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Practical Applications of Economic Mathematics in Business Studies Programmes

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Abstract: *In today's complex and competitive business environment, maths is increasingly becoming an integral part of the business disciplines as a powerful tool. With the rapid development of technology and the explosion of data, the application of mathematical models is not only important at the theoretical level, but also plays a key role in business decision-making.The purpose of this thesis is to delve into the practical application of advanced mathematics in business decision-making, and to assess the effectiveness of mathematical tools such as elasticity functions, marginal functions, first-order differential equations, and other mathematical tools for business decision-making by examining their comprehensive application in economics majors. Through this study, it aims to provide business students with a more practical and insightful perspective on the application of mathematics and to improve their ability to solve practical business problems.*

Keywords: *economic mathematics, first-order partial differential equations, elastic functions, marginal functions*

1. Introduction

In today's globalized and digital business environment, mathematics has become an integral part of the business discipline as a powerful tool. With the rapid progress of science and technology and the explosion of big data, the importance of mathematical models in theoretical research and practical application has become increasingly prominent. Economic mathematics, as an application of mathematics in the field of economics, not only provides in-depth insights at the theoretical level, but also plays a vital role in business decision-making. It helps managers quantify complex issues, conduct risk assessments, optimize resource allocation, and anticipate market dynamics.

This dissertation aims to provide an in-depth look at the practical application of economic mathematics in business decision-making, especially the effectiveness of tools such as elastic functions, marginal functions, and first-order differential equations. By assessing the combined application of these tools in the economics major, we aim to provide business students with a more practical and intuitive perspective on the value of mathematics in solving real-world business problems[1]. In addition, the paper proposes teaching strategies to enhance students' ability to apply mathematics in real-world situations, laying a solid foundation for their future career development.

Through case studies, we show how to use elasticity functions to optimize portfolios, marginal functions for risk management, and first-order differential equations for market forecasting. These cases not only demonstrate the theoretical validity of mathematical tools, but also highlight their usefulness in solving real-world business challenges[2]. Finally, we discussed how to cultivate students' practical application ability through case teaching and mathematical modeling training, with a view to better integrating theory and practice in business education.

The contribution of this paper is to highlight the importance of economic mathematics in the discipline of business and to provide a method for educators to develop business talents with practical problem-solving skills. Through these efforts, we hope to advance business education and better prepare students for the rapidly changing business environment.

2. Effectiveness of mathematical tools in business decision-making

2.1 Application of Elasticity Functions to Portfolios

It is well known that the elasticity function measures the percentage change in one variable relative

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to another. The elasticity coefficient E_{ij} of the asset X_i with respect to X_j in a portfolio can be calculated using the following formula:

$$E_{ij} = \frac{\partial \ln(X_i)}{\partial \ln(X_j)} \tag{1}$$

The application of the elasticity function allows us to quantitatively analyse the correlation between different assets. For two assets in a portfolio X_i and X_j, the covariance $Cov(X_i, X_j)$ can be calculated from the elasticity coefficient:

$$Cov(X_i, X_j) = \sigma_i \cdot \sigma_j \cdot E_{ij} \tag{2}$$

This formula is a good representation of the correlation between assets X_i and X_j. where σ_i and σ_j denote the standard deviation of assets X_i and X_j, respectively.

By maximising the overall resilience of the portfolio, we can adjust the weights of the assets to optimise the portfolio. Using Markowitz mean-variance theory, it is known that the weight of the asset X_i in the portfolio w_i The adjustment can be expressed as:

$$\Delta w_i = \frac{w_i \cdot E_{ij}}{\sum_k w_k \cdot E_{ik}} \cdot \Delta w_j \tag{3}$$

This formula expresses the relationship between the adjustment of asset weights and their elasticity coefficients as well as other asset weights and elasticity coefficients. The elasticity coefficient allows us to calculate the adjusted weights to optimise the portfolio.

In risk-adjusted optimisation, the introduction of an elasticity function allows us to flexibly adjust asset exposure. We can calculate the adjusted asset weights: σ denotes the standard deviation of the whole market.

$$w_i^{'} = \frac{(1 + E_{ij}) \cdot \omega_i + Cov(X_i, X_j) \cdot \sigma}{(1 + E_{ij}) \cdot \omega_i + \omega_j + Cov(X_i, X_j) \cdot \sigma} \tag{4}$$

2.2 Application of Marginal Functions in Risk Management

Marginal functions are used to describe the rate of change of one variable relative to another. In risk management, the marginal function helps us to understand the extent to which decision variables affect risk[3]. The general definition of the marginal function is as follows:

$$MF_{ij} = \frac{\partial Y}{\partial X_i} \tag{5}$$

where MF_{ij} denotes the marginal function of Y to X_i .

In risk management, the marginal risk of an asset X_i can be calculated using the marginal function:

$$MR_i = \frac{\partial \sigma_p}{\partial w_i} \tag{6}$$

This formula represents the marginal risk of an asset X_i in a portfolio as the response of the overall risk of the portfolio σ_p to a change in the weighting of X_i . The calculation of marginal risk allows us to quantify the contribution of each asset to the overall risk and adjust the portfolio accordingly. Next,

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this paper extends the formula to obtain the following equation:

$$MR_A = \frac{e_A \cdot \sigma_p}{\omega_A \cdot E(R_A) + (1-\omega_A) \cdot E(R_B)} \tag{7}$$

This formula can be used to measure the additional risk associated with adding asset A to an existing portfolio. Considering the overall risk and the marginal risk of the asset X_i , we can then adjust the asset exposure through the marginal function:

$$w_i^{'} = w_i - \beta_i \cdot \frac{MR_i}{MR_{all}} \tag{8}$$

Where β_i is the exposure adjustment factor for asset X_i . This formula indicates that in risk management, through the adjustment of the marginal function, we can flexibly change the exposure of each asset in the portfolio according to the degree of marginal impact of each asset on the overall risk, in order to adapt to the changes of market risk.

2.3 Example analysis

1) Issue 1: Analysis of the elasticity function

a) Problem Analysis:

If A is an investor, his current assets are shown in the table 1 below:

Table 1:Asset distribution table

parameters / liabilities	prices	sum of money invested (in a currency or bond)	percentage
Stock A	50	70,000	70 per cent
Equity B	1000	30000	30 per cent

Without loss of generality in the data, initial market volatility can be assumed: $\sigma = 0.02$, standard deviation of asset stocks A $\sigma_A = 0.1$, standard deviation of asset bonds B $\sigma_B = 0.05$.

According to the problem, the elasticity function can be calculated separately E_{AB} :

$$E_{AB} = \frac{\partial \ln(Q_A)}{\partial \ln(Q_B)} \tag{9}$$

Covariance:

$$Cov(X_A, X_B) = \sigma_A \cdot \sigma_B \cdot E_{AB} \tag{10}$$

Weight adjustment formula:

$$w_A^{'} = \frac{(1+E_{AB}) \cdot \omega_A + Cov(X_A, X_B) \cdot \sigma}{(1+E_{AB}) \cdot \omega_A + \omega_B + Cov(X_A, X_B) \cdot \sigma} \tag{11}$$

From this, the data were calculated as shown in the table 2 below:

Table 2:Result table

parametric / liabilities	elasticity function E_{AB}	(statistics) covariance $Cov(X_A, X_B)$	Weighting adjustments ω
Stocks A and B	2.1299	0.010650	0.87957

Based on the data, the following conclusions can be drawn:

The results of the elasticity function $E_{AB} = 2.1299$ show that as the number Q_A of stocks A

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increases by 1% relative to the number Q_B of bonds B, the logarithmic return on stocks increases relative to the logarithmic return on bonds by about 2.123%.

From the covariance $Cov(X_A, X_B) = 0.010650>0$, it is therefore known that there is a positive correlation between stocks A and bonds B.

After calculating the value obtained from the elasticity function, it is possible to obtain $\omega_A^{'} = 0.87957$, this result shows that for a given weight adjustment, it is possible to change the weight of the stock A to 87.96%.

b) Validation of the results: in order to validate the results, the stock market simulation is now carried out through python, and under the above conditions, it can be derived, the change in the total assets with different weights, as shown in the figure1 below:

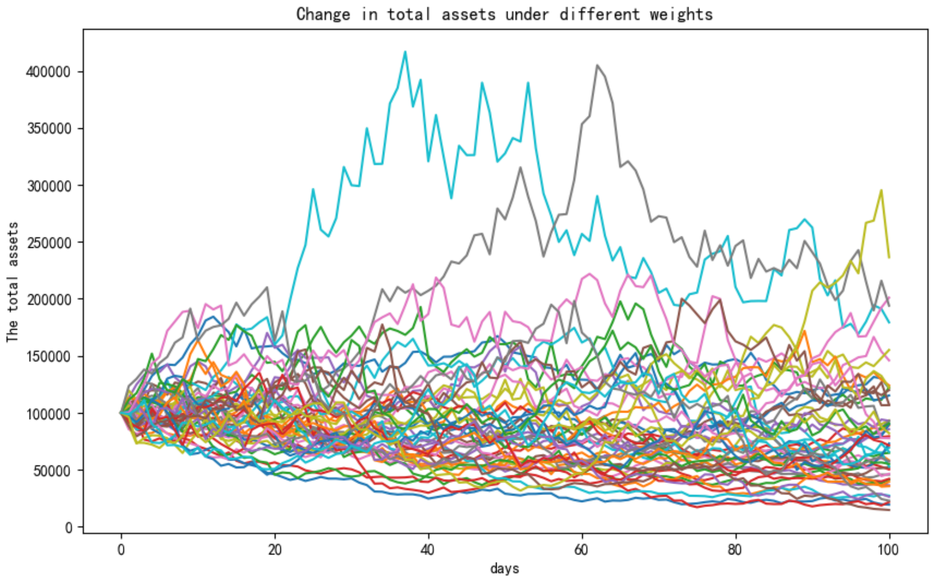

Figure 1:Changes in assets under different weights

The different weights are discussed:

With a weighting of 0.7, the assets of stocks A and B are shown below:(figure2)

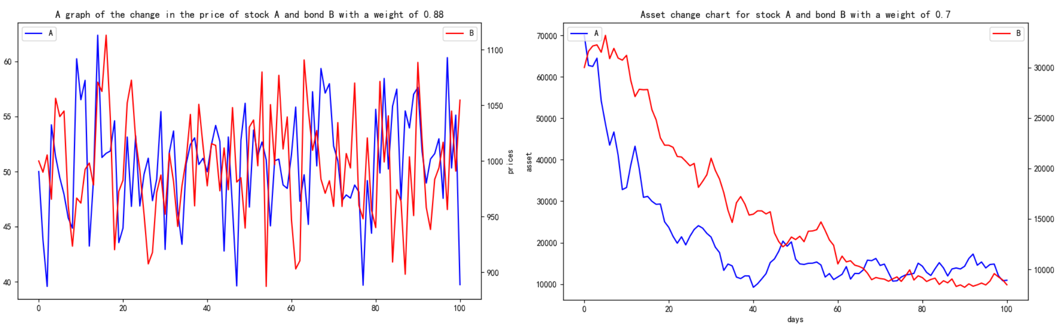

Figure 2:A/B asset and price change charts when 0.7

The assets of stocks A and B under the condition of weighting 0.88 are shown below: (figure3)

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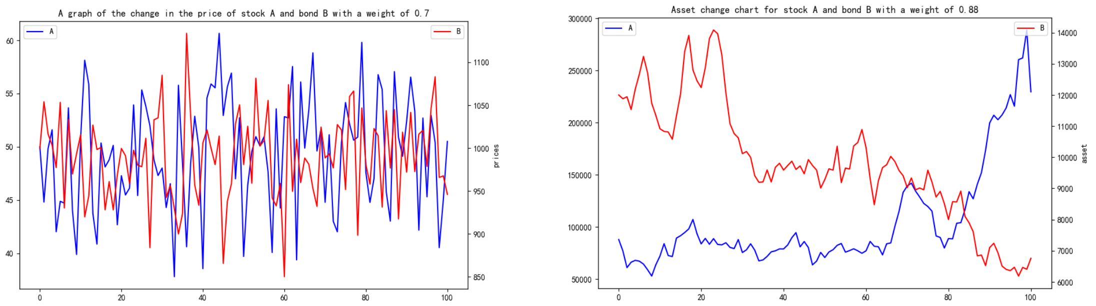

Figure 3: A/B asset and price change charts when 0.88

2) Issue 2: Optimising the resilience of risk adjustment

a) Problem Analysis:

Assume the following the table 3 assets at initial overall portfolio risk (standard deviation) $\sigma_p = 0.02$.

Table 3: Initial asset table

parametric / liabilities	Expected rate of return e	coefficient of elasticity $E(R)$
Asset A	0.05	0.6
Asset B	0.03	0.4

The data is now analysed using the following formula:

Marginal risk formula:

$$\begin{cases} MR_A = \dfrac{e_A \cdot \sigma_p}{\omega_A \cdot E(R_A) + (1-\omega_A) \cdot E(R_B)} \\ MR_A = \dfrac{e_B \cdot \sigma_p}{\omega_A \cdot E(R_A) + (1-\omega_A) \cdot E(R_B)} \end{cases} \tag{12}$$

This formula is used as an important measure of the risk and return of investing in equities. It indicates the impact on the change in risk of the overall portfolio when small changes are made to the weighting of an asset. If the MCTR of an asset is larger, it means that it contributes more to the overall risk and investors should prioritise investing in other assets to reduce the overall risk of the portfolio.

Risk-adjusted marginal analysis formula:

$$\begin{cases} RAM_A = \dfrac{\sigma_p}{\omega_A} \cdot \dfrac{e_A \cdot E(R_A)}{\omega_A \cdot E(R_A) + (1-\omega_A) \cdot E(R_B)} \\ RAM_B = \dfrac{\sigma_p}{(1-\omega_A)} \cdot \dfrac{e_B \cdot E(R_B)}{\omega_A \cdot E(R_A) + (1-\omega_A) \cdot E(R_B)} \end{cases} \tag{13}$$

The risk-adjusted marginal analysis formula measures the contribution of a unit of investment to the expected return, taking into account the risk factor. This helps investors to make more informed decisions by assessing more comprehensively the benefits of the different assets in their portfolios.

Open-ended adjustment formula:

$$\omega_A' = \omega_A - \beta_A \cdot \frac{MR_A}{MR_{all}} \tag{14}$$

Exposure adjustment formulas help investors determine the level of risk they can tolerate and allocate their assets accordingly. By allocating risk appropriately, an investor can earn a reasonable return and reduce the overall risk to which the portfolio is exposed.

Substituting the data into the formula to perform the calculations yields the following results(the table 4):

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Table 4: Result table

liabilities	Marginal risk MR	Marginal analysis of risk adjustment RAM	exposure value ω
Asset A	0.272727272727	0.019480519480519484	0.6448979591836734
Asset B	0.2222222222222	0.03703703703703703	0.3551020408163266

Observe the results, fro b m which the following conclusions can be drawn:

Marginal risk indicates the additional risk per unit of investment and it can be seen that investment A has a higher marginal risk, which means that for investment

Each additional unit of investment in Capital A adds more risk.

These values indicate the contribution of the unit of investment to the expected return after taking into account risk. It can be seen that, after adjusting for risk, the expected return per unit of investment for investment A is relatively low, while the expected return per unit of investment for investment B is relatively high.

This value represents the adjusted change in the percentage of exposure, which means that the percentage of exposure to Asset A is reduced, and after the adjustment, the percentage of Asset A becomes 64.49 per cent, which will allow the risk adjustment objective to be met.

2.4 Application of first-order differential equations to market forecasting

First order differential equations can be used to describe the rate of change of a variable in relation to the variable itself. In market forecasting, first-order differential equations can generally be used to model dynamics in order to more accurately capture changes in the market. The general definition of a first order differential equation is as follows[4]:

$$\frac{dy}{dt} = f(y,t) \tag{15}$$

where y is the variable, t is the time, and $f(y,t)$ describes the rate of change of y against time.

1) First order differential equation modelling of market trends

Consider a market trend model. We can establish the following first order differential equation:

$$\frac{dP}{dt} = r \cdot P + k \cdot e^{-\alpha t} \sin(\beta t) \tag{16}$$

where P is the market price, r is the underlying growth rate of the market, k is the amplitude of the oscillating term, α is the decay rate of the oscillating term, β is the frequency of the oscillating term, and t is time.

This equation incorporates three factors: the underlying growth of the market, oscillations and decay. The first term $r \cdot P$ represents the underlying growth of the market, while the second term $k \cdot e^{-\alpha t} \sin(\beta t)$ describes the oscillatory term, where the exponential function represents the oscillations that decrease gradually over time, and the sinusoidal function represents the cyclical variation of the oscillations. This model allows for a more comprehensive consideration of the diversity of market trends.

2) First order differential equations in price volatility

Considering the real situation, a complex price volatility model can be constructed, containing an exponential function and a stochastic term. In this paper, the following first order differential equation is established:

$$\frac{d\sigma}{dt} = \alpha \cdot (\sigma_{\text{smooth}} - \sigma) + \beta \cdot e^{-\gamma t} \cos(wt) + \sigma \cdot \xi(t) \tag{17}$$

Where σ is the current volatility, σ_{smooth} is the volatility in the smooth state, α is the parameter for the speed of adjustment, β is the amplitude of the oscillatory term, γ is the decay rate of the oscillatory term, w is the frequency of the oscillatory term, t is the time, and $\xi(t)$ is the stochastic term obeying a

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normal distribution. The first term $\alpha \cdot (\sigma_{\text{smooth}} - \sigma)$ indicates the speed of return of volatility towards a smooth state, the second term $\beta \cdot e^{-\gamma t} \cos(wt)$ describes the oscillatory term, containing the effects of amplitude, decay and frequency, and the last term $\sigma \cdot \xi(t)$ is the random term, which takes into account the randomness in the market.

3) Practical cases:

Now for a market and price, assume that the price in that market is 1000 with the following parameters(the table 5):

Table 5: Table of the case with an initial price of 1000

parametric / boyfriend	base rate (e.g. of growth in economics)	Amplitude k	attenuation rate	frequency w
market (also in abstract)	0.03	100	0.01	0.1
parametric / boyfriend	Initial volatility	stable volatility σ_{smooth}	Adjustment of speed parameters α	amplitude decay rate γ
prices	0.1	0.2	0.1	0.01

Substituting the data to bring in the above model, the image can be obtained as shown in Figure 4:

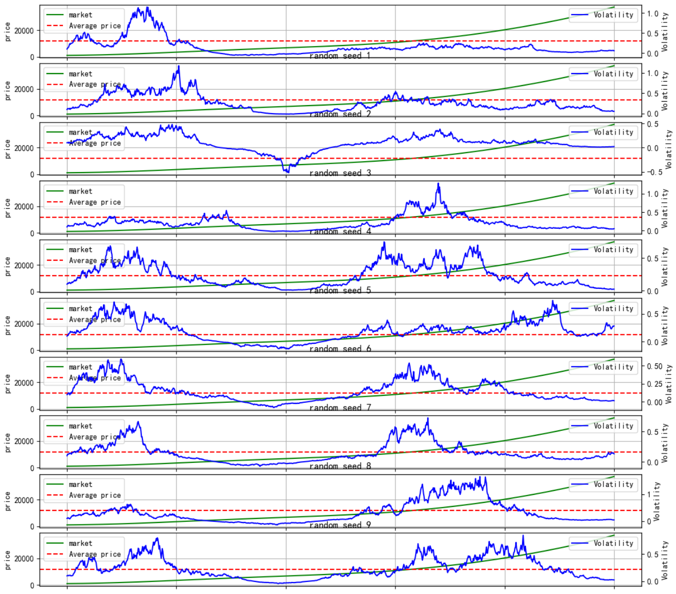

Figure 4: Trends under different random seeds

Looking for 10 random seeds and getting the above graph, you can find that the trend is more stable and has some reference value

3. Development of students' ability to apply mathematics in practice

3.1 Teaching methods for practical case studies

In the process of cultivating students' ability to practically apply mathematics, it can be found that it is crucial to adopt the teaching method of practical case studies. Firstly, in terms of case selection, emphasis should be placed on choosing industry-related cases covering different fields such as finance, marketing, supply chain management, etc., so as to ensure that students can make flexible use of mathematical models when solving practical problems. At the same time, selecting current and real-time cases, such as the latest market trends and financial fluctuations, can help students feel the process of solving practical problems in the classroom. Secondly, in practical teaching, teamwork and simulation situations can be used. Students are divided into groups and each group is responsible for analysing and solving an actual case, which promotes teamwork and joint learning. Meanwhile, through the creation of

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simulated business situations, students are guided to apply mathematical models to the solution of actual problems, so as to improve their practical skills.

3.2 Training programmes in mathematical modelling

In order to effectively cultivate students' mathematical model building ability, the following training programmes can be designed. Firstly, the curriculum design should include three aspects: basic theory teaching, practical skills training and case study. By providing solid basic theories of mathematical model construction, arranging practical course modules and guiding students to study and analyse real cases in depth, students can combine theoretical knowledge with practical problems and improve their problem-solving ability. Secondly, in terms of practical projects, partnerships can be established with relevant industries to allow students to participate in real business projects, apply mathematical models to solve real problems, and deepen their application experience in real scenarios. At the same time, students are encouraged to select and study in depth topics related to mathematical model construction to enhance their independent research and problem-solving abilities. Finally, interdisciplinary integration is also an important part of the training programme. By integrating mathematical model construction into the business curriculum and designing practice-oriented teaching and learning activities, students can experience the practical effects of mathematical models through real-life applications and deepen their understanding of theoretical knowledge, so as to cultivate interdisciplinary and integrated skills[5].

4. Conclusion

The aim of this thesis is to provide an insight into the application of economic mathematics in business majors, with a special focus on the practical utility of mathematical models in business decision-making. The effectiveness of mathematical tools such as elasticity functions, marginal functions, first order differential equations and other mathematical tools for business decision making is assessed by examining their comprehensive application in the economics profession. With this paper, it can be seen that economic mathematics, as an important part of business majors, is not only important in theoretical research, but also plays a key role in business practice. Through in-depth study of the construction and application of mathematical models, students can improve their ability to solve practical business problems and lay a solid foundation for future career development. Therefore, strengthening the teaching and training of economic mathematics is of great significance to the cultivation of business talents with practical application ability.

References

[1] Markowitz, H. M. (1952). Portfolio selection: Efficient diversification of investments. Journal of Finance, 7(1), 77-91.
[2] Samuelson, P. A. (1965). Foundations of economic analysis. Harvard University Press.
[3] Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. Journal of Finance, 25(2), 383-417.
[4] Merton, R. C. (1973). Theory of rational option pricing. Bell Journal of Economics and Management Science, 4(1), 141-183.
[5] Arrow, K. J., & Kurz, M. (1970). Public investment, the rate of return, and optimal fiscal policy. Brookings Papers on Economic Activity, 1970(1), 1-64.