The relationship between approximate number system and mathematical ability of children with mathematical difficulties

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Abstract: Mathematics ability is important for people to live and study. However, some children are trapped in difficulties in mathematics learning and lack of development of mathematics ability, which is not conducive to their good academic achievements and future development. Mathematics learning difficulty refers to the phenomenon that children obviously lag behind their peers in mathematics learning due to the deficiency of mathematics ability. The ANS (approximate number system) is a kind of nonverbal ability to represent number approximately and this intuitive and nonverbal sense of number may be the basis for the development of children's advanced mathematics ability. Previous studies have found that ANS and mathematical ability interact with each other, which can further explain the reasons for the lack of mathematical ability of children with mathematical difficulties, and improve their mathematical ability from the approximate number system.

Keywords: Mathematics learning difficulty, approximate number system, mathematics ability

1. Introduction

Learning difficulties are a kind of hot research topic in the field of special education, and also a common phenomenon in basic teaching. Some students are different from others in daily learning activities such as language acceptance and expression, reading comprehension, mathematical calculation, etc. They are not as good as ordinary students in language expression, concentration and reaction ability when they are not mentally retarded and have no obvious physical defects, but there are obvious phenomena such as insufficient study investment and study lagging comparing with their peers.

Learning Difficulty (LD), also known as learning disability (LD), was first proposed by Kirk, an American scholar, in 1962. He believed that learning difficulty refers to the process of speaking, language, reading, writing, calculation or other school subjects. Difficulties in learning due to possible brain dysfunction or psychological disorders caused by emotional or behavioral disorders, rather than the result of mental retardation, sensory deprivation or teaching culture factors[1]. Learning difficulties have been widely concerned by scholars and the public since they were put forward. According to a survey conducted by the National Center for Learning Disabilities in 2017, one in every five American children has learning disabilities. About 1/16 public schools provide personalized education programs for specific learning disabilities (SLD), such as dyslexia and other health disorders covering ADHD and motor disorders.[2] According to a survey conducted by scholars in China in 2016, the detection rate of children with learning difficulties was 10.09%.[3] It is consistent with the screening rate of most scholars.[4] It can be seen that the number of students with learning difficulties is enough to arouse widespread concern in society, and at the same time, it also urges scholars and front-line educators to continue to explore the clues of psychological obstacles behind learning difficulties and effective practices on how to deal with learning difficulties.

Learning difficulties are mainly divided into dyslexia and mathematical difficulties. Dyslexia refers to the fact that individuals with normal intelligence still have difficulty in accurately recognizing words or spelling fluently under full guidance. Compared with dyslexia, there are few studies focusing on mathematical difficulties. Mathematical difficulties are considered as the difficulties in dealing with numbers[6] or in mapping digital symbols to psychological representations of numbers[7], which individuals with mathematical difficulties need to master number sense arithmetic operation knowledge and reasoning ability in the process of mathematical learning while lacking in quantitative relations, computing ability, spatial ability and problem solving.[8] People tried to find the causes and correction
methods of problems from the perspective of biological and neurological factors in the early days and then turned to the study of defects in psychological processing. After the birth of cognitive psychology, it has gradually become a dominant paradigm to study mathematics learning difficulties from the perspective of information processing, involving attention, sensory input, working memory, long-term memory, executive control and many other aspects of information processing.

ANS (Approximate Number System) and PNS (Precise Number System) together constitute the human quantity representation system,[9] The two of them are independent of each other, and the PNS refers to the system that characterizes small numbers below 3; ANS refers to a system that characterizes large numbers above 3.[10] Individuals approximate abstract quantity and approximate number through ANS, which is a manifestation of mathematical cognitive ability.[11] ANS is measured by accuracy, which is considered to be related to mathematical ability. Compared with general cognitive ability, approximate number system is a special cognitive ability in mathematics, which is more likely to be the basis for children's future mathematical ability development,[12] that is, the higher the accuracy of approximate number system, the better their future mathematical ability.

Mathematical ability is a kind of psychological ability, which can help a person to successfully complete mathematical activities and solve practical problems by using what he has learned,[13] and it can be divided into mathematical operation ability, logical thinking ability and spatial concept.[14]

Mathematics, as the most basic subject of natural science, is an indispensable basic subject in primary and secondary schools, and an indispensable subject for developing intelligence and cultivating students' ability. At the same time, it is also the basic subject of school education, and the study of mathematics directly affects students' study of other subjects, which is the guarantee of students' success in learning. In students' learning, mathematics difficulty has become a problem that puzzles a considerable number of students, and more and more students with mathematics learning difficulties have appeared. Many students are bored with learning and lose confidence in learning because of mathematics learning difficulties. How to effectively transform students with learning difficulties in mathematics and make them get rid of "difficulties" as soon as possible is a very difficult problem for many front-line mathematics teachers, and it is also a big problem that puzzles parents, students and even the education sector.

From our practice of mathematics education, it is not difficult to see that while cultivating a large number of talents, there are also more and more students with learning difficulties in mathematics, which brings considerable pressure to teaching and makes basic education and mathematics teachers face considerable difficulties. Therefore, this paper takes the approximate number system as the starting point to discuss the relationship between the approximate number system and mathematical ability of children with mathematical difficulties, and provide some theoretical basis for improving the practice of mathematical ability of children with mathematical difficulties in the future.

2. The mathematical ability defects of children with mathematical learning difficulties

Mathematics learning difficulties affect children's acquisition of mathematical skills and development of mathematical ability. They have difficulties in understanding simple mathematical concepts and lack intuitive ability to understand numbers. Children with learning difficulties in mathematics show defects in mathematics learning, including low understanding of the principle of counting, difficulty in extracting mathematical facts, inaccurate operation, poor ability to represent mathematical problems and difficulties in solving problems. A large number of studies have found that children with early mathematics learning difficulties show persistent backwardness in the extraction of mathematical facts, and their accuracy and fluency are worse than those of ordinary peers in simple calculation tasks with limited time.

Brain science researches have found that the difficulty in mathematics learning originates from the parietal lobe of the brain, and the core defect of mathematics learning difficulty may be caused by the defect of ANS. Number sense can predict children's future math academic performance, and the comparison between sense number and quantity of 3-6-year-old children is related to representation and operation. For example, if you don't need to count three or less objects, count sets within five and know that the number of the last object represents an integer of the set, some studies have found that children with math learning difficulties show poor number sense,[15] and children with math difficulties show difficulties in number sense. A study on children aged 5-6 whose mathematical ability is less than 10%, and found that these children have defects in the unsigned approximate quantitative representation system.[16] Approximate number system is the basic system for the development of children's number knowledge and computing ability. If children have defects in this system, children will have difficulties
in symbolic representation and non-symbolic representation of numbers.

However, the hypothesis about the relationship between ANS and mathematical ability shows that ANS is not necessarily the basis of mathematical ability. There are four hypotheses between ANS and mathematical ability: (1) ANS is positively related to mathematical ability, and the higher the accuracy of ANS, the better the development of mathematical ability; (2) There is a positive correlation between mathematical ability and ANS. The better the mathematical ability, the higher the accuracy of ANS. (3) The relationship between ANS and mathematical ability is two-way, ANS affects mathematical ability, and mathematical ability also affects ANS; (4) There is no correlation between ANS and mathematical ability, but they are independent systems.

There are many researches on approximate number systems at present, and four hypotheses have been proved. However, there are not many related studies on the group with mathematical learning difficulties. It is very important to clarify the relationship between the ANS and mathematical ability of the group with mathematical learning difficulties, because it is related to whether the follow-up intervention training for this group should be based on the intervention of the ANS.

3. The relationship between ANS and mathematical ability of children with mathematical learning difficulties

Li Ye et al found through research that the accuracy of ANS and mathematical ability of children with mathematical learning difficulties are lower than those of ordinary children, and it was found that the Weber score of children with mathematical learning difficulties was significantly negatively correlated with their logical space ability, mathematical operation and mathematical ability through correlation analysis, that is, the higher the approximate number accuracy of children with mathematical learning difficulties, the better their logical space ability, mathematical operation and mathematical ability\(^\text{[17]}\). Mazoooco found that the accuracy of approximate number system can distinguish children with mathematical learning difficulties from children without it. The lower the accuracy of approximate number system, the higher the possibility of children with mathematical learning difficulties and the lower their mathematical ability\(^\text{[18]}\). An intervention study found that children with dyscalculia have difficulty in obtaining flexible and automatic quantitative representations\(^\text{[19]}\). The causes of children's mathematics learning difficulties are at least partly due to the defects of ANS. This is consistent with the existing research results. Price et al. used functional magnetic resonance imaging (fMRI) to study the neural connection between children with developmental dyscalculia and their peers with normal development in basic digital processing. The symptom of dyscalculia is that although individuals have sufficient educational resources, they still have difficulties in understanding numbers and arithmetic, which is manifested in the inability to allocate numbers and the difficulty in learning multiplication tables. Although children have a normal level of intelligence, dyscalculia will still lead to obvious backwardness in their academic performance\(^\text{[20]}\). It is found that numerical comparison becomes more and more difficult with the decrease of the distance between comparison numbers. The potential approximate expression of this number measures the accuracy of the approximate number system, which has been proved to regulate the brain activity of bilateral parietal lobes in adults and children\(^\text{[21]}\). The results of this study show that children with developmental dyscalculia have less activity in the parietal sulcus than normal children of the same age.

Many studies have found that the numerical information of various formats and modes is mainly processed in the parietal lobe, and the internal groove of the parietal lobe (IPS) is the basis of the ANS\(^\text{[22]}\). Some scholars have compared the children with very low birth weight but normal IQ with those with very low birth weight and normal IQ without calculation defect, and found that the gray matter volume of children with no calculation defect in a region of left IPS is larger than that of children with calculation difficulty. However, this decrease seems to be specifically aimed at calculation, because there is no difference in brain volume between the two groups compared. These findings emphasize the importance of IPS, and show that the atypical development in this area hinders the development of children's ANS and may eventually hinder the development of children's mathematical ability\(^\text{[23]}\). The research shows that the number sense in infancy can predict the mathematical ability of preschool children, and the number preference score of infants in June is positively correlated with the standard math test score and the comparison score of unsigned numbers at the age of 3.5. Even after controlling the intelligence factors, this relationship still exists, which shows that the sense of numbers in preschool contributes to the acquisition of digital symbols and array abilities and has a unique contribution to mathematical ability\(^\text{[24]}\). Piazza et al found that the development of approximate number system of mathematical difficulties is equivalent to that of 5-year-old normal children in intelligence development\(^\text{[25]}\). The lag of approximate
number system will lead to poor number sense, which may have a potential impact on children's early learning of numeral words and Arabic numerals, making their learning situation lag behind normal children\(^{26}\). Even if children with dyscalculia have good counting and arithmetic skills, they will show subdivision dysfunction\(^{27}\).

From the existing research, mathematical ability will also affect the development of the accuracy of ANS. Scholars had investigated the approximate number accuracy of children and adults among Brazilian aborigines, and found that the approximate number representation level of subjects who received formal school education, especially the instruction of computing skills, was much higher than that of indigenous residents who did not receive mathematics education. Education can significantly improve the accuracy of individual perception of quantity\(^{28}\). A study by Nys et al. shows that, compared with adults who have received mathematics education, adults who have never received mathematics education have longer reaction time and lower correct rate in quantitative comparison tasks, which shows that the approximate quantitative representation ability of adults will be significantly improved through the learning of accurate number knowledge in mathematics education\(^{29}\).

Cheng made a meta-analysis between ANS and mathematical ability, and found that ANS and mathematical ability have a two-way relationship\(^{30}\). He believed that on the one hand, individuals with mathematical learning difficulties may have ANS defects, which is not conducive to their formal solution to mathematical problems. On the other hand, the lack of formal mathematical learning by individuals with mathematical learning difficulties may lead to the slow development of their ANS. Therefore, the intervention of children with mathematics learning difficulties based on ANS may support the development of mathematics ability of children with mathematics learning difficulties.

4. ANS is related or not related to the edge of mathematical ability

There are also inconsistent results in the prediction of mathematical ability by the accuracy of ANS. The results of a longitudinal study used for 11 months show that the knowledge of Arabic numerals at the age of 6 has a significant predictive effect on arithmetic skills, while the accuracy of approximate number system has no significant predictive effect on arithmetic skills\(^{31}\). Mathematical language can predict students' mathematical performance, but ANS accuracy or other cognitive fields can't\(^{32}\). The accuracy of ANS can only significantly predict children's math scores when they are 5 years old, and they lose their predictive ability after they enter school to receive formal education. However, a short-term longitudinal study investigated the relationship between the accuracy of ANS and symbolic mathematics ability of children. The results showed that the approximate number accuracy of 5-year-old children had no predictive effect on their symbolic quantity comparison tasks after 6 months, and there was no correlation between the approximate number accuracy score and the quantitative comparison task score after 6 months\(^{33}\). Butterworth thought that the ANS follows Weber's law and represents quantity in a logarithmic way, while mathematical abilities such as addition and subtraction represent quantity in a linear way\(^{34}\). There are essential differences between them in the way of quantity representation, so there is no relationship between the ANS and mathematical abilities.

A key problem in this kind of research is that it does not control other additional variables, such as inhibition control, spatial memory, working memory and other cognitive and executive functions. Therefore, it is impossible to completely determine whether the accuracy of ANS and mathematical ability are influenced by intelligence, language understanding, executive function, etc. But a study of Fuh and Mcneil found that when inhibition ability was added, the relationship between ANS and mathematical ability was no longer significant\(^{35}\).

5. Intervention training of ANS

ANS can be improved by training, especially for children with mathematics learning difficulties, appropriate training can significantly improve their mathematics ability. The training methods of ANS are mostly based on the symbolic characteristics of non-symbols. Park and Brannon developed an unsigned approximate addition and subtraction training task. The training results of a series of studies show that this task not only effectively improves the arithmetic skills of college students, but also significantly improves the math level of preschool children in low-income families. The training task not only improves the accuracy of ANS, but also improves the math ability\(^{36}\). The number sense training based on individual level can improve the accuracy of children's approximate number system and promote children's mathematics performance\(^{37}\). The computer game training results of "approximate
number representation” show that the arithmetic score of the experimental group is better than that of the control group[38]. The accuracy of approximate number system of children with mathematical difficulties is worse than that of ordinary children, and the accuracy of approximate number system predicts children’s mathematical ability. After training, the accuracy of approximate number system and the value-added of mathematical ability in experimental group are greater than those in control group, and the value-added is significantly different[39].

Acute aerobic exercise is found to intervene the ANS. Wilson et al. made a digital race game, which is a computer game training[40]. It includes two parts: "comparison screen" and "chessboard screen". Nine children with dyscalculia aged 7-9 participated in the digital race game training for four days a week, 30 minutes a day, and trained for five weeks. The results showed that the accuracy and reaction time of unsigned number comparison are improved after training, but the sensitivity of approximate number system has not changed. However, there is no control group in this study, and it is impossible to distinguish whether the improvement in performance is the result of training or the result of attention or motivation. The combination of arithmetic and geometry training improves children's non-symbolic mathematics performance. Compared with letter recognition training, approximate arithmetic training significantly improves the students' non-symbolic mathematical ability, but this effect is limited to children with low mathematical performance[41].

To sum up, on the one hand, for the intervention training of children with mathematical difficulties, it is necessary to start from the different characteristics of different children and supplement different teaching methods according to their weaknesses in different aspects of mathematics learning. On the other hand, research shows that exercise can improve the cognitive ability of individuals[42], and it is also a good method to improve the accuracy of ANS for children with mathematical difficulties by using “exercise + cognitive” training.

6. Summary and prospect

Previous studies paid more attention to the relationship between the accuracy of approximate number system and mathematical ability of adults or children, but less attention was paid to the group with several difficulties. As a group with defects in the development of mathematical ability, it is of far-reaching practical significance to explore the causes of mathematical difficulties.

First of all, the research on ANS is rich abroad, involving the development of brain science, theory and various intervention training. However, in China, the research on ANS is still at the theoretical level, and the research on ANS of the number-poor population is even less. Future research can focus on the development characteristics of ANS of China people and explore the physiological characteristics such as genetic factors, EEG components and brain regions behind ANS.

Secondly, there is still controversy about the relationship between approximate number system and mathematical ability of children with mathematical difficulties. Preschool ANS, as a part of number sense, predicts future mathematical ability. After children receive formal education, mathematical ability and ANS interact and develop with each other. During this period, the development of children's ANS and mathematical ability will be developed by various cognitive functions or executive functions such as memory, speech, thinking and inhibition control. For children with mathematical difficulties, the age at which they have the characteristics of mathematical difficulties seems to be more important. At present, there are few studies in China, and most of them focus on the level of educational theory. Future studies can explore whether these functions mediate or regulate the relationship between ANS and mathematical ability in children from preschool to post-school.

Finally, there are various causes of math difficulties, so it is necessary to teach students with math difficulties in accordance with their aptitude, explore the causes of math difficulties of children with math difficulties and then prescribe the right medicine. The intervention of ANS needs to be closely combined with educational practice in order to realize the transformation of poor students.

References


[38] Obersteiner, A., Reiss, K., & Ufer, S. (2013). How training on exact or approximate mental representations of number can enhance first-grade students’ basic number processing and arithmetic skills. Learning and Instruction, 23, 125–135. https://doi.org/10.1016/j.learninstruc.2012.08.004


