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Comparative Analysis of Old and New High School Math Textbooks—Taking "Solving Triangles" in the People's Education Press Version as an Example

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Abstract: *Solving Triangles is a relatively important content in high school mathematics textbooks and a popular topic in the college entrance examination. It is not only closely related to theoretical knowledge such as trigonometric functions, but also frequently reflected in real life, with rich practical significance. The "National High School Mathematics Curriculum Standard" (2017 edition) no longer sets solving triangles as a separate chapter, but classifies it as the application of plane vectors, and requires the use of vector methods to prove the sine rule and cosine rule. Therefore, the 2019 new textbook has also made great changes compared with the old textbook in 2004. This article compares the chapter structure, teaching content, example questions, and other aspects of the solving triangles in the new and old textbooks, hoping to find the differences in the content changes of solving triangles in the new and old versions of the textbooks, and help teachers better analyze the textbooks and conduct instructional design, so that students can better master this knowledge.*

Keywords: *Textbook analysis, Solving Triangles, People's Education Press textbook, Sine and cosine rules*

1. Problem Statement

Textbooks are teaching materials based on curriculum standards, so each edition of a textbook will have its corresponding curriculum standards. It is precisely because of the differences in curriculum standards that the content of textbooks will undergo many changes after each revision. In this high school mathematics textbook reform, solving triangles is one of the contents with significant changes between the old and new textbooks. In addition, the survey results of the new version of high school mathematics textbooks show that most provinces currently use the People's Education Press A version. Therefore, this article selects the 2004 People's Education Press A version of the ordinary high school curriculum standard experimental textbook (hereinafter referred to as the "old textbook") and the 2019 People's Education Press A version of the ordinary high school textbook (hereinafter referred to as the "new textbook") as research objects, summarizes the similarities and differences between the new and old textbooks in solving triangles, and further explores the deep reasons for the changes in textbooks and the mathematical ideas contained therein, to help teachers better understand and use the new textbooks.

With the rapid development of technology and the deepening of educational reform, high school mathematics has an increasingly significant impact on people's daily lives, and the application of solving triangles is particularly widespread. It can not only solve plane geometry problems but also be applied to practical measurements. Therefore, students' mastery of the sine and cosine theorems can not only improve their mathematical calculation and abstraction abilities but also enable them to apply knowledge to practice and solve some problems in life. With the practical implementation of the new textbooks, the differences between the new and old textbooks have caused difficulties for many teachers in teaching. Taking solving triangles as an example, this article compares the new and old textbooks in terms of structure, content, and exercises, and based on this, analyzes and proposes teaching suggestions one by one[1-2].

Solving triangles, as one of the important knowledge points in high school mathematics, is a must for students to master. After adjustment according to the new curriculum standard, solving triangles has undergone significant changes. The old textbook requires students to explore the relationship between the side lengths and angles of any triangle and master the sine and cosine theorems. The new textbook

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requires students to explore the relationship between the side lengths and angles of a triangle through vector operations, master the sine and cosine theorems, and be able to solve simple practical problems. The teaching content and requirements have changed, which requires teachers to conduct in-depth analysis of the new textbooks, compare the differences between the new and old textbooks, and understand the advantages and disadvantages of the changes in the new textbook arrangement for students' mastery of knowledge, improvement of mathematical ideas, and cultivation of core competencies.

2. Research Comparison and Analysis

By analyzing the chapter units of solving triangles in the new and old versions of textbooks, comparing how to introduce, content, teaching difficulties, example problems, and homework exercises, there are similarities and differences as follows.

2.1 Comparison of Textbook Organizational Structure

Analyzing the chapters where solving triangles is located in the new and old textbooks, the specific results are as follows , as shown in Table 1:

Table 1: Comparison and Analysis of New and Old Textbook Chapters

Textbook Version	New People's Education Press A Edition	Old People's Education Press Edition
Textbook	High School Mathematics Volume 2, Grade 1	High School Mathematics Volume 5, Grade 3
Chapter	Sixth Chapter: Plane Vectors and Their Applications	Chapter One: Solving Triangles
Subsections Included	6.4 Application of Plane Vectors 6.4.3 Cosine Theorem, Sine Theorem 1 Cosine Theorem 2 Sine Theorem 3 Examples of Cosine Theorem and Sine Theorem Applications	1.1 Sine Theorem and Cosine Theorem 1.1.1 Sine Theorem 1.1.2 Cosine Theorem 1.2 Examples of Applications 1.3 Practical Exercises

2.1.1 Comparison of Textbook Chapters

As shown in the table above, the changes in solving triangles between the new and old versions of textbooks are significant. Compared to the old textbook's Volume 5, the new textbook has moved the study of solving triangles to Volume 2, where it is studied in the application of plane vectors after learning the basic operations of plane vectors. This content arrangement not only strengthens students' understanding of vector-related knowledge and enhances the coherence of knowledge but also cultivates students' mathematical calculation skills in advance.

Solving triangles is no longer a separate chapter but is inserted into the chapter on vectors as part of the application of plane vectors. Adding it to the vector chapter can also train students' ability to solve simple geometric problems using vector operations, focus on cultivating students' mathematical thinking methods, and highlight the importance and utility of plane vectors.

2.1.2 Comparison of Chapter Structures

Both the new and old textbooks present theorems in the order of proposition, derivation, and application. The difference is that the old textbook teaches the sine theorem before the cosine theorem, while the new textbook teaches the sine theorem after the cosine theorem. This is because in actual teaching, the proof process of the sine theorem is slightly more difficult than that of the cosine theorem. The new textbook follows the principle of "from simple to complex" and adjusts the learning order of the sine and cosine theorems, teaching the cosine theorem first and then the sine theorem, which is in line with the cognitive development of students and makes it easier for them to accept and improve learning efficiency. In the new textbook, both the sine and cosine theorems are proved using vector methods and are considered as one of the applications of plane vectors. However, in actual teaching, students are more likely to think of using vector methods to prove the cosine theorem, but it is more difficult for them to think of using vector methods to prove the sine theorem. Therefore, the new textbook first explores the relationship between the side lengths and angles of a triangle based on vector operations, and obtains the cosine theorem. Then, when guessing the form of the sine theorem, it guides students to discover that the sine theorem, like the cosine theorem, involves the relationship

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between the sides and angles of a triangle, and that the cosine theorem is proved using vector methods, thus helping students to think that the sine theorem can also be proved using vector methods. Therefore, the new textbook introduces the cosine theorem first and then the sine theorem to help students think of using vector methods to prove the sine theorem. The new textbook also deleted the section on practical exercises and placed it in the exploration of homework exercises after class, requiring students to complete practical exercises independently to improve their practical operation ability and better master the knowledge in practice[3-4].

2.2 Comparison of Teaching Content Structures

The horizontal analysis mainly compares how the cosine theorem, sine theorem, and application examples are presented in the new and old textbooks, and what differences there are in the teaching process.

2.2.1 Cosine Theorem

When guiding students to use vector methods to derive the cosine theorem, the old textbook introduced it as follows: "Since it involves the problem of side lengths, we can consider using the scalar product of vectors or the two-point distance formula in analytic geometry to study this problem." The new textbook states it as follows: "Because it involves the two side lengths and their included angle of a triangle, we consider using the scalar product of vectors to explore it." The expressions in the new and old textbooks seem to have little difference, but for the old textbook, students may wonder why the scalar product is used when it involves side lengths, while in the new textbook, since they have just learned the scalar product of vectors, students naturally think of using it to solve problems involving side lengths and angles, which quickly combines the knowledge learned before and after, and is more conducive to students' mastery.

2.2.2 Sine Theorem

For the sine theorem, both the new and old textbooks directly use right triangles and the knowledge of trigonometric functions to derive the conjectural form of the sine theorem $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. First, this form is shown to be valid in right triangles, and then it is verified whether this conjecture holds for general triangles. However, the new and old textbooks use different methods in the verification process of this conjecture.

The old textbook first uses trigonometric functions to prove it by constructing a perpendicular line to transform a general triangle into a right triangle, and then using the knowledge of trigonometric functions to prove the sine theorem. Since the textbook initially based on the knowledge of trigonometric functions to derive the conclusion that the sine theorem holds in right triangles, we can first try to transform a general triangle into a right triangle, and then use the knowledge of trigonometric functions to prove the sine theorem. The proof method of the old textbook for the sine theorem is very good. By constructing a perpendicular line to transform a general triangle into a right triangle, and then using the relationship between the sides and angles of the right triangle and the relevant knowledge of trigonometric functions, we can obtain the relationship that also holds for general triangles, and thus obtain the sine theorem, simplifying the proof process of the sine theorem. In this proof process, teachers can constantly inspire and guide students to think about methods to solve problems, achieve breakthroughs in mathematical thinking, and improve students' ability to apply mathematics.

The new textbook, on the other hand, uses vector methods to prove the sine theorem. After conjecturing the possible form of the sine theorem, we can easily see that the expression of the sine theorem, like the cosine theorem, involves the relationship between the sides and angles of a triangle. Since the cosine theorem is proved using vector methods, we can also consider using vector methods to prove the sine theorem, which ensures the consistency of the derivation of the sine and cosine theorems. By using vectors to derive the sine theorem, students can also review the relevant knowledge of plane vectors, ensuring the continuity of their learning, and allowing them to appreciate the simplicity of vector methods. At the same time, it also exercises students' ability to solve problems using vector operations, highlighting the importance of plane vectors and enabling students to deeply understand that plane vectors are an important carrier of the integration of mathematics and geometry.

2.2.3 Application Examples

In terms of the application examples of the sine and cosine theorems in the new and old textbooks,

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the new textbook provides three example problems, while the experimental version of the textbook provides nine example problems, with the specific types of problems as follows , as shown in Table 2 and Table 3:

Table 2: Analysis of New Textbook Problem Types

Example 1	Example 2	Example 3
Distance between two locations problem	Building height problem	Angle and distance problem

Table 3: Analysis of Old Textbook Problem Types

Example 1&2	Example3,4&5	Example 6	Example7&8	Example 9
Distance between two locations problem	Building height problem Measuring angle problem	Measuring angles	Using formulas to calculate the area of a triangle	Proving using the sine and cosine theorems

Firstly, from the two tables above, we can see that the old textbook repeats the same type of problems with two or three example questions, while the new textbook only provides one example question for each type of problem. This is because although the questions are different, the essence of the problem is the same. Teachers guide students through the explanation of this example question, and if students truly understand and learn how to apply it, they can solve similar problems easily, leaving room for students to think and develop. Secondly, the new textbook removes the use of formulas to calculate the area of a triangle, and reduces the formula for the area of a triangle to a post-lesson exploration exercise. Instead of guiding students to derive the formula for the area of a triangle together, students are required to think and explore on their own after class, which can help students better understand and master the formula for the area of a triangle, and focus on cultivating students' mathematical thinking and methods. Finally, the new textbook also removes the example questions that use the sine and cosine theorems to prove, because the new curriculum emphasizes the practical application of solving triangles and emphasizes the cultivation of students' ability to combine knowledge with practice.

2.3 Analysis of Exercise Problems

Comparing the exercise problems in the new and old versions, it can be seen that the old textbook had 35 problems, while the new textbook has reduced it to 22 problems, and it is found that the example and exercise problems for solving triangles have been greatly reduced.

2.3.1 Analysis of Exercise Problem Changes

In terms of content, the new textbook has the following changes in its focus on solving triangles:

Emphasis on the combination of the sine and cosine theorems with vectors. Solving triangles is included in the chapter on plane vectors according to the new curriculum standards, and there have been changes not only in the proof of the sine and cosine theorems, but also in the focus of this knowledge point, emphasizing the use of vectors to solve problems. In the old textbook, there were no vector-related questions in the A and B exercise groups, while the first four questions in the new textbook all use vector methods to solve problems, indicating that solving triangles is becoming closer to vectors and closely related to them. Secondly, the new textbook has weakened the algebraic operations of trigonometric functions. With the deletion of the formula for the area of a triangle, related exercise and example questions have also been removed. Additionally, because solving triangles is no longer a separate chapter but has been included in the chapter on plane vectors, there are fewer exercises in the new textbook related to algebraic operations of trigonometric functions, and more emphasis is placed on cultivating students' ability to apply knowledge to solve practical problems and on their learning of mathematical thinking and methods.

2.3.2 Analysis of Exercise Difficulty

By selecting 22 questions from the new textbook and 35 questions from the old textbook as samples and using Jiansheng Bao's comprehensive difficulty comparison model, the difficulty of the example and exercise questions in the new and old textbooks was calculated, resulting in the following chart, , as shown in Table 4 and Figure 1.

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Table 4: Quantitative Indicators of Difficulty Factors in New and Old Textbooks

Difficulty Factor	Level	Exercise Quantity Weighted		Average	
		New People's Education Press Version A	Old People's Education Press Version	New People's Education Press Version A	Old People's Education Press Version
Exploration	Recall	0	0	2.68	1.71
	Comprehension	7	10		
	Exploration	15	25		
Background	No practical background	16	16	1.27	1.51
	Personal life	6	18		
	Public knowledge	0	1		
	Scientific scenario	0	0		
Operation	No operation	0	3	2.86	2.94
	Numerical operation	8	10		
	Single-step symbolic operation	9	8		
	Two-step symbolic operation	5	14		
	Three-step symbolic operation	0	0		
Reasoning	No reasoning	6	7	2.45	2.4
	Single-step reasoning	4	8		
	Two-step reasoning	8	19		
	Three-step reasoning	4	1		
Knowledge Content	One knowledge point	5	26	1.95	1.28
	Two knowledge points	13	8		
	Three knowledge points	4	1		
	Four knowledge points	0	0		

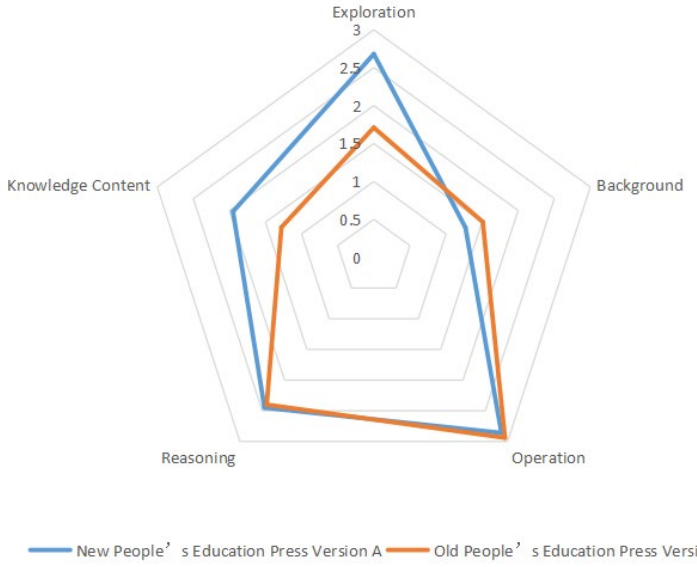

Figure 1: Analysis of Comprehensive Difficulty of Solving Triangle Exercises in New and Old Editions of People's Education Press (PEP) Textbooks

From the above analysis, it can be seen that the new textbook has undergone significant changes in chapter structure, teaching content, example and exercise problems, etc. Firstly, solving triangles has been added to the chapter on plane vectors, and the learning time has been moved up to the second compulsory course, following the principle of starting from simple to general by placing the cosine theorem before the sine theorem. Secondly, in the proof of the sine theorem, instead of using trigonometric knowledge, the vector method is used to strengthen the connection between solving triangles and plane vectors. Finally, the number of example and exercise problems has been greatly reduced, and the formula for the area of a triangle is no longer learned in the main text, but rather students are required to explore it independently, thereby strengthening their understanding and

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mastery of the formula for the area of a triangle[5-6].

3. Research Conclusion and Suggestions

3.1 Research Conclusion

Through the above analysis, it can be concluded that the new textbook has undergone significant changes in chapter structure, teaching content, example and exercise problems, etc. Firstly, solving triangles has been added to the chapter on plane vectors, and the learning time has been moved up to the second compulsory course, following the principle of starting from simple to general by placing the cosine theorem before the sine theorem. Secondly, in the proof of the sine theorem, instead of using trigonometric knowledge, the vector method is used to strengthen the connection between solving triangles and plane vectors. Finally, the number of example and exercise problems has been greatly reduced, and the formula for the area of a triangle is no longer learned in the main text, but rather students are required to explore it independently, thereby strengthening their understanding and mastery of the formula for the area of a triangle.

3.2 Teaching Suggestions

Based on the above analysis, teachers should pay attention to the following points when teaching the topic of solving triangles:

3.2.1 Focus on textbook analysis and understand the intention of the new textbook

The new curriculum standards have new requirements for both teachers and students. Teachers are no longer just cultivating students' mathematical calculation abilities, but also paying more attention to the development of students' deduction and practical abilities. In the section on solving triangles, students should be guided to think continuously, analogize and infer the similarities between the proof of the sine theorem and the cosine theorem, and find a concise proof method for the sine theorem, so that students can understand the sine and cosine theorems more intuitively and clearly.

3.2.2 Strengthen the connection between solving triangles and plane vector knowledge

According to the new curriculum standards, solving triangles and plane vectors are no longer separate and unrelated chapters, but closely related parts. Therefore, the application of plane vectors in solving triangles should be strengthened, and the thinking methods of plane vectors should be integrated into the proof process of the sine and cosine theorems. This will not only help students master the deduction process of the sine and cosine theorems, but also further consolidate their understanding and application of plane vectors in the process.

3.2.3 Exercise selection should focus on quality

For high school students, time is precious, and it is not about quantity but quality when it comes to doing exercises. Therefore, teachers should carefully select exercises, and there is no need to repeat the same type of questions excessively. Teachers should guide students to understand the difficulties and key points of the questions, so that students can grasp the mathematical thinking methods behind the exercises through practice. This is the only way to achieve mastery of a type of question and greatly improve students' efficiency and quality of doing exercises.

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