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Study of optimal damping coefficients for devices based on a single-objective optimization model—Considering two types of damping coefficients

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Abstract: *Wave energy is an important marine renewable energy source, and one of the key issues in its utilization is to improve the energy conversion of wave energy devices efficiency. In this paper, we consider the pendulum motion of the float in the wave, firstly, we establish the coordinate system for the float system and analyze the forces on the float and the oscillator, establish the initial value system model of the second-order coupled ordinary differential equation system about the displacement function of the float and the oscillator, and then calculate the displacement and velocity of the float and the oscillator under the constant damping coefficient by reducing the order and finite difference. Finally, a single-objective optimization model with the output power of the PTO system as the objective function is established, and the damping coefficients in the cases of constant damping coefficient and variable damping coefficient are found out respectively, so as to maximize the power.*

Keywords: *Mechanical analysis, finite differences, coupled systems of ordinary differential equations, wave energy devices*

1. Introduction

Wave energy is an important marine renewable energy source, and one of the key issues for its large-scale utilization is energy conversion using devices. The wave energy device drives the damper to do work by the relative motion of the float and the oscillator, and outputs the work done as energy. The wave energy device is composed of a float, a vibrator, a center shaft and an energy output system (PTO, including a spring and a damper)[1-2]. In which the oscillator, the center shaft and the PTO are sealed inside the float; the float consists of a cylindrical shell and a conical shell with uniformly distributed masses; the two shells are connected with a compartment that serves as a support surface for mounting the center shaft; the oscillator is a cylinder threaded on the center shaft and connected to the center shaft base through the PTO system. It is assumed that seawater is an ideal fluid without viscosity and rotation, and based on the micro-amplitude wave assumption, the float makes oscillatory motion (vertical and longitudinal oscillatory motion) under the action of waves[3-5].

In order to establish the equations of motion and obtain the pendular displacement and velocity of the float and oscillator, this paper first determines the coordinate system, which is represented by a single coordinate system due to the relative motion between the float and oscillator. Assuming that the center of gravity of the float is on the base[6], the coordinate system is established with the center of gravity of the float as the coordinate origin and the vertical static water surface upward as the positive direction[7]. Second, according to the problem, the float and the oscillator are analyzed separately and the initial conditions are determined to establish the kinetic coupling equation model. Again, in the process of solving the set of coupled equations, the numerical solution of the model is obtained by using finite difference and iterative methods[8-9].

In order to determine the mathematical model of the optimal damping coefficient of the linear damper, so that the average output power of the PTO system is maximized, this paper establishes a single objective optimization model with the maximum output power of the PTO system as the objective function[10]. The main variables involved in this paper are shown in Table 1.

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Table 1: Variable descriptions

Symbol	Symbol meaning and description
m	Oscillator mass
M	Float mass
μ	Additional mass
F_{PTO}	Overall force of the energy output system
C	Linear damper damping coefficient
C_0	Scaling factor of linear damper damping coefficient
a	Power index of linear damper damping factor
k	Spring stiffness
x_r	Relative displacement of float and oscillator
$\dot{x}_r$	Relative velocity of float and oscillator
F_e	Wave excitation force
f	Wave excitation force amplitude
ω	Wave frequency
T	Wave period
F_h	Hydrostatic recovery force
ρ	Seawater density
g	Gravitational acceleration
R	Float bottom radius
F_c	Damping force
D	Damping coefficient
s	Float pendulum displacement
$\dot{s}$	Float droop velocity
$\ddot{s}$	Float pendulum acceleration
x	Oscillator pendulum displacement
$\dot{x}$	Oscillator pendulum velocity
$\ddot{x}$	Oscillator pendulum acceleration
l	Original spring length
h_0	Compression of the spring at the equilibrium position at the initial moment
h_1	The height of the oscillator at the equilibrium position at the initial moment
P	Output power
W	Total work
θ	Angular displacement
M_e	Wave excitation moment
M_c	Hydrostatic recovery moment

2. The establishment and solution of the equations of motion

2.1 Scenario 1: Damping factor is constant

2.1.1 Coordinate system establishment

The center of gravity of the float is assumed to be located at the intersection of the cylindrical shell and the cone. Using the position of the float's center of gravity as the coordinate origin and the vertical hydrostatic water surface upward as the positive direction, a coordinate system is established as shown in Figure 1(a). In the case of static water, the float is in equilibrium, subject to gravity, buoyancy and the spring reaction force between the float and the oscillator. At the same time, the oscillator is also in equilibrium, subject to gravity and the spring force between the float and the oscillator, as shown in Fig. 1(b)(c). In equilibrium, the mechanical equations of the float and the oscillator are:

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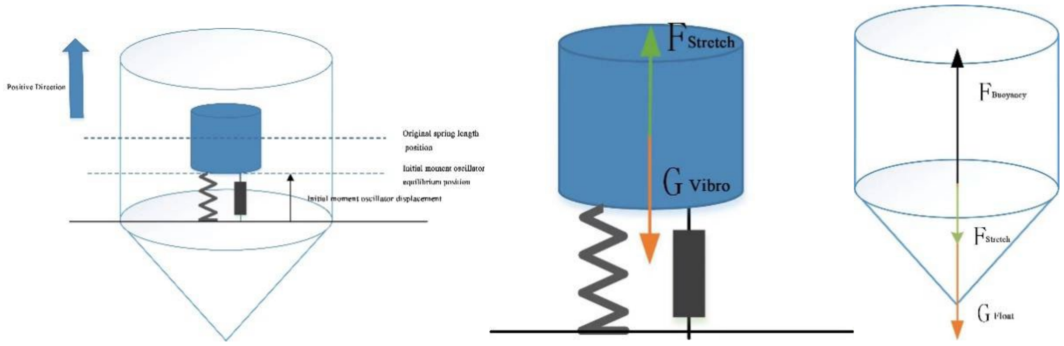
$$\begin{cases} oscillator: mg = kx \\ float: Mg + kx = \rho g V \end{cases} \tag{1}$$

When there are waves, the motion of the float and the oscillator oscillate up and down at the equilibrium state under the action of the wave excitation force, so except for the initial moment to discuss the gravity, and then the oscillation process are no longer considered gravity.



(a) (b) (c)

Figure 1 Composition of wave energy conversion device

2.1.2 Background: Damped and forced vibrations

Vibration under the action of repulsive and resistive forces is called damped vibration. The vibration of an object under the continuous action of a periodic external force is called forced vibration. The motion of the float and the oscillator in this problem belongs to the joint action of damped vibration and forced vibration. The equation of motion of an object under the action of elastic force, resistance and driving force is

$$m\ddot{x} = -kx - c\dot{x} + F_0 \cos \omega t \tag{2}$$

where F0 is the amplitude of the driving force and ω is the angular frequency of the driving force.

let $\omega_0{}^2 = \frac{k}{m}$, $2\delta = \frac{c}{m}$. Then the above equation can be written as $\ddot{x} + 2\delta\dot{x} + \omega_0{}^2 x = \frac{F_0}{m}\cos \omega t$

In the case of small damping, the solution of the equation is $x = A_0 e^{-\delta t} \cos(\sqrt{\omega_0{}^2 - \delta^2 t}$ $+\phi_0{}') + A\cos(\omega t + \phi)$

At the beginning of the driving force is a transient process, it is a reduced amplitude vibration. After a period of time, the first vibration will be weakened to negligible, and the second one is the equal amplitude vibration after the steady state of forced vibration. According to the theoretical calculation, we can get

$$A = \frac{F_0}{m\sqrt{(\omega_0{}^2 - \omega^2)^2 + (2\delta)^2 \omega^2}} \tag{3}$$

$$\tan \phi = -\frac{2\delta\omega}{\omega_0{}^2 - \omega^2} \tag{4}$$

At steady state, the amplitude of the velocity of the vibrating object is

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$$v_m = \frac{\omega F_0}{m\sqrt{({\omega_0}^2 - \omega^2)^2 + (2\delta)^2 \omega^2}} \tag{5}$$

2.1.3 Establishment of the force equation

The float is subjected to wave excitation, additional inertia, wave damping, hydrostatic recovery and the reaction of the PTO system under the action of linear periodic micro-amplitude fluctuations, where the wave excitation is the driving force. The oscillator will be subjected to the force of PTO system. The mechanical equation of the oscillator.

$$m\ddot{x} = F_{PTO} \tag{6}$$

The force of the PTO system on the float and the oscillator is divided into the damping force of the linear damper and the elastic force of the spring

$$F_{PTO} = -C\dot{x}_r - kx_r \tag{7}$$

Mechanical equations of the float

$$(M + \mu)\ddot{s} = F_e - F_h - F_{PTO} - F_C \tag{8}$$

where, from the question, the wave excitation (driving force) is

$$F_e = f \cos \omega t \tag{9}$$

Hydrostatic recovery force refers to the floating body in seawater to do the dangling motion, will make the floating body back to the equilibrium position of the force, in fact, the change of buoyancy The expression is therefore:

$$F_h = \rho g V = \rho g \pi R^2 s \tag{10}$$

The wave damping force refers to the resistance of the wave to the rocking motion of the floating body, and its expression is:

$$F_C = D\dot{s} \tag{11}$$

The relative displacement is

$$x_r = x - s \tag{12}$$

In summary, we establish a coupled system of ordinary differential linear equations

$$\begin{cases} m\ddot{x} = F_{PTO} \\ (M + \mu)\ddot{s} = f \cos \omega t - \rho g \pi R^2 s - F_{PTO} - D\dot{s} \\ F_{PTO} = -C\dot{x}_r - kx_r \\ x_r = x - s \end{cases} \tag{13}$$

The initial condition is derived from expression (1) as

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$$\begin{cases} \dot{s}(0)=0 \\ \dot{x}(0)=0 \\ s=0 \\ x=l-h_0+\dfrac{h_1}{2} \end{cases} \tag{14}$$

In order to discretize the continuous mechanical equation (8) model, the coupling equation is written in matrix form. Although the coupled equation is a two-dimensional normally differential linear equation, it can be discretized by setting $x_1(t)=\dot{x}, s_1(t)=\dot{s}$ 、 $\dot{x_1}(t)=\ddot{x}, \dot{s_1}(t)=\ddot{s}$. Perform a step-down.

$$\begin{pmatrix} m & 0 & 0 & 0 \\ 0 & M+\mu & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{s}_1 \\ x_1 \\ s_1 \end{pmatrix} + \begin{pmatrix} c & -c & k & -k \\ -c & c+D & k & k+\rho g \pi R^2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ s_1 \\ x \\ s \end{pmatrix} = \begin{pmatrix} 0 \\ f\cos\omega t \\ 0 \\ 0 \end{pmatrix} \tag{15}$$

2.1.4 The result of solving the equation

The time range of the solution is 40 wave periods, and the wave frequency is known to be 1.4005s - 1, so the individual period time is 4.4841s. The total time of 40 periods is divided into equidistant intervals of 0.2s steps, and the dip displacement and velocity of the float and oscillator are calculated. The dip displacements and velocities of the float and oscillator at 10s, 20s, 40s, 60s, and 100s are shown in Table 2 below.

Table 2: Heave displacement and velocity of float and oscillator in subproblem 1

Time(s)	**Float**		**Oscillator**	
	displacement (m)	**speed(m/s)**	**displacement (m)**	**speed(m/s)**
10	0.174839	- 0.09038	0.233858	- 0.1911
20	- 0.20796	- 0.55449	0.20038	- 0.62142
40	0.290553	0.887672	0.296362	0.972472
60	- 0.34998	- 0.97848	- 0.368	- 1.07253
100	- 0.21898	- 0.54935	- 0.2431	- 0.59171

2.1.5 Scenario 2: Variable damping factor

The difference between subproblem II and subproblem I is that the damping coefficient of the linear damper is proportional to the power of the absolute value of the relative velocities of the float and the oscillator, where the scale factor C0 = 10000 and the power exponent is a = 0.5. At this point

$$C=C_0\left|\dot{x}_r\right|^a \tag{16}$$

At this point the damping coefficient is not constant and the original set of coupling equations becomes a nonlinear set of equations.

$$\begin{cases} m\ddot{x}=F_{PTO} \\ (M+\mu)\ddot{s}=f\cos\omega t-\rho g\pi R^2 s-F_{PTO}-D\dot{s} \\ F_{PTO}=-C\dot{x}_r-kx_r \\ x_r=x-s \\ C=C_0\left|\dot{x}_r\right|^a \end{cases} \tag{17}$$

Since the damping coefficient contains a relative velocity term, the original equation of motion is a nonlinear ordinary differential equation and is considered to solve its numerical solution. Using the

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relative velocity at the steady state in case I as the initial condition, when the damping coefficient C is constant, the steady state solution at this damping coefficient can be obtained by substituting it into the iterative format of subproblem I. Substitute the new steady-state relative velocity into C to obtain the new damping factor. Repeat several times to set the stopping condition. When satisfied, the calculated value of C is the damping factor, a specific value, and the effect of this action is equivalent to $C = C_0 \left| \dot{x}_r \right|^a$, Substituting C at this point into the iterative format of Subproblem 1, the pendulum displacements and velocities of the float and oscillator can be calculated.

2.1.6 Solution results

The pendular displacement and velocity of the float and oscillator at 10s, 20s, 40s, 60s and 100s are shown in Table 3 below.

Table 3: Heave displacement and velocity of float and oscillator in subproblem 2

Time(s)	Float		Oscillator	
	displacement (m)	speed(m/s)	displacement (m)	speed(m/s)
10	0.043098	- 0.561118	0.431834	- 1.63171
20	- 0.23252	- 0.393622	- 0.10249	- 2.61335
40	0.355317	1.303107	0.066932	0.03011
60	- 0.24011	- 1.40805	- 0.54143	- 0.1641
100	- 0.30223	- 0.6436	- 0.0944	- 0.55892

3. Determination of the optimal damping factor

3.1 Scenario 1: Damping factor is constant

Determine the mathematical model of the optimal damping system for the linear damper such that the average output power of the PTO system is maximized, and the problem is a single objective optimization problem. By reviewing the literature, the output power equation for the linear damper is derived, which is used as the objective function to find the maximum value, and the constraints are the range of values of the damping coefficient. The following single-objective optimization model is established.

$$\begin{cases} o.b = \left\{ \max P = \dfrac{C\omega^2 A^2}{2} \right\} \\ s.t \begin{cases} 0 \le C \le 100000 \\ A = \dfrac{f}{m\sqrt{({\omega_0}^2 - \omega^2)^2 + (2\delta)^2 \omega^2}} \\ \omega_0 = \dfrac{k}{m} \\ 2\delta = \dfrac{C}{m} \end{cases} \end{cases} \tag{18}$$

The solution is

$$C = \frac{m({\omega_0}^2 - \omega^2)}{\omega} \tag{19}$$

Substituting the data to solve for the theoretical values of the optimal damping coefficient is C = 30741.4N - s/m and P = 194.461657W. In addition to finding the analytical solution, we used the numerical solution method, which is needed if the expression of the power is complex or the analytical solution cannot be found. Therefore, the analytical solution can be used as a test of the accuracy of the numerical solution. Using the variable step search method, the value of C that maximizes P is found in the interval [0, 100000]. The algorithmic flow of the variable step search method is shown in Figure 2 below.

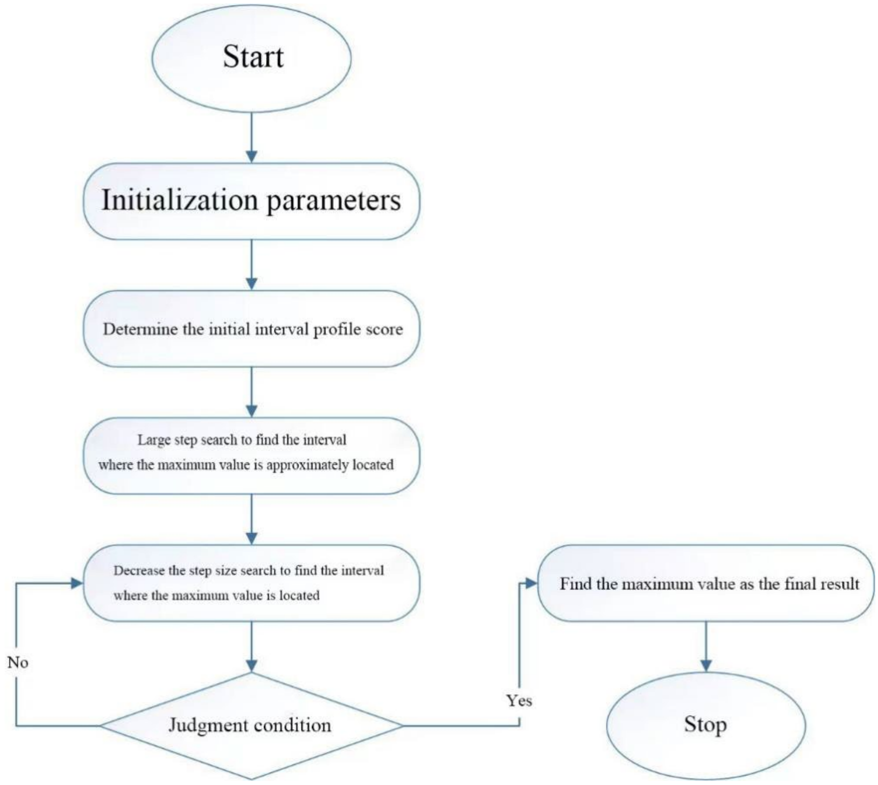

Figure 2: Analysis Process

The numerical solution of the optimal damping coefficient by the variable step search method is C = 30741.4N-s/m and P = 194.461657128522W.

Comparing the analytical solutions, it can be seen that the numerical solutions solved by the variable step search method are reliable.

3.2 Scenario 2: Variable damping factor

The damping coefficient is known to be proportional to the power of the absolute value of the relative velocities of the float and the oscillator, with a scale factor in the interval [0, 100000] and a power exponent in the interval [0, 1], for a single objective optimization model satisfying the following conditions.

$$\begin{cases} o.b = \left\{ \max P = \dfrac{C\omega^2 A^2}{2} \right\} \\ s.t \begin{cases} 0 \le C_0 \le 100000 \\ 0 \le a \le 1 \\ A = \dfrac{f}{m\sqrt{(\omega_0{}^2 - \omega^2)^2 + (2\delta)^2 \omega^2}} \\ \omega_0 = \dfrac{k}{m} \\ 2\delta = \dfrac{C}{m} \\ C = C_0 \left| \dot{x}_r \right|^a \end{cases} \end{cases} \tag{20}$$

This model is difficult to solve analytically, so only its numerical solution is considered, and the variable-step search method is still used. Unlike case 1, there are two constraint variables in case 2, and the values of C0 and a are set to an equally spaced grid, and at each grid lattice point, there is a determined (C0, a). The output power P corresponding to each grid point data is derived by exhaustive enumeration to find the value of C0, a corresponding to the largest P. Next, the search is narrowed down to C0, a and the nearby range, re-equalized, and the above process is repeated until the stopping condition holds. It is

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obtained that P reaches the maximum at a = 0.1 and C0 = 26314.3, which is P = 194.4617W.

4. Conclusions

In this paper, based on the motion analysis of float and oscillator in wave energy device, the initial value problems of coupled constant coefficient (constant damping coefficient case) and variable coefficient (variable damping coefficient case) systems of ordinary differential equations about the displacement function and angular displacement function of float and oscillator are established in the cases of vertical and oscillating motion, respectively. The continuous equations are discretized by finite differences and the results are solved iteratively using the initial conditions. In this paper, a single-objective optimization model is also developed to solve the optimal variable damping coefficient and maximum output power based on the approximate linear, iterative and variable-step search methods under univariate and bivariate variables, respectively, are considered.

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