

# A Survey of Design and Implementation of Phase Noise Optimization Based on OFDM System

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**Abstract:** *Orthogonal Frequency Division Multiplexing (OFDM) is a kind of modulation technology with strong resistance to frequency selective fading and high frequency spectrum utilization. It is a kind of multi-carrier transmission scheme with the lowest implementation complexity and the most widely used. However, the whole OFDM system is particularly strict on the orthogonality between subcarriers. Any small carrier frequency offset will destroy the orthogonality between subcarriers, so it is sensitive to local oscillator phase noise. This paper analyzes and summarizes the existing phase noise optimization research, and a phase noise optimization algorithm based on feedback decision is proposed. Through the simulation of phase noise elimination algorithm at the receiver, it is verified that the phase noise of OFDM system can be suppressed.*

**Keywords:** *Phase noise, OFDM, Feedback decision, MMSE*

## 1. Introduction

In recent years, there have been many researches and analyses on the influence of phase noise on OFDM system, and many methods of phase noise compensation have been proposed. Phase noise affects OFDM signals mainly in two ways: Inter-carrier Interference (ICI) and Common Phase Error (CPE) [1]. Phase noise can be suppressed by using a better oscillator to improve the manufacturing process, but it increases the system cost [2]. Therefore, it is an urgent problem to study the optimization method to accurately estimate and compensate phase noise in OFDM system.

## 2. Phase Noise Elimination Method

### 2.1. Frequency Domain Elimination Algorithm

Phase noise can be decomposed into common phase noise and intercarrier interference in frequency domain. S Wu and Y Bar-Ness [4] first estimates CPE with pilot information, then estimates the phase noise in frequency domain using maximum likelihood estimation method based on the decision information after CPE error correction, and finally uses ICI cancellation method to eliminate the influence of phase noise. S Wu and Y Bar-Ness [5] further simplifies the algorithm in their last article [4], greatly reducing the implementation complexity of the algorithm while ensuring the performance of the algorithm.

### 2.2. Time Domain Elimination Algorithm

The phase noise elimination in time domain is estimated by inserting certain pilot information to further eliminate the phase noise. The methods proposed in J. Bingham [6] and S.L. Biracree, R.A. Casas and A.E. Youtz [7] are basically the same, that is, phase noise is decomposed into a series of linear discrete time domain components, and the least square method can be used to estimate the transmission symbols under time domain conditions. However, for random phase noise, the frequency band utilization is low and the efficiency is poor in fading channels.

## 3. OFDM Phase Noise Optimization

### 3.1. Phase Noise Definition

The ideal state of the signal source is to have an output at a single frequency, that is, in the frequency

domain its spectrum is an infinitely narrow line ( $\delta$  function). But in practice, the spectral line of the signal source has a certain bandwidth, which is accompanied by random phase drift and periodic stray interference, which is called phase noise.

An ideal signal source can be represented by a standard sine wave

$$V(t) = V_0 \sin 2\pi f_0 t \tag{1}$$

Where  $V(t)$  is the instantaneous signal voltage,  $f_0$  is the signal frequency,  $V_0$  is the signal peak voltage amplitude.

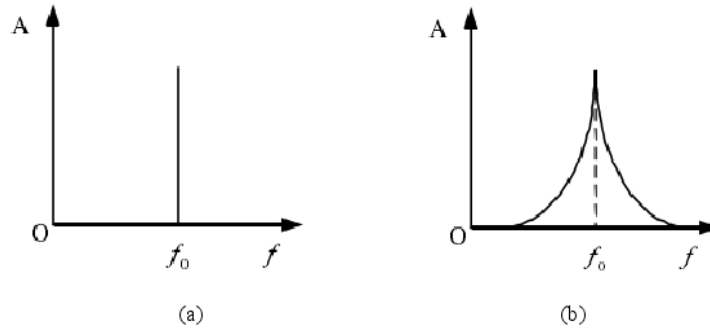


Figure 1: Sinusoidal signal frequency spectrum

In practical application, since any sinusoidal signal is unstable, that is, there are amplitude, frequency and phase fluctuations of signal interference, (actual output) becomes

$$V(t) = [V_0 + \varepsilon(t)] \sin[2\pi f_0 t + \Delta\varphi(t)] \tag{2}$$

In the formula,  $\varepsilon(t)$  is the instantaneous amplitude fluctuation,  $\Delta\varphi(t)$  is the instantaneous phase fluctuation.

The signal spectrum is shown in Figure 1.(b). In general  $\varepsilon(t) \ll V_0$ , then, Equation 2.1-2 can be rewritten as

$$V(t) = V_0 \sin[2\pi f_0 t + \Delta\varphi(t)] \tag{3}$$

In general, phase noise can be expressed as:

$$\Delta\varphi(t) = \Delta\varphi_n(t) + \Delta\varphi_{mi}(t) \cos \omega_{m1} t + \dots \tag{4}$$

In the formula,  $\Delta\varphi_n(t)$  is the random phase noise,  $\Delta\varphi_{mi}(t) \cos \omega_{mi} t (i = 1, 2, 3 \dots)$  is periodic stray interference.

### 3.2. Phase Noise Model

The characteristics of the phase noise in the local oscillator are generally represented by the power spectral density  $S_\phi(f)$ .

$$\begin{aligned} S_\phi(f) &= 10^{-e} + 10^{-a}, |f| < f_1 \\ &= 10^{-e} + 10^{-\frac{(f-f_1)b}{f_2-f_1}-a}, f > f_1 \\ &= 10^{-e} + 10^{-\frac{(f+f_1)b}{f_2-f_1}-a}, f < -f_1 \\ &= 10^{-e} + 0, |f| > f_1 \end{aligned} \tag{5}$$

In the formula, parameter  $a$  is the power Spectrum density (PSD) from the center frequency to  $f_1$ , which determines the phase noise size within the loop bandwidth. The parameter  $f_1$  is the 3dB bandwidth of the phase noise power spectral density;  $b$  represents the roll down factor of phase noise frequency from  $f_1$  to  $f_2$ . When the frequency is greater than  $f_2$  the noise is represented by parameter  $c$ , that is, the level of the white noise base. Typical values of each parameter:  $a = 6.5, b = 4, c = 10.5, f_1 = 1\text{kHz}, f_2 = 10\text{kHz}$ . It can be seen that the energy of phase noise is mainly concentrated in several adjacent subcarrier channels.

### 3.3. Influence of Phase Noise on OFDM System

The basic block diagram of OFDM system disturbed by phase noise is shown in Figure 2. Phase noise is a multiplicative interference in time domain,  $X_m(n)$  is the transmitting end baseband signal;  $Y_m(n)$  is the baseband signal of the receiving end,  $X_m(k)$  represents the modulated signal on the  $k$  subcarrier of the  $m$  OFDM symbol,  $k = 0, 1, \dots, N - 1$ . Assume that the receiver is fully synchronized.

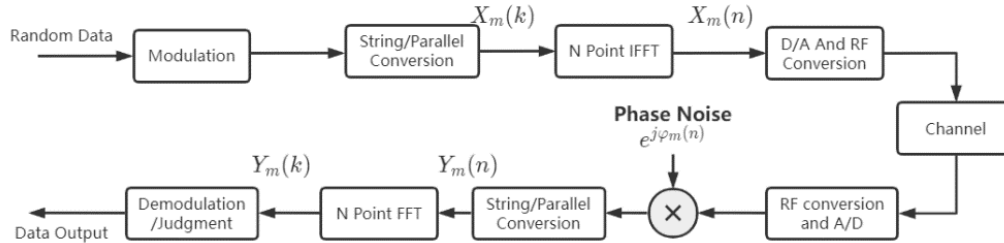


Figure 2: OFDM system with phase noise

Phase noise can be modeled by a phase rotation  $e^{j\varphi(n)}$ . Considering the signal passing through the channel  $h(n)$ , the  $N$  sampling point of an OFDM signal can be expressed as:

$$y(n) = (x(n) * h(n)) \cdot e^{j\varphi(n)} + w(n) \tag{6}$$

In the formula,  $*$  denotes convolution,  $\varphi(n)$  is phase noise,  $w(n)$  is additive White Gaussian noise with variance  $\sigma^2$ . The receiver performs fast Fourier transform (FFT) of  $y(n)$  for OFDM demodulation. The frequency domain of the sample value can be expressed as:

$$Y(k) = X(n)H(n)Q(0) + \sum_{\substack{l=0 \\ l \neq k}}^{N-1} X(l)H(l)Q(l-k) + w(k) \tag{7}$$

According to formula (7), even if the whole constellation of CPE has random rotation  $Q(0)$ , in the same OFDM system,  $Q(0)$ , as a constant complex number, has the same effect on each subcarrier. Therefore, pilot frequency can be used to track and suppress CPE.

### 3.4. Phase Noise Optimization Algorithm Based on Feedback Decision

First introduce the MMSE equalization algorithm, the MMSE algorithm is described in detail in Songping Wu and Y. Bar-Ness [8]. MMSE algorithm uses the minimum mean square error criterion (MMSE) to obtain the equilibrium coefficient  $C_{MMSE}$  in the frequency domain. Then the estimated value of the sent data symbol is:

$$\hat{S}_m(k) = R_m(k) * C_{MMSE}(k) \tag{8}$$

Where,  $C_{MMSE}$  can be obtained as follows:

$$C_{MMSE}(k) = \frac{\hat{P}_m^*(0)H_m^*(k)}{|\hat{P}_m(0)H_m(k)| + \frac{\delta_v^2}{E_x}} \tag{9}$$

Where,  $P_0$  represents the estimated value of CPE,  $\delta_v^2$  represents the channel noise and the power of ICI,  $E_x$  represents the average energy of the signal sent, which can be estimated by the power of the signal on the empty subcarrier at the receiving end.

$$\delta_v^2 = \frac{1}{N_N} \sum_{k \in S_N} |R_m(k)|^2 \tag{10}$$

$N_N$  represents the number of empty subcarriers in OFDM system, and  $S_N$  represents the set of empty subcarriers.

In an OFDM system, CPE has the same effect on each subcarrier in an OFDM symbol period due to the common phase error. In this way, the CPE of the pilot subcarrier can be estimated by using the pilot

signal, and then the CPE estimation corresponding to the pilot signal can be used to compensate other data subcarriers.

The specific implementation process is as follows:

1) Firstly, the pilot subcarrier is used to complete the MMSE equalization algorithm, and the data symbol after equalization is obtained.

2) CPE is estimated by data subcarrier, and the estimated value of data subcarrier of CPE is as follows:

$$Q_D(0) = \frac{\sum_{k \in S_D} Y_D(k) X_D^*(k) H_D^*(k)}{\sum_{k \in S_D} |X_D^*(k) H_D^*(k)|^2} \quad (11)$$

3) The CPE estimation based on pilot frequency and the CPE estimation based on the main clause is used to optimize the CPE, and the optimization factor is p (p is usually 0~0.1). After optimization, the following results are obtained:

$$Q(0) = pQ_p(0) + (1 - p)Q_D(0) \quad (12)$$

4) The optimized CPE value is used to calculate the equalization coefficient in frequency domain, and the data is equalized.

5) Repeat steps (1) to (4) until satisfactory performance is obtained.

#### 4. Results and Analysis

In order to verify the effectiveness of the proposed algorithm, different degrees of simulation experiments are carried out.

Table 1: The parameters of the relationship among modulation methods, coding efficiency and data rate.

Data rate (Mb/s)	Modulation method	Coding efficiency	Encoding bit/subcarrier( $N_{BPSK}$ )	Data bit /OFDM symbol( $N_{DPFS}$ )	Encoding bit /OFDM symbol( $N_{CBPS}$ )
6	BPSK	1/2	1	24	4
9	BPSK	3/4	1	36	48
12	QPSK	1/2	2	48	96
18	QPSK	3/4	2	72	96
24	16	1/2	4	96	192
36	14	3/4	4	144	192
48	64	2/3	6	192	288
54	64	3/4	6	216	288

An OFDM system, data transmission using 48 subcarriers. When 16QAM or BPSK modulation is used, the final coding rate can be provided as  $48 \times 4 \times 1/4\mu s = 48Mb/s$  and  $48 \times 1 \times 1/4\mu s = 12Mb/s$  respectively. The above table shows the corresponding relationship between modulation mode, data efficiency and coding rate.

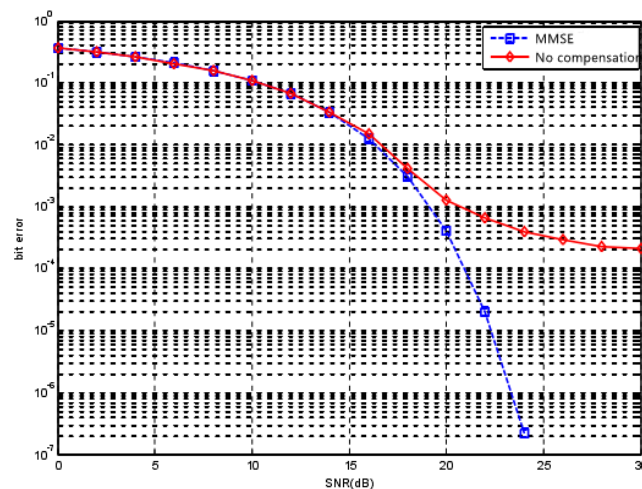


Figure 3: 16 QAM modulation mode OFDM System Performance Comparison [3]

Fig. 3 shows the influence of phase noise on OFDM system performance when modulation mode is

16QAM and the improvement effect of decision feedback algorithm on performance. The simulation results in the figure confirm that phase noise has a considerable influence on the performance of OFDM system. By using the decision feedback algorithm proposed in Songping Wu and Y. Bar-Ness [8], the system performance is greatly improved.

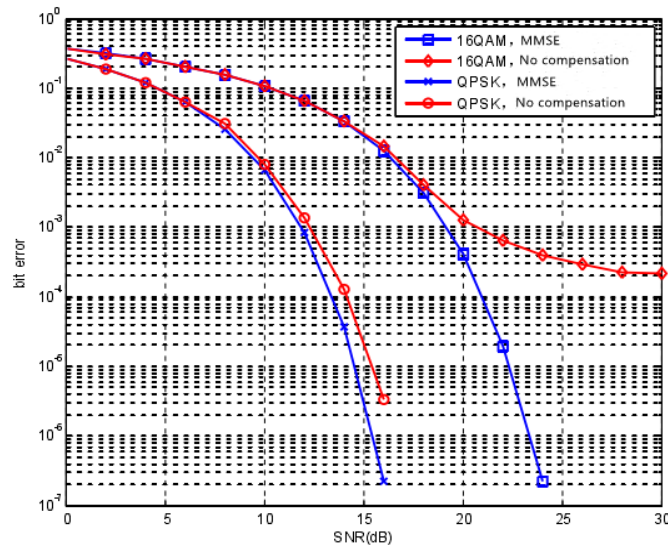


Figure 4: Performance Comparison between Different Modulation OFDM systems [3]

Fig. 4 shows the performance comparison of OFDM systems with modulation modes of QPSK and 16QAM respectively. Obviously, phase noise has a great influence on the performance of OFDM systems, and the influence degree varies with different modulation modes. As shown in Fig. 4, 16QAM is much more severely affected by phase noise than QPSK, which means that the higher the modulation order (that is, the closer the distance between constellation points), the more obviously the OFDM system is affected by phase noise.

## 5. Conclusion

This paper introduces the definition and model of phase noise in detail, discusses the impact of phase noise on OFDM system performance, and studies the previous impact and elimination methods of eliminating CPE and ICI. A phase noise optimization algorithm based on feedback decision is proposed. In simple terms, a CPE estimation module feedback loop is added to the original OFDM system. The decision feedback algorithm has the advantages of low algorithm complexity and simple hardware implementation. In the process of hardware implementation, CPE estimation and correction can be realized only by using simple multiplication module and corresponding summation module.

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