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Research on Application of Multifractal Spectrum and Neural Network in Financial Market

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Abstract: *With the deepening of the financial market's reform and innovation, to better explain the market's stock price fluctuations, this article uses the traditional efficient market hypothesis to study further. This study is showed that the Shanghai Stock Exchange Index, the Shenzhen Stock Exchange Index, and the Nasdaq Index have the characteristics of "tip fat tail ", which indicates that the distribution is not normal. Based on the theory of multifractal spectrum, empirical analysis and comparison results of multifractal characteristics, three indexes are carried out to analyze the impact of the maturity of different financial markets on stock price fluctuations. Finally, the multifractal spectrum parameters and daily return rate as input variables, an average accuracy rate of 99.436%, are obtained for the Shanghai Composite Index forecast for the next 30 days, using a 5-layer neural network model. It has particular practical significance for controlling and managing financial risks.*

Keywords: *Multifractal, MF-DFA, multifractal spectrum, neural network, financial market*

1. Introduction

While the financial market plays a vital role in the smooth operation and development of a country's economy, many studies have proved. The classical efficient market hypothesis is difficult to explain many phenomena in the actual financial market, and the financial market is a nonlinear system [1]. As an essential branch of nonlinear scientific research, the fractal theory has positive significance for explaining and predicting the complex volatility trend of financial markets, which reveals its essential function [2].

At present, researchers at home and abroad have investigated the financial market, especially the stock market, relying on the fractal as a new mathematical tool. Some studies have only studied one fractal dimension to describe the fractal structure of the stock market. Most studies are still in the empirical test on the fractal characteristics of the price fluctuation curve of financial products. Because of the current situation, the main contributions of this article are as follows:

(1) It is applied the multifractal analysis to eliminate trend fluctuations (MF-DFA) [3]. Research on the multifractal characteristics of the financial markets between the United States and China (based on the Nasdaq index, Shanghai Stock Exchange Index, and Shenzhen Stock Exchange Index) are analyzed. The relationship between multifractal characteristics and market maturity has summarized China's financial market deficiencies compared with mature markets in developed countries.

(2) It is obtained the research object's high-frequency data, which is used a neural network and multifractal spectrum to predict the stock price of the next 30 days. It has practical significance for preventing or controlling risks.

2. Principle explanation

2.1 Multiple analysis

The multifractal process describes the local fractal characteristics of time series through time-varying parameters (such as generalized Hurst index, generalized fractal dimension, local Holder index, and multifractal spectrum). It can be described the complex statistical characteristics of price changes in the financial market. It is described in mathematical language as follows:

The incremental sequence of the random process $\{x(t)\}$ satisfies the scale relationship:

$$|x(t+\triangle t)-x(t)| \sim C_t(\triangle t)^{\alpha(t)}$$

Divide the interval [0, T] into b^k sub-interval, $N_k(\boldsymbol{\alpha})$ For include $\boldsymbol{\alpha}$(t) Number of subintervals, if $f(\alpha)=\lim_{k\to\infty}\frac{\log N_k(\alpha)}{\log_b k}$ is larger than the range of a point and takes a positive number, then x (t) is a multifractal process.

2.2 MF-DEA Method

Multifractal elimination of trend fluctuation analysis method (MF-DFA method) is a multifractal analysis method proposed by Kantelhardt. This paper has used this method to verify and analyze the multifractal characteristics of the Shanghai stock index, Shenzhen stock index and Nasdaq index.

It is assumed that the time series corresponding to the research object is a sequence $\{x_i\}$ with length N, the MF-DFA method is used to process it. The specific steps are as follows:

(1) Find the average value of the data, and use the sum of the deviations of each data and the average value to construct a new sequence:

$$y(i)=\sum_{k=1}^{i}[x_k-\bar{x}], i=1,2,3,\dots,N$$

(2) The new sequence is divided into $N_s=int\left(\frac{N}{s}\right)$ disjoint intervals with a length of s. If N is not an integer multiple of s, its data is inversely divided to obtain $2N_s$ intervals to ensure the existence of all data.

(3) The local trend of each interval is fitted by the least square method, and the residual sequence is obtained by eliminating the interval. Then, the average value of the subinterval sequence of eliminating the trend in these intervals is calculated, and the q-order fluctuation function is obtained.

(4) Fix the order q to find its logarithm and it is drawn the functional relationship diagram of $\ln(F_q\ (s))\sim\ln s$, with the slope of $h(q)$

(5) $h(q)$ and Renyi index $\tau(q)$ were obtained by the MF-DFA method. The correlation formula is $\tau(q)=qh(q)-1$

The singular exponent is obtained after Legendre transformation α and multifractal spectrum f(α): $\alpha=h(q)+qh'(q)\quad f(\alpha)=q[\alpha-h(q)]+1$

3. Data source and processing

The analysis data of this paper is selected the Shanghai Stock Exchange composite stock price index (hereinafter referred to as "Shanghai stock index"), Shenzhen Stock Exchange stock price composite index (hereinafter referred to as "Shenzhen stock index") and Nasdaq composite index (hereinafter referred to as "NDAQ"). They are highly influential indexes in China's and the United States stock markets, which represents the general nature of Chinese and American stocks, respectively. The sample data of this study are from the Tushare website. The time range of the Shanghai index and Shenzhen index is from January 2, 1992, to December 30, 2020; The NDAQ time range is from July 1, 2002, to December 30, 2020. Since the statistical nature of the logarithmic return is better than the relative return, the logarithmic return is obtained from the original sequence of daily closing price, which is used for empirical analysis. The conversion formula is: $r_t=\ln(P_t)-\ln(P_{t-1})$ where t is the ranking of trading days in the sample data and P_t is the closing price corresponding to the T trading day.

Here, the normal distribution is taken as the reference standard, and the yield probability density diagram of yield is illustrated in Figure 1.

The returns of the Shanghai stock index, Shenzhen index and NDAQ obey the normal distribution in the interval of (- 0.05, 0.05), (-0.025, 0.025) and (- 0.025, 0.025) respectively, and there is an essential difference from the normal distribution outside the interval. The left lower end of the curve tilts upward, and the right upper end tilts downward, which is in line with the intuitive result of "sharp peak and fat tail" in the probability density diagram, as shown in Figure1. In addition, the right tail of the Shanghai stock index is significantly fatter than the left tail, and it is also significantly faster than the tail of the yields of the other two indexes.

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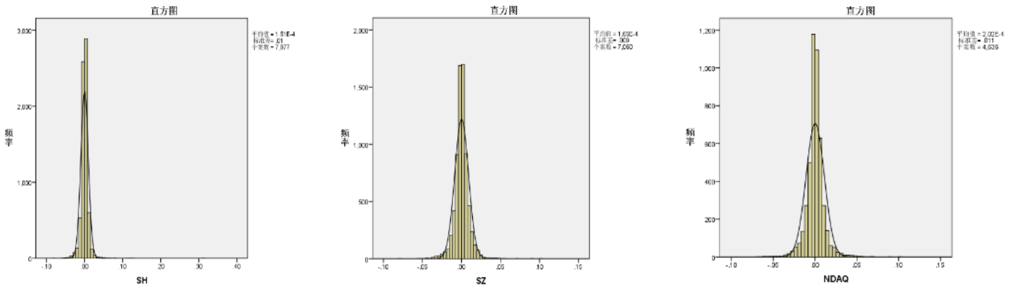

(a) Shanghai stock index (b) Shenzhen stock index (c) NDAQ

Figure 1: Histogram of Daily Yield

To sum up, in the sample period, China's stock market volatility is greater than that of the United States, and the volatility of stock index return of China's representative and well-developed enterprises is similar to that of the United States. The "fat tail" of China's stock market may attract more obvious risk preference.

4. Empirical analysis of multifractal characteristics

4.1 Degree test of multifractal features

(1) q is the order of multifractal structure, and different q is to produce different h(q). If h (q) is a constant, the sequence is a single fractal. If h (q) is a function of q, the sequence is multifractal with the change of q. The more remarkable change of h(q), the more significant its multifractal characteristics are.

(2) Singular index $\boldsymbol{\alpha}$(t) is the local Holder index, which describes the uneven price fluctuation in different parts. The difference between its maximum and minimum can well describe the intensity of multifractal characteristics. f (α) is a multifractal spectrum. The larger its width is, and the more significant its multifractal characteristics are.

4.2 Result analysis of the multifractal spectrum

The fractal parameter results and rearrangement results of the three stock index returns are as follows:

Table 1: Results of fractal parameters of the index return

parameters	$\boldsymbol{\alpha_{min}}$	$\boldsymbol{\alpha_{max}}$	$\boldsymbol{\Delta\alpha}$	$\boldsymbol{f(\alpha_{min})}$	$\boldsymbol{f(\alpha_{max})}$	$\boldsymbol{\Delta f}$
Shanghai stock	0.1700	0.9040	0.7340	0.3330	-0.1040	0.4370
Shenzhen stock	0.4555	0.7813	0.3258	0.6090	0.2600	0.3490
NDAQ	0.2784	0.6318	0.3534	0.4930	0.2430	0.2500

Table 2: Index return rate rearranges the results of each fractal parameter

parameters	$\boldsymbol{\alpha_{min}}$	$\boldsymbol{\alpha_{max}}$	$\boldsymbol{\Delta\alpha}$	$\boldsymbol{f(\alpha_{min})}$	$\boldsymbol{f(\alpha_{max})}$	$\boldsymbol{\Delta f}$
Shanghai stock	0.1540	0.6990	0.5450	0.6350	-0.1190	0.7540
Shenzhen stock	0.4044	0.6783	0.2739	0.4650	0.4070	0.0580
NDAQ	0.2775	0.6853	0.4078	0.2320	0.3100	-0.0780

From the initial data results, it can be found that the h(q) of the three indexes are functions of q and are not a constant value, which can explain that the three indexes have multifractal characteristics. The change and h(q) of Shanghai stock index $\boldsymbol{\Delta\alpha}$ is compared with the Shenzhen index and NDAQ, and it is shown that the Shanghai index's multifractal characteristics are significant, and its risk is also considerable. Since the Shanghai stock index includes all the stocks of companies listed on the Shanghai Stock Exchange, and Shenzhen stock exchange is the stock of 500 representative companies listed on the Shenzhen Stock Exchange, the risks of the two are large, and the verification results are in line with the actual conjecture. Compared with NDAQ, which is a developed stock market, China's stock index lacks effectiveness. It belongs to a new financial market, and the risk should be more significant [4].

The fractal parameters of the three indexes have changed from the rearrangement results, and the multifractal characteristics are relatively weakened. The changes of fractal parameters of the Shanghai

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stock index are greater than those of the Shenzhen stock index and NDAQ. It can be seen that the correlation of the Shanghai stock index series is weak, which also shows that the sequence correlation is one of the reasons caused by multifractal characteristics.

5. Prediction of the stock price index based on neural network

5.1 Design of neural network structure

Based on the above empirical analysis, it is built the following neural network model to predict the stock index:

(1) Model type: it is selected the 5-layer neural network with 128, 512, 256, 128, and 64 nodes, respectively.

(2) Transfer function: Leaky ReLUs function is selected as the activation function, and Leaky ReLUs gives a non-zero slope to all negative values.

(3) Input and output variables: according to the results of empirical research studies, multifractal spectrum parameters and stock index return are selected as input variables;

(4) Training and prediction data: The first 441 data of Shanghai and Shenzhen indexes from January 2, 2009, to December 31, 2020, for training, and the last 14 data for prediction to predict the data of the next day.

5.2 Network training and prediction results

The training effect and prediction results of the neural network are shown in the Figure 2.

Figure 2: Training effect and prediction result of neural network

It can be seen from the Figure 2 that the prediction effect of this prediction model is good in the short term. The overall trend is the same as the actual value, but it is also almost completely fitted with the actual value at some peaks. Due to the complex influencing factors of the stock market, it is difficult to accurately predict the stock price's long-term trend. There is a particular deviation from the long-term prediction results, resulting in higher errors. It is predicted that the 30 days mean-variance of the Shanghai Stock Exchange is 630.792246743667, and the Shenzhen Stock Exchange is 16502.892873302826. In order to verify and quantify the accuracy of the prediction results, the Shanghai stock index is taken as a

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typical example. The output value of the neural network is processed. The arithmetic average of the accuracy is about 99.436%, and the arithmetic average of the error rate is about 0.564%. The neural network model based on the multifractal spectrum has achieved good predicting the stock price index. The arithmetic average of the accuracy is about 99.436%, the arithmetic average of the error rate is about 0.564%, and the lowest accuracy is 98.071%. It is shown that the model can be used better to simulate the short-term overall stock market trend and predict with a certain probability.

6. Conclusion

The operation of the financial market is unpredictable and intricate. However, this article proves that it has multifractal characteristics and has specific laws to follow, which can be predicted to some extent. Based on many actual data and multifractal empirical research results, it is constructed a five-layer neural network model with good forecasting effect, high accuracy and a small error to predict the country's representative stock indexes, which has achieved good empirical results. To further increase the accuracy and timeliness of prediction, it is depended on the research and application of new methods. For example, the further combination of wavelet technology, time series, neural network, and multifractal is one of our future research directions.

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