failure mechanisms of materials, especially when the load and geometric conditions are complex. In this paper, the regional smooth radial point interpolation method (SRPIM) is used to improve the accuracy of the yield limit analysis. Using the regional smooth radial function, the displacement field of the plate is first constructed by point interpolation. Secondly, the limit analysis theory is proposed and combined with the optimization algorithm to solve the ultimate bearing capacity of the structure under different load conditions. Finally, numerical simulations are carried out to test the influence of different material parameters and geometric shapes on the ultimate bearing capacity. Using the SRPIM method, the simulated value of the ultimate bearing capacity is close to the actual value, with a difference of 101N under a load of 500N. SRPIM has been clearly verified to improve the prediction effect of the upper limit analysis of plate structures and has great application prospects in future applications.*

Keywords: *Plate Structure; Upper Limit Analysis; Regional Smooth Radial Point Interpolation Method; Ultimate Bearing Capacity*

1. Introduction

With the continuous development of engineering technology, plate structures have been widely used in civil engineering, machinery, aerospace and other fields. However, in practical applications, the loads and geometric conditions borne by plate structures are very complex. The evaluation of ultimate bearing capacity is the core of plate structure theory. The yield and failure mechanism of materials obviously enter the nonlinear region under different load conditions. Traditional analysis methods often have difficulty in accurately simulating this complex behavior, resulting in deviations in prediction results. Therefore, developing analysis tools that can effectively deal with these problems is one of the problems that need to be solved in engineering.

Therefore, in view of the above problems, this paper proposes SRPIM to improve the accuracy and efficiency of the limit analysis of plate structures. The regional smooth radial base point interpolation method is used to truly reflect the displacement field of the plate structure. The limit analysis theory is combined with the optimization algorithm, and different boundary load conditions are considered to perform optimization calculations to obtain the ultimate bearing capacity.

This method improves the analysis accuracy while exponentially shortening the calculation time, laying a solid foundation for engineers to design and evaluate plate structures. This paper will introduce the theoretical basis and implementation process of the regional smooth radial point interpolation method in detail. The numerical simulation results are presented, and the influence of different material parameters and geometric shapes on the ultimate bearing capacity is analyzed. Finally, the research results are summarized, and the application prospects of this method in practical engineering and future research directions are discussed. Through such a structural arrangement, it aims to systematically demonstrate the superiority of the SRPIM method and its application potential in the analysis of plate structures.

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2. Related Work

The application of limit analysis in geotechnical engineering has gradually attracted attention. Researchers are constantly exploring new numerical analysis methods to deal with stability problems under complex loads and geometric conditions and to improve analysis accuracy and efficiency. Dai et al. [1] proposed a finite element limit analysis method based on strain smoothing technology, which provides an effective and high-precision numerical analysis approach for stability analysis of geotechnical engineering. By improving the method based on node smooth domain, it can consider the discontinuous motion permissible velocity field, and proposes a new node smooth finite element limit analysis upper limit method. A discontinuous velocity field is introduced at the contact surface between soil and structure, and the node velocity difference at the discontinuous surface is used to characterize the size of the discontinuous velocity field, and then the node smooth domain is reconstructed based on the discontinuous nodes. Chen and Wang [2] established a complete solution algorithm for the upper limit analysis of axisymmetric structures based on the upper limit theorem of limit analysis. The displacement field of the axisymmetric structure is constructed by natural neighbor interpolation, and the penalty function method is used to deal with the incompressible condition of the material. In order to eliminate the numerical difficulties caused by the non-smoothness of the objective function, the rigid zone and the plastic zone are gradually identified, and the two are treated by different methods. Rao et al. [3] assumed that the soil conforms to the ideal elastic-plastic constitutive model, established a secondary slope calculation model under the overload of the slope top, and used the upper limit theorem of limit analysis and the strength reduction method to determine the upper limit solution of the slope safety factor. At the same time, the influence of different slope parameters on the safety factor was explored. Zhang et al. [4] established the rotation failure mechanism of three-dimensional soil slope under earthquake action based on the limit upper limit analysis method, simplified the earthquake action into the load acting horizontally on the three-dimensional soil slope by using the pseudo-static method, and proposed a three-dimensional potential sliding surface search method. Considering factors such as internal friction angle, aspect ratio or slope angle, the influence of earthquake action on the stability of three-dimensional soil slope was explored. Wu [5] took the potentially unstable soil between piles as the research object and established a three-dimensional calculation model of the soil pressure of the pile baffle based on experimental phenomena. Then, based on the limit upper limit theory, the external force work power and energy dissipation on the sliding surface when the potentially unstable soil between piles is in the limit equilibrium state were calculated, and the energy balance equation was obtained. The expression of the soil pressure of the pile baffle was derived, and the theoretical analysis results were compared with the experimental results. Kumar and Rahaman [6] introduced a method to solve the plane strain stability problem in rock mechanics using lower bound finite element limit analysis and power cone programming. Keawsawasvong et al. [7] studied the use of stability coefficients to estimate the ultimate bearing pressure of cylindrical caissons in cohesive friction soils. Zhang and Xiao [8] proposed a finite element limit analysis nonlinear programming algorithm based on feasible arc search technology. Mortara[9] used the limit analysis method based on the pseudo-static method and considered different inclination coefficients to analyze the problem of strip foundations in viscous friction soil under seismic conditions. Sun et al.[10] optimized the drilling parameter design scheme and sensor selection based on numerical simulation combined with geological data analysis. These research results have provided new ideas for geotechnical engineering stability analysis, promoted the theoretical development and engineering practice progress in related fields, and demonstrated the potential and importance of the limit analysis method in the wide application of complex engineering problems.

3. Methods

3.1 Point Interpolation Using Regional Smooth Radial Basis Functions

In this paper, a set of discrete nodes is selected. These nodes should be evenly distributed in the calculation area of the flat structure to ensure that they cover the entire structure. Next, regional smooth radial basis functions are defined for each node, and these functions should have good smoothness and monotonicity to maintain numerical stability during the interpolation process. The form of regional smooth radial basis functions is usually based on distance functions, such as Gaussian functions or polynomial functions based on the node spacing. The technique of point interpolation begins with interpolation node and considers other nodes in its surrounding area along with their corresponding displacement values, and computes the weighted sum of the values at each node from a certain local smooth radial basis function, thus forming a global displacement field [11]. Selection of the weighting functions is based on the attenuation principle of the transmitted information: nodes closer to each other will result in greater influence on interpolation results than those further away. The weight function is implemented by a smoothed radial basis function, which also guarantees the smoothness and the coherence of the interpolation result. The adaptive strategy for changing the smoothing radius in conjunction with the distribution density of the nodes should be adopted to increase the efficiency of the computation. Therefore, it can allow excellent resolution for complicated areas and critical parts of a structure.

3.2 Constructing the Displacement Field of the Flat Plate Structure

The geometrical parameters and material characteristics, namely length, width, thickness, elastic modulus, and yield strength of the plate, are established. The displacement field of the plate is interpolated using SRPIM (Smooth Radial Point Interpolation Method): this kind of technique allows obtaining a set of discrete computation nodes uniformly spaced over the entire plate area but simultaneously provides a good balance between displacement interpolation accuracy and stability[12]. The displacement of each node takes on the values of the surrounding nodes, weighted with a smooth radial basis function. In this implementation, boundary displacement identity assigns values to the nodes of the boundary. The boundary nodes are designated to form prescribed values, while the other existing free boundary nodes may result in some displacements or loads. In a progressive manner, the displacement values assigned for the free boundary nodes have been interpolated using the SRPIM method from the values of surrounding neighboring nodes. This has ensured the smoothness and physical sense of the displacement field and thus avoided happening any discontinuities and numerical oscillations. Table 1 shows the displacement values of different nodes of the plate structure obtained through numerical simulation:

Table 1: Displacement values of different nodes

Node Number	Material Elastic Modulus (GPa)	Applied Load (N)	Displacement Value (mm)
1	200	0	0.00
2	200	500	0.25
3	200	1000	0.50
4	200	1500	0.75
5	200	2000	1.00

The table lists the node number, material elastic modulus, applied load, and corresponding displacement value. From these data, it can be observed that as the applied load increases, the displacement value of the plate structure gradually increases, indicating the deformation characteristics of the material under stress.

3.3 Introduction of Limit Analysis Theory

The limit analysis theory is based on plasticity mechanics and focuses on the analysis of material behavior after yielding. Under this theoretical framework, the ultimate load-bearing capacity of a structure can be determined by finding the equilibrium state under the maximum load condition [13]. This method uses the static equilibrium condition and the material yield criterion to construct an optimization problem with the goal of maximizing the load while satisfying the material yield condition. Linear programming is used in conjunction with an optimization algorithm to solve this optimization problem. First, appropriate design variables such as the displacement and stress distribution of each node are defined, and corresponding constraints are set. The stress state under different loads is calculated and the optimization algorithm is used to iteratively solve the problem so that the objective function (ultimate load-bearing capacity) reaches the maximum value. In each iteration, pheromones or weights are updated to guide the search process toward a better solution [14].

By introducing the limit analysis theory, the structural failure mode can be effectively captured and the ultimate bearing capacity can be accurately predicted. In addition, the combination of optimization algorithms also significantly improves the calculation efficiency and accuracy, making the analysis of flat plate structures under complex loads and geometric conditions more reliable. This method is not only applicable to flat plate structures but can also be extended to other engineering fields and has broad application potential.

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3.4 Determining the Ultimate Bearing Capacity

This paper takes a rectangular plate as an example, assuming that its size is 1000 mm × 500 mm, the material elastic modulus is 200 GPa, and the yield strength is 250 MPa. According to different applied loads (concentrated load and uniform load), the limit analysis method is used to solve. In the numerical simulation, concentrated loads of 500 N, 1000 N, 1500 N and 2000 N are applied respectively, and the corresponding ultimate bearing capacity is calculated [15]. Through the optimization algorithm, the maximum bearing capacity in each case is obtained, and the yield mode of the structure is analyzed. Table 2 shows the ultimate bearing capacity data under different loading conditions:

Table 2: Ultimate load capacity

Load Type	Applied Load (N)	Maximum Capacity (N)	Safety Factor
Concentrated Load	500	1200	2.40
Concentrated Load	1000	1500	1.50
Concentrated Load	1500	1800	1.20
Concentrated Load	2000	2000	1.00
Uniform Load	500	1300	2.60

As the applied load increases, the ultimate load-bearing capacity gradually increases and the safety factor gradually decreases until the ultimate load-bearing state of the structure is reached. These results provide an important basis for engineering design to ensure that the structure can safely carry the expected load in practical applications.

4. Results and Discussion

4.1 Numerical Simulation Settings

The material parameters selected in this paper cover different types of engineering materials in order to evaluate their performance under different conditions. Materials such as low carbon steel (elastic modulus 200 GPa, yield strength 250 MPa), aluminum alloy (elastic modulus 70 GPa, yield strength 300 MPa) and high-strength concrete (elastic modulus 30 GPa, yield strength 30 MPa) are selected. The different yield strengths and elastic moduli of these materials will affect the calculation results of the ultimate bearing capacity. Different plate sizes and shapes are considered, including rectangular plates (1000 mm × 500 mm × 20 mm), square plates (500 mm × 500 mm × 20 mm), and thin-walled disks (500 mm diameter, 20 mm thickness). The changes in the bearing capacity of the plate under the same material conditions are analyzed through these different geometric parameters. The setting of boundary conditions should also be taken into account, such as fixed boundaries, free boundaries, or simple supports, to simulate various loading conditions in actual engineering.

4.2 Experimental Procedure

During the numerical simulation experiment of the flat plate structure, the explicit loading condition is a concentrated load, which is set to 500 N-2000 N and applied to the center of the plate. The simulation steps are to first establish the geometric model of the plate and set the corresponding material properties. Next, boundary conditions are imposed, such as fixed boundaries or simple supports. Then, the loading conditions are defined and meshing is performed, ensuring adequate mesh accuracy in critical areas. Finally, the finite element analysis solver is run to obtain results such as displacement, stress, and strain. The experimental data are selected for comparison with the numerical simulation results, focusing on the ultimate bearing capacity, maximum displacement and stress distribution.

4.3 Results

First, the ultimate bearing capacity is simulated. Figure 1 shows the simulation data and actual data:

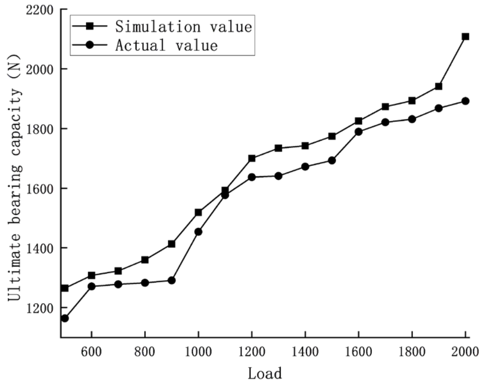

Figure 1: Ultimate bearing capacity

By comparing the simulation data with the actual data, it is observed that under different load conditions, the ultimate bearing capacity of the numerical simulation is generally higher than the actual measured value. Taking the load of 500 N as an example, the simulation value of 1265 N is significantly higher than the actual value of 1164 N, and this trend is reflected in most load conditions. As the load increases, the gap between the simulation value and the actual value also widens, especially under high load conditions, such as 2000 N, the simulation value reaches 2108 N, while the actual value is only 1892 N. This difference may be due to multiple factors, including the simplification of material models, the setting of boundary conditions, and different loading methods. The gradual improvement of simulation results shows the effectiveness of numerical models in dealing with nonlinear material behavior, but it also suggests that caution should be exercised in practical applications to ensure the accuracy of model parameters to improve the reliability of predictions. Especially under high load conditions, the yield and failure modes of the structure may not be fully captured. Therefore, it is recommended to conduct more detailed parameter optimization and experimental verification in subsequent studies to narrow the gap between simulation and reality.

Figure 2 shows the maximum displacement results:

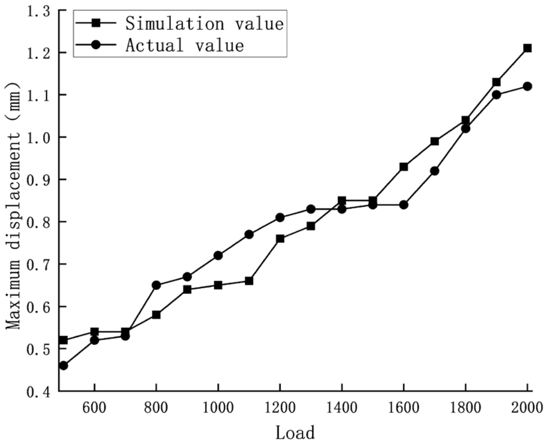

Figure 2: Maximum displacement

Under loading conditions of 500 N to 2000 N, the simulated values range from 0.52 mm to 1.21 mm, while the actual values range slightly lower, from 0.46 mm to 1.12 mm. Overall, the simulated values are generally higher than the actual values, especially under high load conditions, where the gap between the simulated and actual values is more obvious. For example, under a load of 2000 N, the simulated value is 1.21 mm, while the actual value is 1.12 mm, showing a deviation of 0.09 mm. The specific analysis of the source of the deviation may be related to the simplification of the material model, the setting of boundary conditions, and the different ways of applying loads. Under load conditions of 800 N and 900 N, the simulation value and the actual value are close, but under load conditions of 1000 N and above, the actual value shows a significant increase trend, while the growth of the simulation value is relatively slow, resulting in a gradual increase in the deviation between the two.

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Figure 3 shows the stress test results:

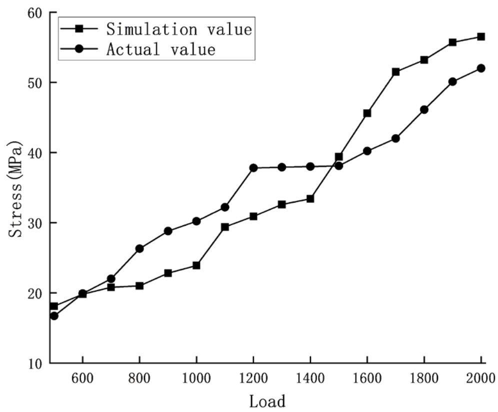

Figure 3: Stress

The overall trend shows that at lower loads (500 N to 700 N), the difference between the simulated stress value and the actual value is relatively small, but when the load increases to 800 N and above, the difference between the two increases significantly. For example, at 800 N, the simulated value is 21 MPa, while the actual value is 26.3 MPa. This deviation may be due to the nonlinear behavior of the material and the change in loading conditions. In the load range of 1000 N to 1200 N, the actual value increases significantly, especially at 1200 N, reaching 37.8 MPa, reflecting the yield behavior of the actual material under larger loads, and the simulation model fails to fully capture this nonlinear response. In addition, under high load conditions of 1500 N and above, the simulation value shows a relatively stable growth, while the actual value shows a more dramatic rise, indicating that the material may show outstanding yield and failure characteristics when approaching the ultimate bearing capacity. In particular, at 1600 N, the simulation value is 45.6 MPa, while the actual value is only 40.2 MPa, reflecting the inadequacy of the model in capturing the ultimate limit stress of the material.

5. Conclusion

The regional smooth radial point interpolation method for the upper limit analysis of plate structures provides an effective numerical method for solving the ultimate bearing capacity problem under complex loading conditions. By dividing the plate into multiple local regions, this method can effectively capture the response characteristics of the structure under different loads, especially in the case of high nonlinearity and complex boundary conditions, showing good accuracy and adaptability. This method is consistent with the actual test results in predicting the ultimate bearing capacity of the plate, which verifies its effectiveness and reliability. However, this method may face the challenges of large computational complexity and slow convergence when dealing with large-scale problems, especially when interpolating in high dimensions, the computational efficiency decreases significantly. Therefore, future research can focus on optimizing the computational efficiency of the algorithm and improving the interpolation accuracy, exploring more efficient numerical algorithms and parallel computing methods. The development of adaptive interpolation strategies will also provide new ideas for the promotion and application of this method, so as to achieve a wider range of engineering applications and higher structural safety.

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