ISSN 2522-6398 Vol. 8, Issue 4: 8-14, DOI: 10.25236/FER.2025.080402

Research on problem-driven inquiry-based teaching for high school mathematics—Taking "determining the perpendicularity of a line and a plane" as an example

Chang Jin

Henan Normal University, Xinxiang, Henan, 453000, China

Abstract: *Problem-driven inquiry-based teaching is a student-centered teaching model that emphasizes stimulating students' curiosity and inquiry desire by posing authentic and meaningful questions. It guides them to actively think, cooperate in exploration, and construct knowledge, thus developing their abilities. Taking "Determining the Perpendicularity of a Line and a Plane" as an example, this paper demonstrates how problem-driven inquiry-based teaching can help students actively build knowledge, improve their core mathematical literacy, and provide a reference for frontline teachers.*

Keywords: *Problem-driven; Inquiry-based learning; High school mathematics; Classroom teaching*

1. Introduction

The new curriculum reform in secondary school mathematics requires teachers to shift their mindset, study the curriculum standards, textbooks, and students, and guide them to engage in autonomous, cooperative, and inquiry-based learning. Traditional teaching methods often focus on teacher-centered instruction, where students passively receive knowledge, neglecting their subjectivity and creativity cultivation[2]. In recent years, inquiry-based teaching, a student-centered teaching model, has gradually attracted attention in the mathematics education field. The design of problem-driven teaching reflects the idea that, apart from learning specific knowledge and skills, students should also gain more on a higher level[1]. This approach not only arouses students' curiosity but also encourages them to actively seek the logical consistency behind mathematical problems, thus enhancing their mathematical thinking and problem-solving abilities.

2. Theoretical basis of problem-driven inquiry-based teaching

Problem-driven teaching is a teaching method that is guided by problems. It creates authentic and meaningful problem scenarios to stimulate students' learning interest and inquiry desire. In mathematics teaching, problem-driven teaching emphasizes integrating mathematical knowledge with real-world problems, allowing students to actively construct a knowledge system as they solve problems. This method not only increases students' learning interest but also cultivates their problem-solving abilities and innovative thinking.

Inquiry-based learning theory emphasizes students' initiative and participation in the learning process. It considers learning as an active process of constructing knowledge, where students discover knowledge and understand concepts through observation, experimentation, reasoning, and other methods. In mathematics teaching, inquiry-based learning can foster students' mathematical thinking abilities and inquiry spirit, enhancing their mathematical literacy.

The integration of problem-driven and inquiry-based teaching provides new insights for high school mathematics teaching. By carefully designing a problem chain, teachers can guide students to progressively explore the essence of mathematical concepts, and, through solving problems, they actively construct a knowledge system. This method aligns with students' cognitive development and can effectively improve teaching outcomes, cultivating students' core competencies.

ISSN 2522-6398 Vol. 8, Issue 4: 8-14, DOI: 10.25236/FER.2025.080402

3. Principles of problem-driven inquiry-based teaching design

3.1 Problem-oriented principle

The core of problem-driven inquiry-based teaching lies in stimulating students' curiosity and desire for knowledge through carefully designed problems. Problems should be challenging but not overly difficult, beyond students' current level but within their zone of proximal development, ensuring they can solve the problem with effort. Additionally, problems should be related to students' life experiences or prior knowledge to enhance relevance and guide deeper thinking.

3.2 Progressive level principle

The design of problems should follow a logical sequence, progressing from simple to complex, from easy to difficult, helping students gradually construct their knowledge system. For example, in teaching "Determining the Perpendicularity of a Line and a Plane," a simple problem (such as the relationship between a gnomon and the sundial surface) can be introduced first, followed by a more complex one (how the ancients ensured the perpendicularity of the gnomon and the sundial surface for accurate timekeeping). Each problem should connect with the previous and lead into the next, reinforcing previous learning while laying the foundation for future knowledge, forming a complete problem chain that guides students to delve deeper into the exploration.

3.3 Practical and investigative principle

Inquiry is one of the most effective ways to promote understanding in mathematics. Mathematics learning should emphasize practice and exploration through hands-on activities, experiments, and observations to deepen understanding of the knowledge. For instance, in teaching "Determining the Perpendicularity of a Line and a Plane," students can explore the characteristics of perpendicular lines and planes by experimenting with a chalk box and triangle ruler to observe and summarize the conditions for perpendicularity. These practical activities not only enhance students' intuitive understanding but also cultivate their scientific inquiry spirit and hands-on abilities.

3.4 Evaluation and feedback principle

Teaching evaluation should run throughout the entire process, combining both formative and summative evaluations. For instance, in teaching "Determining the Perpendicularity of a Line and a Plane," classroom observations, student conjectures, and group discussions can be used to provide timely feedback, helping students improve their learning strategies.

4. Problem-driven "determining the perpendicularity of a Line and a plane" inquiry-based classroom teaching

4.1 Clarifying goals and guiding learning

By intuitive perception and hands-on confirmation, summarize the theorem for determining the perpendicularity of a line and a plane, and explain the theorem from the perspective of definitions.

Apply the theorem to real-life situations and solve simple problems involving the perpendicularity of lines and planes.

Appreciate the scientific and applied value of knowledge, incorporate mathematical abstract thinking and inductive reasoning into the proof process, and foster a rigorous, evidence-based learning attitude to develop mathematical core literacy, including abstract thinking, intuitive imagination, and logical reasoning.

4.2 Creating problem scenarios to introduce the topic

Teacher's question: As shown in Figure 1, please carefully observe the sundial video played by the teacher. What is the relationship between the gnomon and the sundial surface?

ISSN 2522-6398 Vol. 8, Issue 4: 8-14, DOI: 10.25236/FER.2025.080402

Figure 1: Dynamic video

Teacher-student activity: Through an ancient poem, introduce the sundial and play a sundial time-keeping video to help students intuitively grasp the principles of sundial timekeeping. Students observe the relationship between the gnomon and the sundial surface and respond to the question.

Teacher's question: To accurately measure time, how did the ancients ensure the perpendicularity between the gnomon and the sundial surface when making the sundial?

Teacher-student activity: The teacher asks the question and introduces the topic, encouraging students to think.

Design intent: Questions are the source of thinking. This lesson stimulates students' sense of national pride by showcasing the achievements of the ancients in using the sundial to measure time. The observation leads to further questioning, arousing students' curiosity and desire for knowledge.

4.3 Reviewing Old Knowledge to Seek Solutions

Question 1: What is the definition of perpendicularity between a line and a plane? Can anyone explain it in their own words?

Definition of perpendicularity between a line and a plane:

In general, if line l is perpendicular to every line in a plane, then we say the line is perpendicular to the plane, denoted as $l \perp \alpha$.

Teacher-student activity: The teacher uses multimedia to display an animated image of a flagpole's shadow, guiding students to recall the definition of the perpendicularity between a line and a plane. Students respond and derive the first method of determining perpendicularity: the definition method.

Follow-up question: Is it convenient to determine the perpendicularity between a line and a plane using the definition?

Teacher-student activity: Students observe that a plane extends infinitely. While theoretically, it is possible to use the definition to determine the perpendicularity between a line and a plane, it is practically difficult. The teacher leads students to recognize the need for a more feasible method of determining perpendicularity.

Design intent: By revisiting the knowledge, the focus of the lesson narrows down to the relationship between the perpendicularity of a line and a plane. The definition method for determining the perpendicularity is introduced, allowing students to recognize its limitations, thereby setting the stage for learning the theorem for determining perpendicularity.

Question 2: How can we use a more feasible method to determine the perpendicularity of a line and a plane?

Teacher-student activity: The teacher guides the students to review the process of deriving the theorem for determining parallelism between a line and a plane. By analogy, the teacher proposes a conjecture: determining the perpendicularity between a line and a plane could involve testing if the line is perpendicular to a finite number of lines within the plane, encouraging students to think critically.

Design intent: Through reviewing the derivation of the parallelism theorem, the teacher helps students identify the approach to solving the perpendicularity problem. This encourages students to think critically, recognizing how mathematical principles can be applied analogically to new situations.

Question 3: How many lines within a plane must be perpendicular to a line to determine the perpendicularity of the line to the plane?

Teacher-student Activity: In order to explore feasibility, the teacher converts the infinite problem into

ISSN 2522-6398 Vol. 8, Issue 4: 8-14, DOI: 10.25236/FER.2025.080402

a finite one. The students discuss in groups and hypothesize that one line alone cannot determine perpendicularity.

Follow-up question: Can you provide real-life examples that illustrate this phenomenon?

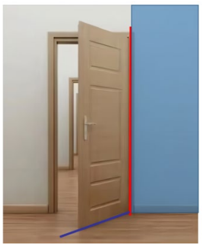

Figure 2: Classroom door

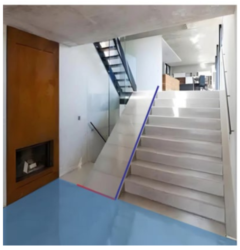

Figure 3: Staircase ramp

Teacher-student activity: As shown in Figure 2 and 3, the teacher encourages students to observe real-life phenomena such as the relationship between the bottom edge of a door and the plane of the door hinge or the relationship between a staircase ramp and the ground. These examples lead students to the conclusion that one line cannot determine perpendicularity, and the teacher provides timely feedback and positive reinforcement.

Follow-up question: If one line isn't sufficient, can two lines determine perpendicularity?

Teacher-student activity: As shown in Figure 4, in examining the perpendicularity of two lines, the teacher guides students to observe a chalk box in the classroom. Through hands-on observation, the teacher emphasizes the importance of models. By demonstrating with a rectangular model (Figure 5), the teacher leads students to hypothesize that if a line is perpendicular to two intersecting lines within a plane, then the line is perpendicular to the plane.

Figure4: Chalk box

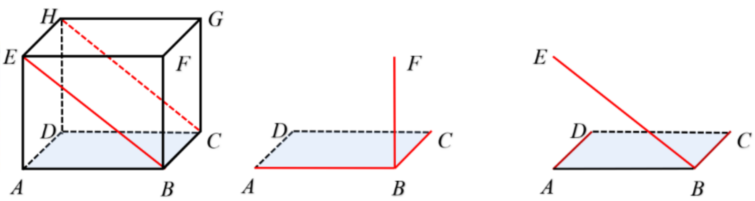

Figure 5: Rectangular model

Design intent: The teacher facilitates group discussions where students make conjectures, fostering independent inquiry. By incorporating real-life examples, the teacher helps students intuitively understand why one line alone doesn't determine perpendicularity. Using a rectangular model, the teacher highlights the importance of modeling and sparks further exploration in the lesson.

4.4 Hands-on confirmation and deriving the theorem

Experimental exploration: As shown in Figure 6, take the pre-prepared triangular paper piece ABC. Fold the paper along vertex A to form crease AD. Place the folded paper vertically on the table (with edges BD and DC touching the surface). Students perform the operation themselves.

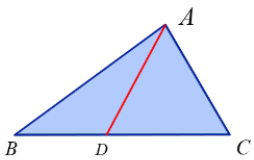

Figure 6: Triangular Paper Piece

Follow-up question: Is the crease AD perpendicular to the table surface?

Teacher-student activity: The teacher guides students to perform the folding operation. Students notice that the crease may not always be perpendicular to the surface, depending on the fold.

Follow-up question: How should the paper be folded to ensure crease AD is perpendicular to the table surface?

Teacher-student activity: Students discuss in groups, and when folding along the height of the BC

ISSN 2522-6398 Vol. 8, Issue 4: 8-14, DOI: 10.25236/FER.2025.080402

side, as shown in Figure 7, they observe that the crease AD becomes perpendicular to the table surface.

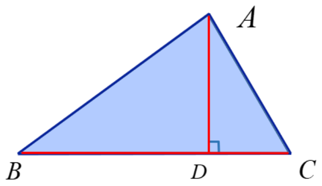

Figure 7: Paper Folded Along BC

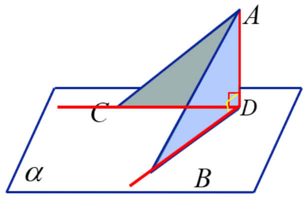

Figure 8: Observation

Follow-up question: Why is the crease AD perpendicular to the surface when folded along the height of BC? What can you deduce from this?

Teacher-student activity: The teacher displays the triangular model with crease AD perpendicular to the surface (Figure 8). Students observe that when AD is the height of side BC, the perpendicular relationship between AD and the base does not change. The teacher further demonstrates using a Sketchpad animation (Figure 9), explaining the concept from the definition and deriving the theorem for determining the perpendicularity of a line and a plane.

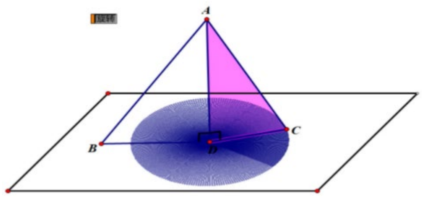

Figure 9: Dynamic Video

Design intent: Through hands-on activities and visual confirmation, students deepen their understanding of the theorem for determining perpendicularity. Group collaboration and inductive reasoning help students link theory to practice, enhancing their mathematical skills such as reasoning, analogy, and logical deduction.

4.5 Summarizing and deepening the concept

Question 4: Can you summarize the theorem for determining the perpendicularity between a line and a plane in your own words?

Follow-up question: Can you express the theorem using mathematical symbols?

Teacher-student activity: The teacher writes down students' symbolic representations on the board, verifying their accuracy.

Follow-up question: What are the three conditions that must be met to apply the theorem for determining the perpendicularity between a line and a plane?

Teacher-student activity: The teacher guides students to answer:

The lines must be within the plane.

The lines must be two intersecting lines.

The lines must be perpendicular to both the line lll and the two intersecting lines. All three conditions must be met.

Follow-up question: Can the two intersecting lines in the theorem be replaced by an infinite number of lines?

Teacher-student activity: The teacher leads students to analyze the concept, explaining that an infinite number of lines does not suffice to determine perpendicularity because they could be parallel lines. This leads to the realization that it's not just about multiple intersections; simplicity is key in mathematics.

Design intent: The teacher encourages students to summarize the theorem and connects mathematical language and symbols. By exploring the concept's history, students learn the importance of clarity and simplicity in mathematical definitions.

ISSN 2522-6398 Vol. 8, Issue 4: 8-14, DOI: 10.25236/FER.2025.080402

4.6 Applying new knowledge

Question 5: Can you now explain the "sundial" problem that the teacher posed before the lesson?

Teacher-Student activity: Students provide their answers, applying the new knowledge learned in class.

Question 6: As shown in Figure 10, can you understand the principle behind the example of "a worker using a plumb line to build a wall"?

Teacher activity: The teacher demonstrates using tools, guiding students to analyze the example.

Follow-up question: Can you transform this real-life problem into a mathematical problem and prove it?

Teacher activity: Through questioning, the teacher helps students convert the problem into mathematical language. Students then prove the statement, writing down the known facts and what needs to be proven.

Proof: As shown in Figure 11, if one line in a set of parallel lines is perpendicular to a plane, then the other line is also perpendicular to the plane.

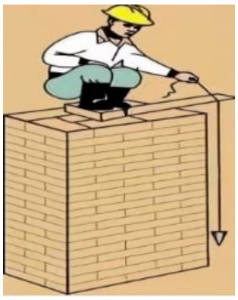

Figure 10: Worker with Plumb Line

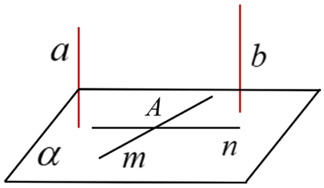

Figure 11: Proof Diagram

Teacher activity: The teacher guides students through the proof, analyzing the connections between the mathematical concepts. The teacher also explains how the students can apply the theorem to prove simple geometric problems, enhancing their understanding of symbolic language.

Design intent: By applying mathematical knowledge to real-life problems, students see the practical use of mathematics in daily life. This also deepens their understanding of mathematical symbols and improves their proof-writing skills.

4.7 Summary and ideological elevation

Teacher activity: The teacher prompts students to review the key points from the lesson, guiding them to summarize the main concepts.

Mathematical knowledge: The theorem for determining the perpendicularity of a line and a plane.

Thinking methods: Transforming the perpendicularity between a line and a plane into the perpendicularity between lines, turning an infinite number of lines into a finite number.

Learning process: The students have followed a process of questioning, discussion, experimentation, analysis, and summary. Along the way, core mathematical literacy has been embedded.

Design intent: By reflecting on knowledge and methods, students deepen their understanding and elevate their conceptual thinking. The teacher guides the students through the review process, encouraging them to make connections between the learned concepts, mathematical methods, and their core competencies.

Homework assignments:

Required exercises: Pages 152, Questions 1, 2, and 3 from the People's Education Press, A-version of the high school mathematics textbook.

Exploratory question: Are two lines that are perpendicular to the same plane parallel? If so, how would you prove this conclusion?

ISSN 2522-6398 Vol. 8, Issue 4: 8-14, DOI: 10.25236/FER.2025.080402

5. Conclusion

As socrates once said, "Education is not the filling of a pail, but the lighting of a fire." The most effective way to teach is not by providing the answers directly but by guiding students to think through questions. Questions are the source of thinking activities, transforming the initial state of a problem into its goal state[3]. This process requires teachers to guide students through a sequence of questions, moving from simple to complex. By transforming real-world problems into mathematical problems, students not only engage in abstract mathematical thinking but also connect their learning with real-life applications.

References

[1] Zheng, Yuxin. "Three Teaching Forms for 'Mathematical Problem Posing': Comparison and Analysis." Journal of Mathematics Education, 2025, 34(01): 1-5 + 63.

[2] Zhong, Weidong. "Exploring the 'Problem-Driven Student Participation' Teaching Mode—Review of 'Interactive, Heuristic, and Inquiry-Based Classroom Teaching Methods Based on Problem Orientation.'" Educational Theory and Practice, 2024, 44(08): 65.

[3] Fang, Lixin, Liu, Xinchun. "Promoting High-Order Mathematical Thinking through Problem-Driven Teaching—Taking the First Review of 'Function Monotonicity' as an Example." Mathematics Bulletin, 2023, 62(04): 49-52.