

Comparison and Analysis of Measurement Methods of Total Factor Productivity

Dandan Li^{1,*}, Doudou Li²

¹ Business School ,Northwest University of political science and law, Xi'an 710122, China

²Business School ,Northwest University of political science and law, Xi'an 710122, China

*Corresponding author e-mail:happydans@163.com

ABSTRACT. *The traditional method of measuring TFP is mainly to use provincial or industrial data from the macro level. However, when estimating the total factor productivity of an enterprise, the traditional methods will produce endogenous problems, which will lead to inconsistencies in the estimation results. Through sorting out and comparing the advantages and disadvantages of the latest estimation methods of micro production function, the results show that: OP model and LP model of semi parametric method can solve the problems of simultaneous bias and sample selection bias; ACF model setting is more general than OP model; De Loecker model can solve the estimation of TFP in incomplete competitive market. Therefore, it is very important to choose a suitable TFP estimation method.*

KEYWORDS: TFP, enterprise; estimation method, production function

1. Introduction

Since the reform and opening up, China's economy has grown rapidly and entered the new normal development. According to the accounting framework proposed by Solow (1956), the improvement of economic performance is actually the decision of total factor productivity (simply called TFP) on economic performance. TFP growth has become the source of economic growth in the new normal. Scholars at home and abroad have studied TFP in various regions of China from different perspectives (Hsieh and Klenow, 2009; Qiwei Zhao et al., 2016; Tang Li et al., 2016; Qiao Fan and Aijun Guo, 2019) [1-4]. However, due to the difference of the estimation methods, the estimation results are not the same. Therefore, it is very important to choose a suitable measurement method for TFP research. Most

scholars generally estimate TFP from the macro level. Xilin Yang (2013) used Solow residual method to estimate TFP of China as a whole and provinces[5];Wei Wang and Liangqun Qi (2018) used the stochastic frontier model to estimate the TFP growth rate of China's equipment manufacturing industry segments, and on this basis, calculated the technology progress bias and factor mismatch index[6]; Li Yang and Huimin Cao (2019) used DEA Malmquist index model to estimate TFP of China's nonferrous metals industry in 2005-2016[7].As can be seen from the above literature, most of TFP estimates use provincial and industrial data to calculate the total productivity (such as country, region, industry).However, this static macro method ignores the heterogeneity and dynamic direction between different enterprises, and the factor compensation and technology selection tendency between enterprises are not considered, which leads to the endogenous problems such as joint biased error and selection biased error in the estimation results, and the estimation results are not accurate. Therefore, scholars study various methods to solve the endogenous problem of TFP estimation. The relevant researchers divide the residual term into deterministic factors and non-deterministic factors. Deterministic influencing factors can be observed by enterprises, and the relationship between them and the studied phenomena can be clearly expressed in a certain functional relationship. The uncertainty is the real residual term, which includes the unobservable technical impact and measurement error. By depicting the deterministic and non-deterministic factors that affect the output at the same time, the model is closer to the reality [8-9]. The main difference between the TFP method and the traditional method is that it fully considers the production decision-making of the enterprise, including the entry and exit of the enterprise from the market, and at the same time, it considers the deterministic and non-deterministic relations that affect the output. Therefore, how to measure the TFP of enterprises reasonably from the micro level is the focus of research.

With the rapid development of microeconomic theory and the gradual availability of micro data, it is particularly important to choose a reasonable and effective TFP estimation method for micro enterprises. Through the evaluation and comparison of the traditional methods and the new methods of measurement enterprises, the advantages and disadvantages of various methods are further clarified, so as to help the relevant departments to choose the appropriate TFP measurement methods in the study of various economic problems, and then draw

accurate conclusions.

2. Traditional Methods of Measuring TFP

2.1 Solow residual method of parameter method

In the neoclassical production function $Y_{it} = A_{it}K_{it}^{\beta_k}L_{it}^{\beta_l}M_{it}^{\beta_m}$, i represents the enterprise, t represents the time, Y_{it} , K_{it} , L_{it} and M_{it} represent the output, capital, labor and intermediate input respectively, β_k , β_l and β_m represent the elasticity of capital, labor and intermediate input respectively. A_{it} represents technological progress, that is, the part that cannot be explained by factor input. Solow remainder measures the part of economic growth other than the growth of factor input [10]. The growth of TFP equals to the rate of technological progress. Solow remainder is called TFP. After the logarithm transformation of neoclassical production function, the following equation can be obtained, namely:

$$y_{it} = \beta_0 + \beta_l l_{it} + \beta_k k_{it} + \beta_m m_{it} + \omega_{it} + \varepsilon_{it} \quad (1)$$

In equation (1), β_0 represents the average efficiency of an enterprise, and ω_{it} represents the productivity of an enterprise in the period of t year. This part cannot be observed by researchers, but an enterprise can observe or realize, for example, its own management level, quoted technology and other factors. ε_{it} represents the random disturbance term, which is not related to the input factors, and it represents the unpredictable technology impact of the enterprise itself.

The estimates of ω_{it} obtained from equation (1) are:

$$\hat{\omega}_{it} = y_{it} - \hat{\beta}_l l_{it} - \hat{\beta}_k k_{it} - \hat{\beta}_m m_{it} \quad (2)$$

Then, the productivity of an enterprise is $\hat{\Omega}_{it} = \exp(\hat{\omega}_{it})$.

As the Solow residual method is in line with economic principles and most representative, it establishes the analytical framework of neoclassical growth theory, and separates the role of technological progress and other different factors in improving productivity. This method has the following defects and deficiencies. The first is the assumption of non-embodied technological progress. This kind of technological progress is regarded as a kind of public goods and external technological progress, which does not need any new labor and capital input. The level of technological progress is determined by factors outside the economic system. However, Yuxin Zheng (2007) thinks that it is unreasonable to separate technological progress from capital investment, which is hard to be true in reality [11]. Any research and development, technological progress and innovation are all dependent on the input of labor and capital. It needs cost to acquire knowledge. It is hard to be persuasive to assume that technological progress is exogenous. The second is the assumption of constant returns to scale. Since China's reform and opening up, the optimal allocation of resources has been improved, and the process of marketization has been improved. Therefore, the market is not the same scale reward. The output elasticity of the third factor remains unchanged. It is assumed that the elasticity of capital and labor output is the same in each year, which negates the influence of the change of factor structure on output. Zhiyun Zhao (2006) believes that the growth rate of China's capital input is higher than that of labor input. The output elasticity of China's capital input and labor force did not remain unchanged from 1979 to 2004, in which the former showed a declining trend year by year, while the latter showed a rising trend year by year, so it is questionable to assume that the output elasticity of factors remained unchanged [12].

2.2 Stochastic frontier production function method of parameter method

The stochastic frontier production function method can separate the influencing factors of TFP from the rate of productivity change, making the model closer to the reality [13], and the commonly set model form is:

$$\ln y_{it} = f(x_{it}, t, \delta) + (v_{it} - u_{it}), i = 1, 2, \dots, N, t = 1, 2, \dots, T \quad (3)$$

In equation (3), $\ln y_{it}$ represents output (logarithm), $f(x_{it}, t, \delta)$ represents random frontier production function, and it represents the most frontier production technology in economic system; x_{it} represents input of an enterprise in the year of t , δ is parameter vector, v_{it} is random error term and obeys normal distribution, u_{it} is a random variable and represents technical inefficiency. The model considers that in the short term, the utilization degree of the existing production capacity of the enterprise represents the technical efficiency, and in the long term, the reference material for measuring the technical efficiency represents the technical progress rate. By estimating the two parts and multiplying them, TFP can be estimated. The advantage of this method in estimating TFP is mainly reflected in the processing of error term. Compared with the error term set by other production functions, the error term in this model has more technical inefficiency, that is, the error term can not only explain the statistical and measurement errors, but also measure the gap between the actual production and the efficiency frontier. By considering the actual situation of economic development, it can better simulate the following influence of machine error on economic growth [14]. The disadvantages are as follows: (1) It is necessary to set the form of production function and behavior constraints in advance to determine the probability distribution form of random error term, which may result in a large deviation of productivity measurement. (2) The measured leading-edge technological progress is an average value, which may be very large without considering the leading-edge technological progress of a few enterprises, so that the technological progress and technological efficiency may change in the opposite direction, which leads to dramatic changes in the measured total factor productivity results.

2.3 Data envelopment analysis of nonparametric method (DEA)

Data Envelopment Analysis (DEA) is a kind of efficiency evaluation method, which is based on the concept of relative efficiency and allows the research object to have technical inefficiency. Suppose the input-output relationship of an enterprise is:

$$Y_{it} = F_{it}(L_{it}, K_{it}, M_{it}) \tag{4}$$

In equation (4), F_{it} represents the corresponding relationship among labor input, capital input, intermediate input and output. Using DEA to measure the change of TFP is mainly to use it to solve the distance function. In DEA method, TFP is equal to output divided by weighted input, in which the input weight of each enterprise is estimated by maximizing the efficiency of the enterprise under the constraint that the efficiency calculated by the weight is less than or equal to 1. The advantage of this method is to allow the heterogeneity of production technology. That is to say, the research object is allowed to have technical inefficiency, that is, the production function in equation (4) is not fixed for different enterprises, and the same enterprise is different in different periods, so it is not necessary to estimate parameters, but to obtain TFP measurement from input and output data by means of linear programming technology. However, the DEA method considers the efficiency measurement of each enterprise, so the measurement error of individual enterprise will affect the accuracy of each enterprise's measurement results, making the random error sensitive to the enterprise's estimation results.

2.4 DEA-Malmquist index method of nonparametric method

There are many methods of index measurement, Malmquist index is the most widely used. The index method can deal with panel data and reduce the dimension of input and output vectors. At the same time, index method can decompose TFP index to find out the factors influencing TFP change, such as DEA Malmquist index.

DEA-Malmquist index method is a method developed on the basis of DEA to measure TFP, and Malmquist productivity index under the technical conditions of t period and $t+1$ period is respectively expressed as

$$M_t(X^{t+1}, Y^{t+1}, X^t, Y^t) = \frac{D_c^t(X^{t+1}, Y^{t+1})}{D_c^t(X^t, Y^t)} \text{ and } M_{t+1}(X^{t+1}, Y^{t+1}, X^t, Y^t) = \frac{D_c^{t+1}(X^{t+1}, Y^{t+1})}{D_c^{t+1}(X^t, Y^t)}. \text{ In the above}$$

formula, X 、 Y and t respectively represent the vector functions of input, output, period and input-output. The Malmquist productivity index from t period to $t+1$ period measured by their geometric average is

$$M(X^{t+1}, Y^{t+1}, X^t, Y^t) = \left(\frac{D_c^t(X^{t+1}, Y^{t+1})}{D_c^t(X^t, Y^t)} \times \frac{D_c^{t+1}(X^{t+1}, Y^{t+1})}{D_c^{t+1}(X^t, Y^t)} \right)^{1/2}. \text{ The advantage of DEA}$$

-Malmquist index method is that it does not need to set the specific production function form in advance, and it can decompose the total index into different sub-indexes, and find out the reasons for the change of TFP according to the specific policy meaning. The disadvantage of this method is that the factor inputs are all static inputs, that is, the price information is missing, all the input factors have no adjustment cost in the sample period, and the impact of random error on TFP is not considered [15].

3. New Methods to Measure Total Factor Productivity at the Enterprise Level

Parametric method and nonparametric method are the traditional methods to measure TFP. These traditional methods estimate the joint errors and selection errors of TFP at the enterprise level. Loecker (2011) [16] believes that the joint biased error refers to the fact that the decision-making of various factor inputs of an enterprise is related to the impact of productivity, which leads to the overestimation of estimated labor force and intermediate input, and the underestimation of capital elasticity; the selective biased error refers to the successful enterprises with high productivity remaining in the market instead of those with low production rate and poor management. Therefore, the sample enterprises are not randomly selected. The probability of exiting the market of incumbent enterprises is small when facing the impact of low productivity, so the capital coefficient will be underestimated and TFP will be overestimated (Xiaodong Lu and Yujun Lian ,2012) [17]. Generally speaking, the traditional macro method is not suitable for the study of TFP in micro enterprises. The latest research will make full use of the information of enterprise behavior and decision-making structure, and focus on TFP of enterprises using control production function method, such as OP model and LP model of semi parametric method, ACF model and De-Loecker model.

3.1 Olley and Pakes method of semi-parametric estimation (OP method)

In the Olley and Pakes model, investment is regarded as an agent variable of productivity, which is a strictly monotonic increasing function of productivity [18].

That is to say, $k_t = (1 - \delta)k_{t-1} + i_{t-1}$. Investment is not related to the current

productivity ω_{it} and random disturbance u_{it} , so as to solve the problem of simultaneous bias. The production function of an enterprise is:

$$y_{it} = \beta_l l_{it} + \beta_k k_{it} + \beta_m m_{it} + \omega_{it} + \varepsilon_{it} \quad (5)$$

In equation (5), in the case of given capital, the function of production rate with respect to investment can be obtained, that is $\omega_{it} = \omega(k_{it}, i_{it})$, and substitute it into equation (5) to get the following equation:

$$y_{it} = \beta_l l_{it} + \beta_m m_{it} + h_{it}(k_{it}, i_{it}) + \varepsilon_{it} \quad (6)$$

In equation (6), $h_{it}(k_{it}, i_{it}) = \beta_k k_{it} + \omega(k_{it}, i_{it})$. the sum of investment function i_{it} is not related to the random disturbance term u_{it} , and the endogenous problem does not exist. By returning equation (6), we can get estimates of labor input and intermediate input and non-parametric parts, which are respectively β_l , β_m and h_{it} . By considering the withdrawal of samples, the survival status of enterprises is recorded as d_{it} , whether enterprises withdraw from the market depends on the critical value $\bar{\omega}_{it}(k_{it})$. when $\omega_{it} > \bar{\omega}_{it}(k_{it})$, $d_{it} = 1$, enterprises continue to operate; Otherwise, $d_{it} = 0$, enterprises withdraw from the market, the survival probability of enterprises can be obtained by Probit model as follows:

$$Pr(d_{it} = 1 | \omega_{it}(k_{it}), J_{it-1}) = h_{it-1}(i_{it-1}, k_{it-1}) \equiv P_{it-1} \quad (7)$$

In this way, the survival probability is used to solve the selective bias problem. For the estimation of capital coefficient β_k , for any given production rate β_k , since the production rate follows the first-order Markov process, the next production rate is expressed by the formula:

$$\omega_{it} = E(\omega_{it} | \omega_{it-1}) + \xi_{it} = f(\omega_{it-1}) + \xi_{it} \quad (8)$$

In equation (8), $f(\omega_{it-1})$ is about the ω_{it-1} unknown form of expected productivity function, and ξ_{it} is the innovation of productivity in t period. By searching different values of β_k , the minimum value of the sum of squares of residuals is its estimated value. Therefore, the most important breakthrough in the study of Olley and Pakes is to consider the dynamic and selection problems, and pay attention to the problem of

the linkage between input and unobservable productivity.

3.2 Levinsohn and Petrin method of semi-parametric estimation (LP method)

Levinsohn and Petrin (2003) believe that the following problems exist in the OP model when using investment as an agent variable [19]. Because the elements are dynamic inputs, they can be adjusted at any time, and there are adjustment costs. When the amount of investment is zero, because the relationship between investment and total output is assumed to be monotonous and increasing, there is a lot of missing investment information. If the intermediate input is used as an agent variable, the intermediate input is a monotonic increasing function of productivity, that is $m_{it} = m_t(k_{it}, \omega_{it})$, we can reduce the loss of sample size and solve the deviation of investment as input. That is to say, it solves the problem of sample selection deviation. Under the given conditions of k_{it} , the inverse function is $\omega_{it} = h_t(k_{it}, m_{it})$, ω_{it} is an unobservable state variable. Compared with OP method, LP method considers the survival probability of the company, while OP method does not consider sample withdrawal, so LP method is not significantly better than OP method [20-22].

3.3 ACF method

Akerberg et al. (2006) believe that in OP model and LP model, the basic recognition conditions of semi parametric model are not satisfied [23]. In the first stage, it is estimated that the elasticity of static input factors in production function will encounter collinearity problem, which makes the change of static input and non-parametric part cannot be separated, so the parameters of labor force and intermediate input cannot be identified. The ACF model solves the endogenous and simultaneous problems according to the order structure of enterprise input decision. Before estimating the current productivity, the capital will be determined first, then the labor input decision of the enterprise, and finally the intermediate material input decision. This model is an extension of OP model. In order to avoid endogenous and simultaneous problems, the first stage does not estimate the elastic parameters, but is

used to separate the productivity from the pure disturbance. In the second stage, according to the first-order Markov process of productivity, non-parametric estimation method (GMM) is used to estimate labor, capital, intermediate input coefficient and productivity at the same time.

3.4 De-Loecker model

Based on ACF model, Loecker (2011) developed a model with productivity heterogeneity and demand heterogeneity in incomplete competitive market [16], assuming that the demand function is:

$$Q_{it} = Q_{jt} (P_{it} / P_{jt})^{\eta} \exp(u_{it}^d + \varepsilon_{it}) \quad (9)$$

In equation (9), Q_{it} and Q_{jt} respectively represent the product demand of enterprise I and industry J . P_{it} and P_{jt} respectively represent the product price of enterprise I and industry J , u_{it}^d represent the demand shock faced by enterprises, η represent the substitution elasticity of different product demand in the same industry, and ε_{it} represent the demand shock. This method assumes that the intermediate input in the production function must monotonically increase with the productivity, and the productivity is an unobservable state variable, and its proxy variable is investment. By controlling the price of the omitted products, we can get the consistent estimation of the input factors in the incomplete competitive product market, and calculate the coefficient of the input factors, the scale reward and the substitution elasticity of the products, so as to solve the problems of the linkage between productivity and production input and the deviation of sample selection.

4. Research Conclusion

The core of TFP is to control the substitution of input factors allowed by production technology. The parameter method sets the structure of the error item applied, and it does not involve the decision-making behavior of the enterprise. When there is an emergency, this abnormal change of productivity growth cannot be accurately described. DEA is essentially a mathematical programming method, which regards the production and operation process of an enterprise as an unobservable black box. DEA-Malmquist index method simply sets the profit maximization behavior of enterprises, without further discussion and utilization of

the decision-making structure of enterprises. However, the development of the world economy is changeable, which is greatly influenced by external factors. It is very important to adopt dynamic research method because it is assumed that the parameters of each period are not consistent with the actual situation. OP model takes investment as agent variable and LP model takes intermediate investment as tool variable. They can not only deal with simultaneous bias, but also deal with sample selection bias. However, LP model does not consider sample withdrawal, so it is not significantly better than OP model. ACF model setting is more general than OP model, which solves the problems of endogenous and simultaneous. De-Loecker model mainly aims at TFP measurement of incomplete competitive market, and solves the missing variable deviation in production function estimation. In the future research, the important research field and development direction of TFP estimation is to use new methods, make full use of enterprise behavior and decision-making, and further open the market structure of enterprises.

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