

# Unmanned Vehicle Parking Path Planning Research Based on Ackermann Steering Model

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**Abstract:** *In this study, an unmanned passenger car is used as an example to realize the function of automatic parking in a parking lot. The unmanned vehicle is an Ackermann structured passenger car with front wheel steering and rear wheel drive; the minimum turning radius of the vehicle is calculated based on the parameters in the given unmanned vehicle model. When the maximum acceleration of the vehicle is limited to 20m/s, the shortest distance to accelerate to the maximum limited speed when the unmanned vehicle travels along a straight line. As well as when the vehicle speed is 20km/h, the unmanned vehicle starts to turn from the state of driving along a straight line, this paper will calculate how much the size of the rate of change of curvature on the path relative to the path length needs to be limited. Besides, on the basis of establishing the mathematical model, the initial position of the unmanned vehicle is the entrance of the garage, which requires us to make the parking trajectory and visualize the trajectory map from the initial position to the designated parking space. The trajectory includes the path length, vehicle orientation, velocity, acceleration, acceleration plus, angular velocity, angular acceleration, etc. of the unmanned vehicle at each moment, while considering three different parking space situations.*

**Keywords:** *MATLAB; Ackerman steering model; Optimal solution*

## 1. Introduction

With the rapid development of automobile industry and technology, intelligent driving car has become one of the important development directions of future automobile recognized at home and abroad. And in the process of car intelligence, automatic parking is a very challenging and practical technology. Automatic parking system is a kind of intelligent car safety assistance system, a new intelligent driving technology, which enables car drivers to complete the parking task quickly and conveniently in the complex urban environment. The automatic parking system can obtain the distance of the parking space relative to the car through various sensors, and control the vehicle by controlling the front wheel rotation angle and instantaneous speed of the car.

This paper addresses the function of automatic parking in the present parking lot. The unmanned car is a passenger car of Ackermann structure, with parameters of front wheel steering and rear wheel drive; the body can be regarded as a rectangle, 4.9m long and 1.8m wide; the wheelbase of the car is 2.8m, and the wheel spacing is 1.7m; the maximum throttle acceleration is 3.0m/s, and the limit maximum deceleration is -6.0m/s, and the plus acceleration does not exceed 20.0m/s as appropriate; the steering wheel The maximum turning angle is 470°, the ratio of steering wheel to front wheel turning angle is 16:1 (steering wheel turning 16°, front wheel turning 1°), and the maximum speed of steering wheel is 400°/s of unmanned vehicle for Ackermann structure of passenger car in parking lot automatic parking problem for planning. In this paper, the Ackermann steering model and the curvature calculation formula are established so as to calculate the minimum turning radius and path curvature, and the model is modeled by segmentation according to the vehicle driving state, and at each stage it is necessary to explore various motion states appearing in the motion by segmentation, and the data are solved by the physics formula and the calculated data so as to make a visual trajectory map of the unmanned vehicle parking path [1].

## 2. Assumptions and notations

### 2.1. Assumptions

We use the following assumptions.

- (1) Only one vehicle is allowed to enter the parking lot
- (2) The time interval between leaving and entering vehicles is the same
- (3) The vehicle cannot leave the parking space until it enters the parking lot and finishes parking
- (4) No microscopic speed change is considered when performing macroscopic modeling

## 2.2. Notations

The primary notations used in this paper are listed as Table 1.

*Table 1: Notations*

Serial number	Symbols	Meaning
1	$\alpha$	Inner wheel steering angle
2	$\beta$	Outer wheel steering angle
3	$w$	Car width
4	$l$	Wheelbase
5	L	Car Captain
6	$\varphi$	Front wheel steering angle
7	$\Phi$	Steering wheel cornering
8	$j$	Acceleration
9	$k$	Curvature

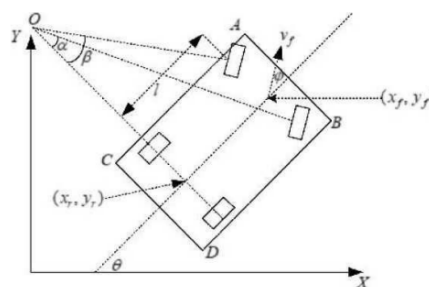
## 3. Model construction and solving

### 3.1. Establishment of Ackermann steering model

We found that the unmanned vehicle is consistent with the Ackermann structure for passenger cars, and by reviewing the information we found that the Ackermann steering model can be used. The kinematic model is the basis for studying the motion process of the car [2]. The parking process is a low-speed driving and almost no lateral force process, the four wheels can be considered not to occur in the process of vehicle slippage and in a pure rolling state, in the steering, the body is rigid, the unmanned car steering axis must intersect at a common point, this intersection is the unmanned car steering center, at this time the unmanned car to meet the Ackermann steering principle, as shown in Figure 1 As shown in Figure 1.

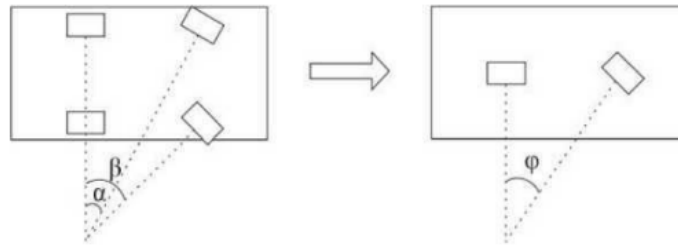
Considering the vehicle as a rigid body, the relationship between the inner and outer wheel steering angles is.

$$\cot \alpha = \frac{w}{l} + \cot \beta \quad (1)$$



*Figure 1: Vehicle kinematics model*

Since the unmanned vehicle can be considered to be driving at low speed when parking in general, this paper can assume that the turning process of the unmanned vehicle is a circular motion process and simplify the four-wheel Ackermann steering geometry to a two-wheel bicycle model. The respective two wheels of the front and rear axles of the unmanned vehicle are transformed into one wheel at the center of the front and rear axles, respectively, before the rear axle wheels are all in pure rolling motion, as shown in Figure 2.



*Figure 2: Equivalent front wheel angle*

The relationship between the wheel rotation angle before and after the model simplification is.

$$\cot \alpha + \cot \beta = 2 \cot \phi \quad (2)$$

Turning angle and steering wheel turning angle  $\phi$  can be approximated as a linear proportional relationship, the ratio of  $k$  relationship is as follows.

$$\phi = k \cdot \alpha \quad (3)$$

According to the simplified model, the turning radius  $R$  at the center point of the rear axle of the car can be introduced as.

$$R = \frac{l}{\tan \phi} \quad (4)$$

Since the equivalent front wheel turning angle has a maximum value, the minimum turning radius of the car is.

$$R_{\min} = \frac{l}{\tan \phi_{\max}} \quad (5)$$

Our analysis shows that the acceleration is determined by the amount of change in acceleration and time. The acceleration can be found by the following equation.

$$\vec{j} = \frac{d\vec{a}}{dt} = \frac{d^2\vec{v}}{dt^2} = \frac{d^3\vec{r}}{dt^3} \quad (6)$$

The combination of the given conditions and the formula for acceleration at any given moment yields.

$$a_{(t)} = a_0 + \int_0^t j(t)dt \leq 3.0\text{m} / \text{s}^2 \quad (7)$$

The equation for velocity at any moment is as follows.

$$v_{(t)} = v_0 + \int_0^t a_{(t)}dt \quad (8)$$

The equation for the distance at any given moment is as follows.

$$s_{(t)} = \int_0^t v(t)dt \quad (9)$$

We analyzed the data and added the uniformly accelerated linear motion displacement formula to the above formula.

$$s_{(t^2)} = v_0t + \frac{1}{2}at^2 \quad (10)$$

By reviewing the information, we learned the equations involved in the curvature of the unmanned vehicle during the turn.

The curvature at point M is as follows,

$$K = \lim_{\Delta s \rightarrow 0} \left| \frac{\Delta \alpha}{\Delta s} \right| = \left| \frac{d\alpha}{ds} \right| = \frac{|y''|}{\sqrt{(1+y'^2)^3}} \quad (11)$$

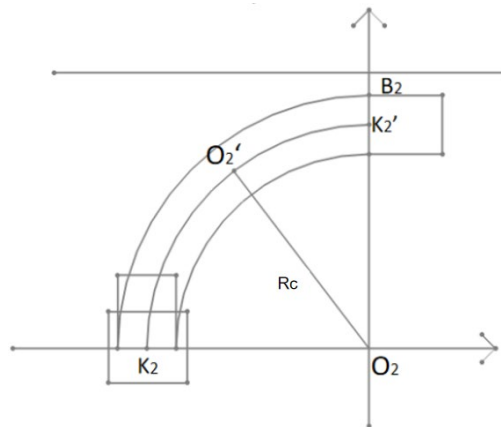
### 3.2. Ackermann steering model solving

By using the formula in Section 3.1 and substituting the base data for the solution, it is calculated that the time required for the first stage of the journey is 0.15 s, when the speed is 0.225 m/s[3]. The time required for the second stage of the journey is 1.777 s, and the displacement is about 5.136 m. The shortest distance required to accelerate to 20 km/h during the entire motion is 5.148 m. The final curvature is calculated as  $1/\alpha$ .

### 3.3. UAV motion modeling

We analyzed the path diagram and found that the unmanned vehicle needs to go through several stages, and we need to segment the model. We need to build the model in stages.

Phase 1: Establish the unmanned vehicle right-turn model, as shown in Figure 3.



*Figure 3: Unmanned vehicle right turn path map*

When the steering wheel of the unmanned vehicle reaches the maximum turning angle, it happens to pass the right-angle turn without collision. At this point we plot the unmanned sketch of the right-turn movement of the vehicle and analyze the relationship between the physical quantities required in the diagram.

The equation for the radius of the outer wheel trajectory of the unmanned vehicle is as follows.

$$R_{i\bar{a}} = \frac{L}{2 \sin \varphi} \quad (12)$$

The formula for the distance of the unmanned vehicle wheels from the edge of the nearest parking space is as follows.

$$A_1 D_1 = R_{i\bar{a}} - w - O A_1 \quad (13)$$

The formula for the control point of the unmanned vehicle is as follows

$$R_C = A_1 D_1 + O A_1 + \frac{1}{2} W \quad (14)$$

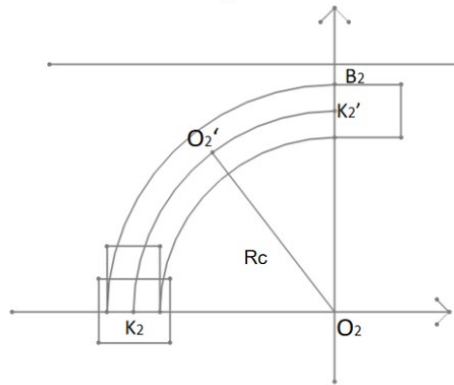
The distance from the vehicle control point to the right edge of the mile to determine the best starting position, the formula is as follows

$$KA_1 = A_1D_1 + \frac{1}{2}w = \left(1 - \frac{\sqrt{2}}{2}\right)R_{\text{ra}} + \left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right)w \quad (15)$$

The distance of the control point at the end position from the lower edge of the formula is as follows

$$B_1D_1' = A_1D_1 \quad (16)$$

The second stage: establish the unmanned vehicle vertical entry model, as shown in Figure 4.



*Figure 4: Unmanned vehicle vertical parking path map*

We used a polar equation to analyze the vehicle trajectory change when the vehicle is parked vertically. We consider the unmanned vehicle as a movable rectangle, and the parking is considered to be completed when the diagonal intersection point of the unmanned vehicle coincides with the diagonal intersection point of the parking space. In the whole process, the unmanned vehicle starts to reverse into the parking space from the starting position with the speed of 0, and then does the acceleration circular motion, then does the linear deceleration motion, and decelerates to 0 just to complete the parking [4,5].

$$\widehat{A_2B_2} = \begin{cases} x = -R_C \sin \theta \\ y = R_C \sin \theta \end{cases} \quad (17)$$

The above polar equation is the change of the control point coordinates in the circular trajectory at each moment though the angle of the circle center.

The initial position of the unmanned vehicle is derived by the following equation.

The equation of the radius of the outer wheel trajectory of the unmanned vehicle is as follows.

$$R_w = \frac{L}{2 \sin \varphi} \quad (18)$$

The formula for the distance traveled when slowing down in reverse is as follows.

$$K_2K_2' = v_0t + \frac{1}{2}at^2 \quad (19)$$

The equation for the distance of the initial position control point of the unmanned vehicle from the lower edge of the road is as follows.

$$B_2O_2' = R_C - L \quad (20)$$

In the above two phases of the problem, we can consider that the unmanned car first accelerates, then moves at a uniform speed, and finally decelerates to the initial position in the second phase.

After the above two stages, the speed of the car should not exceed 10km/h in the 5m range before and after the speed bump, and a deceleration motion is required. The above speed change formula is used.

$$v_{(t)} = v_0 + \int_0^t a_{(t)} dt \quad (21)$$

The formula in model one can find out the initial position of the unmanned vehicle control point is 1.836m from the right edge of the road, according to the image, the control point of the end position of the unmanned vehicle is 2.259m from the lower edge of the road, the position of the unmanned vehicle at this time is exactly through the right angle turn, the speed of the vehicle at the end position is 3.505m / s, and the whole process used time 1.635s. Formula, you can find out the unmanned car to do circular motion of the radius of 4.095m, we also need to consider the reversing process is a uniform acceleration of the initial velocity of 0 circular motion, reversing the speed of the car does not exceed 10km / h, we can use the known path arc length of the reversing trajectory, the use of circular motion formula to find out the process of circular acceleration of the time used 0.643s, and then through the speed change formula to find out The above data can be obtained from the initial position of the unmanned vehicle control point is 2.479m from the lower edge of the road, and the total time is 1.106s. When the above data are satisfied, the trajectory of the reversing vehicle is the best and the time used is the shortest [6]. Through the comparison between model one and model two, we can see that the unmanned car is not moving along a horizontal straight line from model one -13- movement to model two, we can use two different control points to connect the line, and calculate the movement process in the line is the overall movement process of the unmanned car, according to the shortest distance between the two points. The distance between the two points 24.944m, know the initial velocity in k1 3.505m/s, by the formula to calculate the acceleration time used 0.684s and distance 3.099m, and then deceleration time used 1.852s and distance 5.145m, you can get the middle of the uniform distance 16.7m, the uniform process time used 3.006s, the whole process of the total time 5.542s. In the middle of the second stage and the third stage there is a deceleration process before the deceleration belt, this process from the maximum speed deceleration to the maximum speed limit of the deceleration belt 20km / h, and the information in the question can be known from the limit of the maximum speed of 10km / h, the application of the formula in the model can be derived from the deceleration process used in the time 5/3s.

#### 4. Conclusion

In this paper, we use Ackermann steering model structure to analyze the unmanned vehicle parking trajectory i.e. various data to improve the application capability of the model so that the final goal can be well quantified. The software such as Matlab and Python are fully utilized to program the solution, the error of which is small and the data is accurate and reasonable. The formulaization of all data makes the model more clear, easy to analyze the kinematic model, and convenient to calculate the continuous change of data during the motion. The model is practical and has a strong guiding meaning to reality. The formulae used in the established model are simple to understand, easy to calculate and feasible. It is convenient to analyze the best solution in the motion process, and the optimal solution of the whole process can be derived. The problem modeled in this thesis is a feasible practical problem that can be well applied and generalized to similar motion models. However, in order to make the model easier to build, some error conditions are ignored, which do not facilitate the application to practice.

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