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Pure azimuth and passive positioning of the UAV in formation flight

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Abstract: *In order to ensure that when the UAV cluster carries out the formation flight, keep the electromagnetic silence as much as possible to reduce the emission of electromagnetic wave signal, the position of the UAV can be adjusted by the method of pure azimuth and passive positioning. In this paper, by analyzing the different formation conditions of UAV, the pure azimuth passive positioning method in unmanned formation flight is given. Trionometric function and cosine theorem are used to determine the position of each UAV, and the corresponding position adjustment strategy. This paper studies how to use the method of target decomposition and greedy strategy to use the limited information.*

Keywords: *Cross positioning; Greed strategy; Target decomposition; Adjustment strategy*

1. Problem Background (introdution Introduction section)

Fixed-wing uav (unmanned aerial vehicle, UAV) cluster collaborative execution task has become an important development trend of uav system application, small multi-uav with high synergy, multi-tasking, low cost and other advantages have attracted much attention. UAV formation control technology has been widely studied as a sub-problem of multi-UAV collaboration, and the goal of UAV formation flight is to achieve the ideal formation by controlling the behavior of each UAV [1-3]. Common formation flight strategies include master and slave method, virtual structure method, behavior-based method, consensus theory and so on [4-6].

Since the UAV cluster should avoid external interference as much as possible when executing the formation flight, it is necessary to keep the electromagnetic silence as far as possible to reduce the emission of electromagnetic wave signal [7-8]. In order to maintain the formation, it is planned to adjust the position of the UAV, that is, several UAV in the formation transmit the signals, while the rest passively receive the signals, and extract the direction information for positioning to adjust the position of the UAV. Among them, the direction information received by the UAV is the Angle between the UAV and any two transmitting signal drones. Each UAV in the formation has a fixed number, and the relative position relationship with the other UAV in the formation remains unchanged [9-10].

2. Problem analysis

2.1 A positioning model of a passive receiving UAV was established

For the problem of UAV positioning proposed in this paper, this paper do the following analysis one by one:

The premise of realizing accurate cross-positioning is that multiple direction-finding base stations measure the accurate azimuth angle of the UAV (and pitch Angle), and then solve the plane (or space) coordinates of the UAV by positioning algorithm. Next, this paper studies the common direction finding algorithms such as triangulation positioning method and spatial analysis method to ensure the positioning performance of UAV in passive areas, and analyzes the factors affecting the accuracy of each positioning algorithm. After summarizing and analyzing the advantages and disadvantages of the above algorithm, this paper proposes a pure azimuthal passive positioning based on the triangulation positioning method. Cross-positioning, adjustment strategy and target decomposition are analyzed to determine the best azimuth positioning algorithm, and the feasibility of the algorithm is verified to provide theoretical and

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technical preparation for engineering realization. At present, it is difficult to ensure the positioning accuracy of uav in small areas without positioning source, and the use of time difference positioning in small areas is more demanding on the accuracy of time difference measurement. Combined with the above research, this paper believes that the cost of cross-positioning in the actual engineering is lower, the system is easier to implement, and the accuracy can be guaranteed

3. Model hypothesis

For the problems raised in this paper, this paper have made the following model assumptions:

Hypothesis 1: When the UAV in the topic is slightly deviated, it is still generally in the ideal position, and there is no UAV with an Angle deviation beyond the range $\frac{2\pi}{9}$.

Hypothesis 2: Suppose that each drone has a fixed label and clearly knows which drones emit signals.

Hypothesis 3: Suppose that the deviation position of the UAV does not change during the flight.

Hypothesis 4: Suppose that the adjustment time does not need to be considered when adjusting the deviation position of the UAV.

Hypothesis 5: If the UAV emission signal overlaps, it will not affect the direction information of the UAV.

Hypothesis 6: The flight trajectory cannot be recorded, and the UAV cannot receive and transmit the signal simultaneously.

Hypothesis 7: Suppose that the drone can only accept the influence of its own Angle, and can not accept the signals of other drones.

Hypothesis 8: Suppose the drone does not change in altitude.

Hypothesis 9: The transmission and acceptance of the signal will not be affected by the environment.

Common symbols in this paper are shown in the following Table 1, and other symbols are shown in the text:

Table 1: Description of the symbol

symbol	Symbolic meaning
α_k , $k=1,2,3,...$	The Angle between the receiving signal UAV and the transmitting signal UAV
$(x_i, y_i), i=1,2,3,...$	External coordinates of the center of the circle
$r_i, i=1,2,3,...$	circumradius
ρ	Passive receiving signal to the UAV
β β_1 β_2 λ_1 λ_2 λ_3 λ_4 θ_1 θ_2 θ_3	∠FY03FY 00FY 06 ∠FY00FY09FY06 ∠FY00FY09FY03 ∠FY00FY01FY03 ∠FY00FY01FY06 ∠FY00FY01FY09 ∠FY06FY00FY09 ∠FY00FY0Z FY0X ∠FY0XFY0ZFY0Y ∠FY00FY0ZFY0Y

4. Modeling and solution

4.1 Modeling and solution of the passive receiving UAVs are established

(1)Cosine-string theorem method

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Suppose that the radius of the circumference is $R\alpha = 2\pi/9$. The polar coordinate system is established with the UAV FY00 as the pole, FY00 as the endpoint and the ray passing through FY01 as the polar axis. Without loss of generality, assume that one of the transmitting signal drones on the circumference is FY01, after being converted into polar coordinates, the corresponding polar coordinates are $(R, 0)$. Since the position of the transmitting UAV is accurate, the position of another transmitting UAV is fixed according to the number of FY0K. Since all the drones are evenly distributed on the circumference, the corresponding coordinates are $(R,\ (k-1)\alpha)$, where $K \neq 0,1$.

Then, assuming that the polar coordinate position of the UAV receiving the signal is $\rho(x, \theta)$, because the position of the remaining UAV is slightly different, so both parameters need to be determined. Next, let's assume the Angle between it and the three transmitting drones. Suppose that the angle between FY00 and FY01 is α_1, the angle between ρ and FY00 and FY0K is α_2, and the angle between ρ and FY01 and FY0K is α_3. Known information is needed to determine the polar coordinates of the receiving UAV(x, θ).

This paper connect the UAV with slight deviation to the determined UAV position, and solve the polar coordinates (x, θ) by using the sine theorem. The different values of K are discussed due to the perspective involved:

Case i: When K=2, there is no other UAV between the two peripheral determined drones: The distribution is visualized as shown in Figure 1 below:

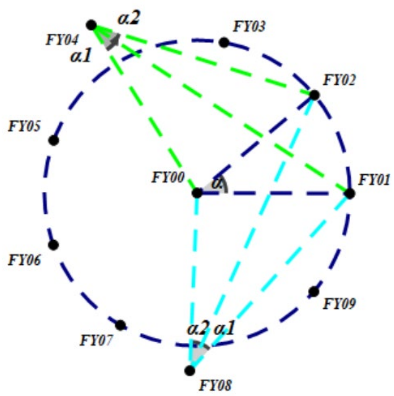

Figure 1: There are no other UAV map between the two peripheral identified UAVs

It can be seen from Figure 1: if $\alpha_1 \leq \alpha_2$(as shown in green), this paper investigate the triangle composed of FY00, FY01 and FY0K and the triangle composed of FY00, FY02 and FY0K, according to the sinusoidal theorem:

$$\begin{cases} \frac{sin(\alpha_1)}{R} = \frac{sin(\pi-\theta-\alpha_1)}{x} \\ \frac{sin(\alpha_2)}{R} = \frac{sin(\pi-(\theta-(k-1)\alpha+\alpha_2))}{x}, \end{cases} \tag{1}$$

Solution is given by Equation (1):

$$\begin{cases} \theta = arctan(\frac{sin(\alpha_1)*sin(\alpha_2)-sin(\alpha_1)*sin(\alpha_2-(k-1)\alpha)}{sin(\alpha_1)*cos(\alpha_2-(k-1)\alpha)-sin(\alpha_2)*cos(\alpha_1)}) \\ x = \frac{sin(\theta+\alpha_1)*R}{sin(\alpha_1)} \end{cases}; \tag{2}$$

According to Figure 1, if$\alpha_1 > \alpha_2$ (the position shown in blue), it investigate the triangles of FY00, FY01 and FY0K and the triangles of FY00, FY02 and FY0K, according to the sine theorem:

$$\begin{cases} \frac{sin(\alpha_1)}{R} = \frac{sin(\pi-\theta-\alpha_1)}{x} \\ \frac{sin(\alpha_2)}{R} = \frac{sin(\pi-((2\pi-\theta)+(k-1)\alpha+\alpha_2))}{x}, \end{cases} \tag{3}$$

Based by equation (3):

$$\begin{cases}\theta = arctan(\frac{sin(V)*sin(\alpha_2)+sin(\alpha_1)*sin(\alpha_2+(k-1)\alpha)}{sin(\alpha_1)*cos(\alpha_2+(k-1)\alpha)-sin(\alpha_2)*cos(\alpha_1)}) \\ x = \frac{sin(\theta+\alpha_1)*R}{sin(\alpha_1)}\end{cases}; \quad (4)$$

Case ii: When K=3,4,5, this paper need to judge whether the biased UAV is sandwiched between the two determined positions according to the size α_3: the visual analysis of the distribution is shown in Figure 2

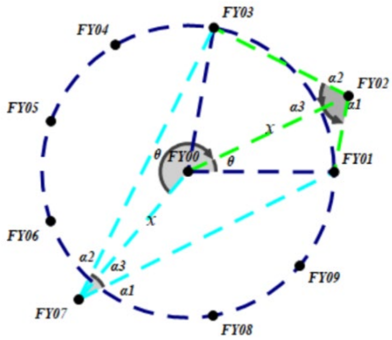

Figure 2: Case ii a visual analysis of the distribution of the UAV

When $\alpha_3 < \frac{\pi}{2}, \alpha_2 = \alpha_3 + \alpha_1$,Figure 2: If the UAV is not between the two aircraft at this time, consistent with the previous situation, the equation is listed as:

$$\begin{cases}\frac{sin(\alpha_1)}{R} = \frac{sin(\pi-\theta-\alpha_1)}{x} \\ \frac{sin(\alpha_2)}{R} = \frac{sin(\pi-(\theta-(k-1)\alpha+\alpha_2))}{x}\end{cases} \quad (5)$$

Solution:

$$\begin{cases}\theta = arctan(\frac{sin(\alpha_1)*sin(\alpha_2)-sin(\alpha_1)*sin(\alpha_2-(k-1)\alpha)}{sin(\alpha_1)*cos(\alpha_2-(k-1)\alpha)-sin(\alpha_2)*cos(\alpha_1)}) \\ x = \frac{sin(\theta+\alpha_1)*R}{sin(\alpha_1)}\end{cases}; \quad (6)$$

When $\alpha_3 < \frac{\pi}{2}, \alpha_3 = \alpha_2 + \alpha_1$,Figure 5: If the UAV is not between the two aircraft at this time, consistent with the previous situation, the equation is listed as:

$$\begin{cases}\frac{sin(\alpha_1)}{R} = \frac{sin(\pi-\theta-\alpha_1)}{x} \\ \frac{sin(\alpha_2)}{R} = \frac{sin(\pi-((k-1)\alpha-\theta+\alpha_2))}{x}\end{cases} \quad (7)$$

When $\alpha_3 \geq \frac{\pi}{2}$,Figure 2: If the UAV is not between the two aircraft at this time, it investigate the triangle composed of FY00, FY01, FY0K and the triangle composed of FY00, FY02 and FY0K. According to the sinusoidal theorem:

$$\begin{cases}\frac{sin(\alpha_1)}{R} = \frac{sin(\pi-\theta-\alpha_1)}{x} \\ \frac{sin(\alpha_2)}{R} = \frac{sin(\pi-(2\pi-(\theta+(k-1)\alpha)+\alpha_2))}{x}\end{cases}; \quad (8)$$

Case iii: When $5 < K \leq 9$,When the case corresponds to 11-K (1 <K<5), just use $(K-1)\alpha$ and $(10-K)\alpha$ to replace the equation at 1 <K<5.

(2)Establishment and analysis of the UAV in formation flight model

1) Establishment of the effective localization model

First of all, when there is only one unidentified UAV transmitting signal, it cannot meet the conditions this paper need, so now the paper is considering the two aircraft.

In the second small question, the number of the UAV on the circumference except FY00 and FY01 is unknown, so the two-station cross-positioning calculation method is used. The two drones on the

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circumference are FY0K1, FY0K2 their coordinate are $(R\cos(k_1\alpha), R\sin(k_1\alpha))$ and $(R\cos(k_2\alpha), R\sin(k_2\alpha))$ ($k_1 \neq k_2$ $k_1, k_2 \neq 0,1$), Its coordinates are respectively. In order to find the coordinates of the receiving signal UAV, paper consider the triangulation method for the four transmitting signals shown in the following table respectively, which can get the coordinate expressions of the receiving signal UAV about, and, respectively. The specific location, classification situation and the number of the UAV are shown in the following Table 2:

Table 2: Classification situation

classify	UAV number		
1	FY00	FY01	FY0K1
2	FY00	FY01	FY0K2
3	FY00	FY0K1	FY0K2
4	FY01	FY0K1	FY0K2

2) Analysis of the model

The problem is solved according to the above classification. The solution steps are as follows:

1) According to FY00 (0,0), FY01(0,R) and coordinates and angles between FY00 and FY01 is α_1, center is O_1, coordinate is (x_1, y_1) and radius is r_1, a system of equations:

$$\begin{cases} (x_1 - 0)^2 + y_1^2 = r_1^2 \\ (x_1 - R)^2 + y_1^2 = r_1^2 \\ R^2 = 2r_1^2(1 - cos2\alpha_1) \end{cases} \tag{9}$$

So as to solve the solution with (x_1, y_1) and r_1.

$$\begin{cases} r_1^2 = \frac{R^2}{2(1-cos(2\alpha_1))} \\ x_1^2 = \frac{R^2}{4} \\ y_1^2 = \frac{R^2}{2(1-cos(2\alpha_1))} - \frac{R^2}{4} \end{cases} \tag{10}$$

Similarly, by the UAV FY00, FY0K1, ρ And points with FY00, FY0K1 can determine α_2, O_2, (x_2, y_2) and r_2 .The size of the circle, whose coordinate and radius are, the following equations can be listed:

$$\begin{cases} (x_2 + 0)^2 + y_2^2 = r_2^2 \\ (x_2 - Rcos(k_1\alpha))^2 + (y_2 - Rsin(k_1\alpha))^2 = r_2^2 \\ R^2 = 2r_2^2(1 - cos2\alpha_1) \end{cases} \tag{11}$$

And from this can obtain (x_2, y_2) and r_2 .

Similarly, according to the number FY0K of two drones that emit signals at the exact location but are not clearly numbered1, FY0K2, K_1 K_2 At this time, both are unknown, similar to the above process, then can have:

$$\begin{cases} (x_3 - Rcos(k_1\alpha))^2 + (y_3 - Rsin(k_1\alpha))^2 = r_3^2 \\ (x_3 - R)^2 + {y_3}^2 = r_3^2 \\ (R - Rcos(k_1\alpha))^2 + (1 - Rsin(k_1\alpha))^2 = 2r_3^2(1 - cos2\alpha_1) \end{cases} \tag{12}$$

The center coordinates can be obtained with (x_3, y_3) and r_3 .

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$$\begin{cases} r_3^2 = \dfrac{R^2(1-\cos(k_1\alpha))^2 + (1-\sin(k_1\alpha)R)^2}{2(1-\cos(2\alpha_3))}, \\ y_3 = \dfrac{1-\cos(k_1\alpha)}{\sin(k_1\alpha)} x_3, \\ x_3 = \dfrac{R}{1+\dfrac{(1-\cos(k_1\alpha))^2}{\sin(k_1\alpha)^2}} + \sqrt{\dfrac{R^2}{1+\dfrac{(1-\cos(k_1\alpha))^2}{\sin(k_1\alpha)^2}} - \dfrac{r_3^2}{1+\dfrac{(1-\cos(k_1\alpha))^2}{\sin(k_1\alpha)^2}}} \end{cases} \tag{13}$$

Similarly, the available and radius correlation equations are the same as (12): thus available and radius.it can be obtained with (x_4, y_4) and r_4.

2) In order to obtain the coordinates of the points, the elements in our three equations:

$$\begin{cases} (x_1 - x)^2 + (y_1 - y)^2 = r_1^2 \\ (x_2 - x)^2 + (y_2 - y)^2 = r_2^2 \\ (x_3 - x)^2 + (y_3 - y)^2 = r_3^2 \\ (x_4 - x)^2 + (y_4 - y)^2 = r_4^2 \end{cases} \tag{14}$$

3) Since the position of the UAV is only slightly deviated, the Angle between them can be judged by the signals emitted by K1 and K2, so as to get their corresponding center Angle. Although it is a slight deviation, but can judge the difference between, so as to get the relationship between.

Finally, the process of establishing of the effective localization model can be transformed into solving the least squares problem, and the above equations can be solved. Using the uniqueness of coordinates, the paper can eliminate the coordinates, the paper can determine the effective positioning. Therefore, in addition to FY00 and FY01, two other signal transmitting drones are needed to determine the effective positioning of the passive receiving signal UAV.Finally, the accuracy of the least-squares method is verified.

(3)Establishment and analysis of the UAV geometry model

1) Establishment of the geometric model

The target splitting method is used to discuss the classification, using only one of the cases as an example, and finally to verify the universality of the results.

This problem requires that when the position of the UAV deviates, the position information received by the UAV can be adjusted several times, so that except for the number 0, the other nine UAVs are finally evenly distributed on a certain circumference. According to the position information sent by the UAV, if the UAV is very limited, adjust the position to a certain circumference through the received position information, and evenly distributed, requiring the UAV to transmit more signals to help adjust. In order to control the number of drones to adjust the position of the UAV with less signal, paper choose the method of target decomposition to solve this problem. According to the question setting, the solution target of the establishment of UAV geometry model is divided into two parts: 1.One and nine drones are eventually distributed in a certain circle. 2.Make the nine drones evenly distributed on the circumference.

2) Analysis and use of geometric models

Step 1, determine the adjustment direction. Looking closely at the data, the UAV 3,6,9 orientation, 2,5,8 orientation is already on a circle, so the source is FY03, FY06, FY06, FY09, FY00 or FY02, FY05, FY08, FY00, while the rest passively receives information to the circumference with a radius of 112 or 98. The description of this question takes an example of 112 radius circumference. The unof drone UAV is as below Figure 3.

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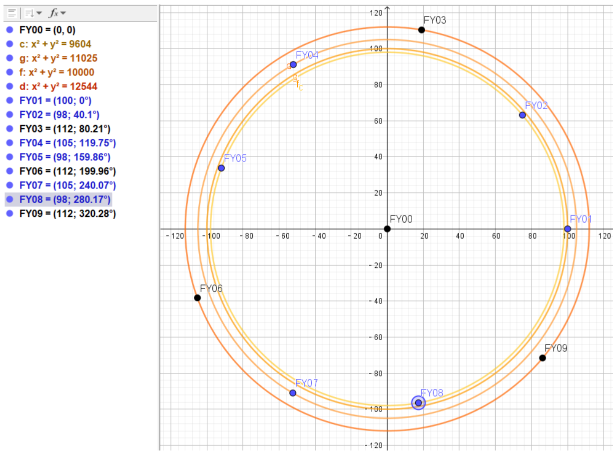

Figure 3: Map of the location of the unmobilized drone

Step 2, Acquisition of the emission source direction information. In the circle as the launch source, pairwise choose to transmit signals together with FY00, while the rest passively receive the information. Get the orientation information of FY03, FY06, and FY09 as shown in the Figure 4.

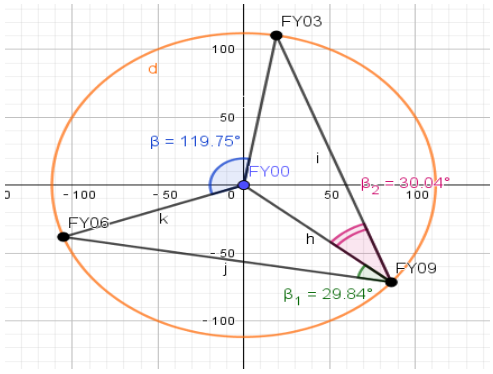

Figure 4: Direction infographic of FY03,FY06,FY09

Take FY09 as an example, the available information is $\beta_{1,}\beta_2,\beta_3\ and\ \beta = \beta_1 + \beta_2$.

Step 3, Limit variables to moving. FY00, FY03, FY06, FY09 are used as the signal transmitting source to control the movement of other UAVs. This question takes the movement of FY01 as an example. The control remains constant, and FY01 moves on the circumference of FY03, FY00, and FY01 (initial position). The circumference of the 112 radius is the largest circumference, so the control FY01 to move in the direction of the decrease. When $\lambda_3 = \frac{1}{2}\lambda_4$, Movement was stopped and YF01 reached a radius of 112 as shown in Figure 5.

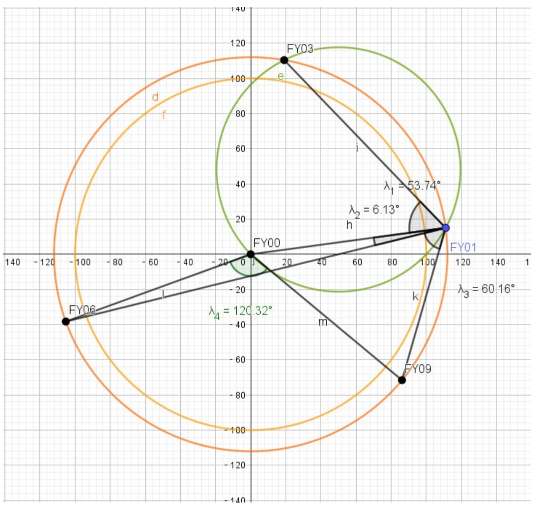

Figure 5: Map of FY00, FY03, FY06, FY09 are used as signal emission

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This question also needs to discuss the selection of the invariant Angle. Since the symmetry of the circle only needs to discuss both FY01 and FY02, if the UAV in both positions can move directionally while only knowing the direction information, then the rest of the UAV can also move.

One of FY03, FY06, and FY09 was randomly selected with FY00 and FY01. The three points determine a circle. Since the newly obtained circle already intersects the circle with radius 112, and△FY00FY01FY0X cannot be a right triangle, the two circles are tangent. That is, the selection of FY03, FY06, and FY09 is not restricted. FY01 moved accurately using the center angle of the circle in a 112 radius of the remaining two UAVs as shown in Figure 6.

From the above discussion, one of the choices from $\lambda_1, \lambda_2, \lambda_3$ to do constants is valid. For the convenience of code writing, specify the angle of FY0X, FY0(X+1) or FY0X, FY0(X+2) and FY00, FY01 as a constant.

Step 4, nine aliquot on the circumference. Three drones, FY0X, FY0X and FY0 Z, are selected. The three UAVs can be arranged counterclockwise on the circle. Keep the same, first judge whether it is greater than 70 degrees. When greater than 70 degrees, move FY0Z to decreased angle to stop to 70 degrees. When less than 70 degrees, ask FY0Z to move in the direction of increased angle to stop to 70 degrees.

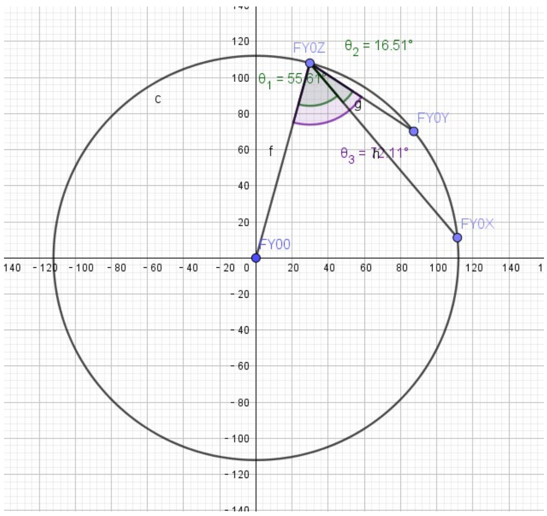

Figure 6: Map of the designated locations of the nine drones

When the position of FY0Z is determined, FY0Z becomes the new FY0Y, as FY0Y '; FY0Y as FY0X, as FY0X'; FY0(Z + 1) becomes the new FY0Z as FY0Z '. Repeat the process four, nine drones in turn to reach the designated position.

According to the above method, this paper need at most three UAVs to transmit signals in circumference, so that when the position of the position information can be adjusted to a certain circumference and evenly distributed as shown in Figure 7.

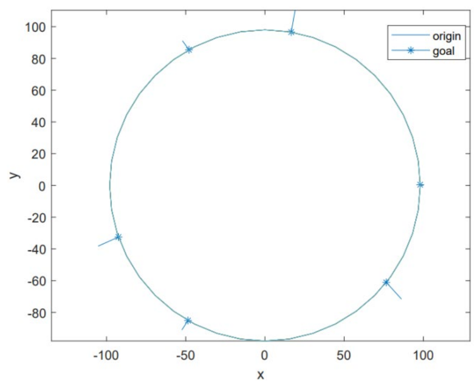

Figure 7: Simulation diagram of the transmission signals of three UAVs

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The paper conduct a simulation process of this process, and remember the schematic diagram of the above process.

5. Conclusion

This paper examines the problem of how to form with limited information in the silent case of the UAV. Firstly, the cosine theorem is used to obtain the function relationship between distance and direction information; in the case of unknown number, the cross positioning is used to determine the number and then locate; Finally, a simple geometric model is established to limit the flight, so that the angle between the UAV and any two transmitting signal drones is fixed, and the relative position relationship with other drones in formation is unchanged.

To sum up, it can be concluded that when the UAV group is at different flying altitude and no transmitting station nearby provides signals, three UAVs with launching function should be determined in the UAV group, so that the position information of each uav in the UAV group can be accurately determined. Thus, the coordinate of each UAV is further adjusted to ensure the accuracy of the UAV formation flight.

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