

The Study of Energy and Hyperloop Design

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Abstract: Hyperloop is a new form of high-speed mass transportation system for both public and goods transport that will set the standard for 21st century travel by connecting cities in minutes. It is expected to explore a potential route between Los Angeles and San Francisco, which is able to slash the eight hour bus ride, four hour train journey and convoluted three hour air experience between the two cities to just 30 minutes. This concept combines two important principles. The first is magnetic levitation, a technology used to repel the passenger pod from the track to lift it upwards and move the floating pod along the track at a considerable speed with little rolling resistance. And the second is the pod travelling in a near-vacuum environment, which enables to reduce the aerodynamic drag to the least. Under such conditions, the pod face scant resistance and can deliver the passengers between cities at supersonic speed and with little energy consumption. Like train stations, Hyperloop stations are planned to locate within inner cities with easy links to existing transport infrastructure.^[1] This gives Hyperloop system a distinct advantage over air travel, where airports tend to be located beyond city limits with fewer accessibility options.

Keywords: Hyperloop; travel trajectories; solution; different phases

1. Introduction

One way of realizing the Hyperloop at low capital cost is electrifying only as short portion of the track to highly accelerate the capsule out of a station and then coast to the next station. So a typical journey for Hyperloop travelling consists of three consecutive phases, which are acceleration phase, coasting phase and deceleration phase respectively. This essay will provide a solution to find the travel trajectories of Hyperloop.^{[2][3]}

2. Modeling

2.1. Acceleration phase

During the acceleration, the Hyperloop is propelled at a constant acceleration rate of a_1 , within given time of t_1 . So the speed and distance covered during this phase can be determined as below.

Speed at the end of this phase (v_1) is

$$v_1 = a_1 t_1 . \quad (1)$$

The distance covered during this phase (d_1) is

$$d_1 = \frac{1}{2} a_1 t_1^2 . \quad (2)$$

2.2. Coasting phase

The acceleration is followed by a period of coasting, in which the speed would be slightly reduced due to air drag and rolling resistance. According to Newton Law, the deceleration rate is

$$mgf_r + \frac{1}{2} \rho C_d A v^2 = -ma_2 = -m \frac{dv}{dt} , \quad (3)$$

where m is the mass of the capsule, g is the gravitational acceleration, f_r is the coefficient of rolling resistance, ρ is the air density, $C_d A$ is the product of coefficient of air drag and frontal projected area, and a_2 is the deceleration rate. Assume that the time for coasting is t_2 , the speed at the end of this phase (v_2) and the distance covered during coasting (d_2) are

$$v_2 = v_1 - \int_0^{t_2} a_2 dt \quad (4)$$

and

$$d_2 = \int_0^{t_2} a_2 dt. \quad (5)$$

2.3. Deceleration phase

After the coasting phase, the Hyperloop begins to decelerate in order to reach the destination, and the deceleration rate is also given as a_3 . So the time needed (t_3) and the distance covered when decelerating are

$$t_3 = \frac{v_2}{a_3}, \quad (6)$$

and

$$d_3 = \frac{1}{2} a_3 t_3^2. \quad (7)$$

If the total distance is known as d , it is equal to the sum of d_1 , d_2 and d_3 , which could be expressed as functions of t_2 . By solving the equation of

$$d_1 + d_2 + d_3 = d, \quad (8)$$

t_2 could be determined, and the rest of other unknown values could also be found.

However, the method mentioned above involves complicated computation of integration and differentiation, as well as non-linear equations containing trigonometric functions and logarithms. Solving such equations could be both difficult and time-consuming for a computer. This essay will provide an accurate approximation way to compute the travelling process of Hyperloop through the concept of integration and circulation iteration.

3. Strategies

3.1. Acceleration phase

The equation (1) and equation (2) could still be used to compute this process easily and quickly. So d_1 could be determined and the range left for the following two phases is

$$d_2 + d_3 = d - d_1. \quad (9)$$

And it is simple to find the value of $d_2 + d_3$.

3.2. Coasting phase and deceleration phase

Due to magnetic levitation and the near-vacuum environment, the actual deceleration of this period is relatively small compared to a_1 and a_2 , resulting in the distance covered when coasting would take up most of the whole range, so does t_2 in relative to the whole travel time. If the Hyperloop keeps coasting to the destination without decelerating, the speed at the destination will be approximately equal to v_2 , which is the speed at the end of the coasting phase when taking deceleration into consideration. Slightly error as it may introduce, such method is worth attention since it simplifies the computing process. Based on this concept, it makes sense to compute the travelling process according to the process below.

(1) Assume that the capsule coasts all the way to the destination, and simulate the coasting process through iteration to find the speed at the destination. As the initial speed of the deceleration phase, which is v_2 in this case.

(2) Find the time consumed and the distance covered during deceleration by solving equation (6) and equation (7).

(3) Compute the range the coasting phase needs to cover:

$$d_2 = d - d_1 - d_3. \quad (10)$$

(4) Simulate the coasting phase again to find the time consumed and the speed variation during the

coasting phase, which are t_2 and v_2 respectively.

And the iteration simulation process can be broken down into several steps.

- (1) Set a time resolution.
- (2) Initiation of the speed, time and distance.
- (3) Set the condition for quitting the simulation loop.
- (4) Plug the speed of every moment in equation (3) to find the deceleration rate, which is a_2 .
- (5) Update the speed and distance using the formula of constant acceleration motion.
- (6) Update time.
- (7) Repeat the steps above until the condition for quitting the circulation is achieved.

3.3. Simulation Sample

The following will show a simulation sample to verify the validity of such approach.

If a capsule weighs 20,000kg, with a cdA product of 1.0 m^2 and a rolling resistance of approximately 0.0001 due to maglev technology. Assume a constant acceleration of 4 m/s^2 for 50 seconds, followed by a period of “coasting”, followed by braking with -4 m/s^2 . Also assume air density is approximately $0.05 \text{ kg/cubic meter}$. The Hyperloop capsule will cover distances of 100km, 300km, 500km in this way respectively. The travelling process will be simulated through Matlab to plot the associated speed vs time trajectories and distance vs time trajectories. Take 100km as an example.

- (1) During acceleration, the distance covered (d_1) can be determined as below:

$$d_1 = \frac{1}{2} a_1 t_1^2 = \frac{1}{2} \times 4 \times 50^2 = 5000 \text{ m} = 5 \text{ km}, \quad (11)$$

and the speed at the end of this phase (v_1) is:

$$v_1 = a_1 t_1 = 4 \times 50 = 200 \text{ m/s}, \quad (12)$$

where a_1, t_1 are acceleration rate and time, respectively.

- (2) During coasting phase, according to Newton Law, the relationship between speed and time can be determined as:

$$mg_{fr} + \frac{1}{2} C_d A \rho v^2 = -ma = -m \frac{dv}{dt}. \quad (13)$$

- (3) By assuming that the capsule would not decelerate but would continue coasting to the destination, the speed at the destination can be found.

If the distance to be covered is 100km, which means the distance covered during the latter two phases is $100 - 5 = 95 \text{ km}$, simulate the coasting process via Matlab. From the simulation, the speed at the destination is $v_2 = 177.13 \text{ m/s}$.

- (4) Assume it to be the initial speed of the deceleration phase, compute the distance travelled and the time consumed:

$$t_3 = \frac{v_2}{a} = 44.28 \text{ s} \quad (14)$$

$$d_3 = \frac{v_2^2}{2a} = 3.922 \text{ km} \quad (15)$$

- (5) Thus it leaves $95 - 3.922 = 91.078 \text{ km}$ for coasting phase. Revise the distance in the code to find the time needed for this phase, and the time is

$$t_2 = 455 \text{ s}. \quad (16)$$

Plot the speed vs time trajectory and speed and distance trajectory as below *Figure 1*.

Similarly, plot the two trajectories for the condition when the capsule travels through 300km and 500km respectively in *Figure 2* and *Figure 3*.

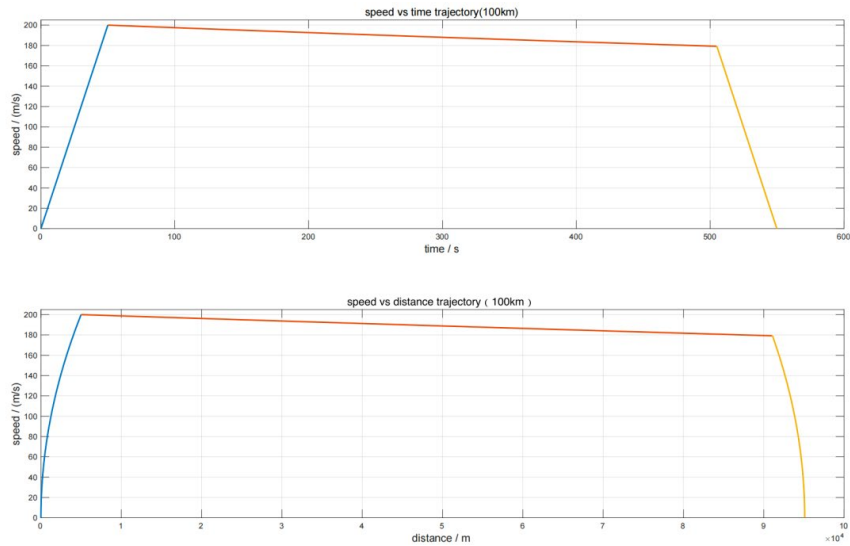


Figure 1: Speed vs time trajectory and speed and distance trajectory(100km) .

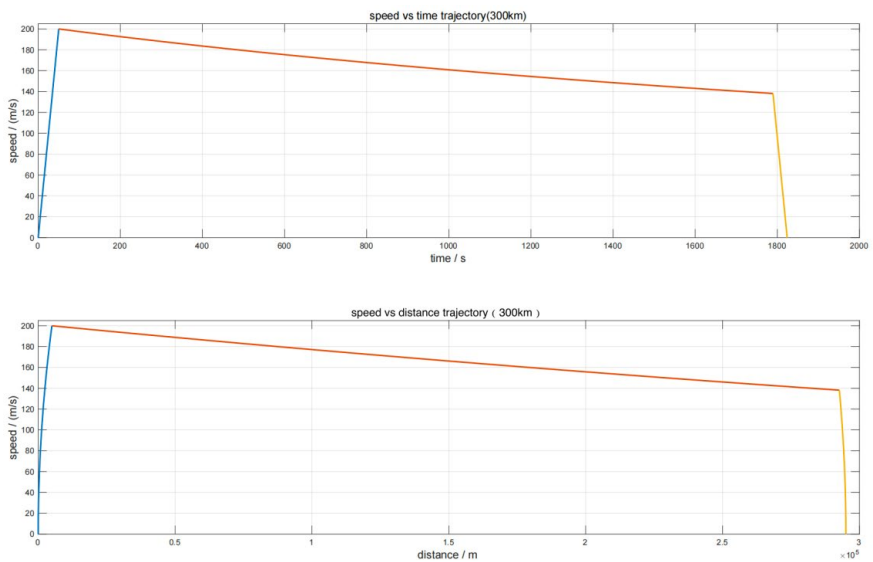


Figure 2: Speed vs time trajectory and speed and distance trajectory(300km) .

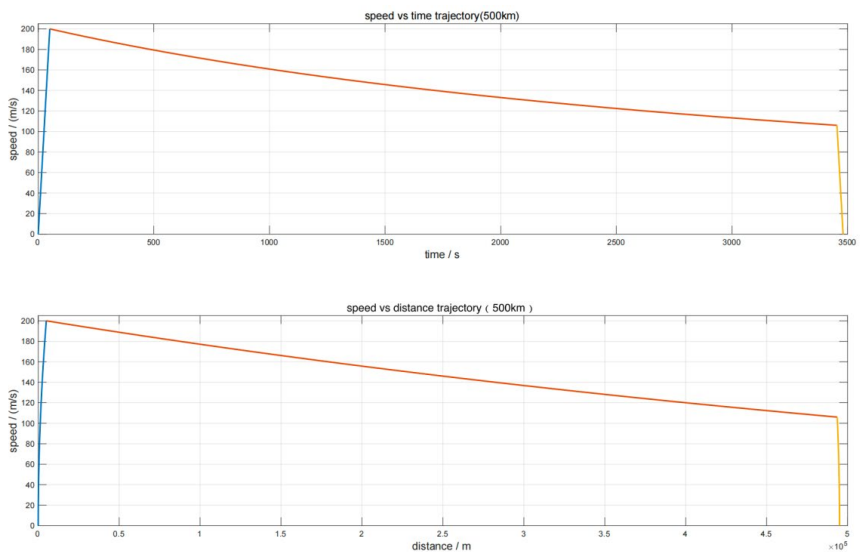


Figure 3: Speed vs time trajectory and speed and distance trajectory(500km) .

4. Evaluation and Conclusion

As can be seen in the distance trajectories, the three curves do not ultimately intersect with x-axis. This is because for the deceleration phase, the speed employed for the initial speed is the speed after the second coasting simulation; otherwise the curve will become discontinuous. For example, when travelling across 100km, the speed used to compute the deceleration time is 177.13m/s whereas the speed after the second coasting simulation is 179.17m/s. And the latter speed is used to plot the curve, which means in this way the coasting distance is shortened, resulting in the curve intersecting with x-axis at a point less than 100km/300km/500km. This is the main error of this method.

However, as the total distance increases, the difference between the actual distance to be covered and the simulation result will decrease. The differences drop from 0.701% to 0.141%, and 0.051% eventually, corresponding to 100km, 300km and 500km respectively. This makes sense since it not only satisfies demands for error control in the engineering, but also the Hyperloop is invented to cover over long distance, just like approximately 600km between Los Angeles and San Francisco which the Virgin Hyperloop was designed to connect.

In conclusion, this method simplifies the complicated non-linear computation into relatively simple linear circulation iteration. It would introduce some errors for certainty, but it is worth considering for computing the travelling process because its high accuracy and compatibility to the real scene.

References

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