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Poisson Flow and Queuing Theory Model Based on OBE Theory

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Abstract: *On the basis of in-depth analysis of OBE theory, the basic idea of integrating mathematical modeling into the course of probability theory and mathematical statistics is put forward. The reform of course content should not only depict the regularity of random phenomena and form the concept of probability, but also pay attention to expounding the application background of mathematical statistics, which is closely combined with professional learning and application practice. Actively guide students to learn to use probabilistic language to describe and express random phenomena, learn to use probabilistic model for application modeling. Based on Poisson flow theory, this paper establishes the queuing theory model, which guides students to analyze problems, put forward hypotheses and build models during the modeling process, improves students' innovation ability and highlights the application function of the course.*

Keywords: *OBE; Mathematical modeling; Poisson flow; Queuing theory*

1. Introduction

Outcomes-based Education (OBE) first appeared in the basic Education reform of the United States and Australia. This model is deeply studied in the book Output-based Education model: Controversy and Answers written by American scholar David Speidi. The book defines OBE as "clearly focused and organized education systems around ensuring that students gain the experience to achieve substantial success in later life." He believes that OBE has achieved a paradigm shift in education. In OBE education, what students learn and whether they succeed is far more important than how and when they learn. Education Western Australia defines OBE as: "An educational process based on achieving a specific learning outcome for the student. Educational structures and curricula are seen as means rather than ends. If they do not contribute to developing students' specific abilities, they have to be rebuilt. Student output drives the education system." OBE is a structure and system in which learning outputs drive the entire curriculum activities and students' learning outputs are evaluated.

In an OBE education system, educators must have a clear vision of the competencies and levels students are expected to achieve upon graduation, and then seek to design appropriate educational structures to ensure that students achieve these desired goals. Student output, rather than textbooks or teacher experience, is the driving force behind the operation of the education system, in sharp contrast to the traditional content-driven and input-focused education. In this sense, OBE education model can be regarded as an innovation of education paradigm

2. The OBE Theory

The key points, or key steps, of OBE implementation are as follows:

Determine learning outcomes. The final learning outcome (peak outcome) is both the end and the beginning of the OBE. Learning outcomes should be clearly stated and directly or indirectly measured, so they are often converted into performance indicators. The determination of learning outcomes should fully consider the requirements and expectations of education stakeholders, including the government, schools and employers, as well as students, teachers and parents.

Construct the curriculum system. Learning outcomes represent a structure of competence, which is mainly achieved through curriculum teaching. Therefore, the construction of curriculum system is

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particularly important to achieve learning results. There should be a clear mapping relationship between the ability structure and the curriculum system structure, and each ability in the ability structure should be supported by a clear curriculum. In other words, each course in the curriculum system should make a definite contribution to the realization of the ability structure. The mapping between curriculum system and ability structure requires students to have the expected ability structure (learning outcome) after completing the curriculum system.

Determine the teaching strategy. The OBE places particular emphasis on what the student has learned rather than what the teacher has taught, on the output of the teaching process rather than its input, on research-based teaching rather than indoctrination, and on personalized teaching rather than "carriage" teaching. Personalized teaching requires teachers to accurately grasp each student's learning trajectory, timely grasp each person's goal, basis and process. According to different requirements, develop different teaching programs, provide different learning opportunities.

3. The Main Strategies of Integrating Probability Theory and Mathematical Statistics into Modeling

Probability theory and Mathematical Statistics is a discipline that studies the statistical regularity of random phenomena. Course makes every effort to apply for the purpose, strive to make the students understand the basic concepts of probability and mathematical statistics and the basic theory, the preliminary master the basic idea and method of dealing with random phenomenon, trains the student to use probability and statistics methods to analyze and solve actual problem ability, learning other professional course for the future lay a solid mathematical foundation. The basic knowledge of mathematical statistics is the basic knowledge students must master, so this course is the theory and method of mathematics and other branches of mutual penetration, and has a wide range of applications in science and technology and other fields.

The course construction of Probability theory and Mathematical Statistics should follow the principles of "occupation", "future", "reality" and "practice", combine the basic content of probability theory and mathematical statistics and students' concrete reality, systematically teach the basic concepts, basic theories and basic methods of probability statistics to students. The main task of "probability theory and mathematical Statistics" teaching is to cultivate students' probability theory thinking, that is, random thinking and the ability to solve practical problems. The methods and steps of integrating modeling into the course to cultivate students' problem solving ability under the condition of randomness thinking include: the transition from "certainty" to "uncertainty" to cultivate students' consciousness of randomness thinking; Show the reasoning process, train the logical reasoning ability under the randomness of thinking; Pay attention to the connection between probability and statistics, train the ability of using randomness thinking flexibly; With the idea of mathematical modeling, improve the ability of solving problems under random thinking.

4. Poisson Flow and Poisson's Theorem

A constant stream of particles is called a flow. Here, the particles can be spatial particles, customers in the mall, and vehicles at the intersection, thus forming particle flow, customer flow, traffic flow, etc., one demand, one piece of information or instruction all constitute flow.

Suppose $\xi_{(0,t]}, t \geq 0$ is poisson flow, there exists a positive number λ, so

$$p_k(t) = P\left(\xi_{(0,\mathrm{t}]} = k\right) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, k = 0,1,\cdots \tag{1}$$

By the theorem of know

$$\xi_{(0,t]} \sim P(\lambda t) \tag{2}$$

That is, The Poisson flow can be regarded as the Poisson distribution with the parameters of λt. When (unit time) $t = 1$ is taken from the above formula, the Poisson distribution of parameter is obtained; when $t = 2$, the Poisson distribution of parameter is 2λ obtained. Therefore, Poisson flow is a direct and important background for generating Poisson distribution.

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Poisson flow and its continuous distribution: exponential distribution and distribution

Suppose $\xi_{(0,t]}, t \geq 0$ is The Poisson flow, and the intensity is λ, then according to the Poisson flow theorem

$$p_k\left(t\right) = P\left(\xi_{(0,\mathrm{t}]} = k\right) = \frac{\left(\lambda t\right)^k}{k!} e^{-\lambda t}, k = 0,1,\cdots \tag{3}$$

Let η is the time when the first particle in poisson flow arrives, because

$$\left(\eta > t\right) = \left(\xi_{(0,\mathrm{t}]} = 0\right), \forall t > 0 \tag{4}$$

The probability $P\left(\eta > t\right) = \exp\left(-\lambda t\right)$, $F_\eta\left(t\right) = 1 - \exp\left(-\lambda t\right)$,the density function of the arrival time $t \leq 0$ of the first particle, $F_\eta\left(t\right) = 0$, in poisson flow η is obtained by taking the derivative.

$$f_\eta\left(t\right) = \begin{cases} \lambda e^{-\lambda t}, & t > 0 \\ 0, & t \leq 0 \end{cases} \tag{5}$$

The parameter here λ is the intensity in poisson flow.

According to the definition of the exponential distribution, it is easy to know η that the exponential distribution of the parameter is λ, i.e. $\eta \sim E\left(\lambda\right)$

If η is the arrival time of the first r particle in poisson flow, because

$$\left(\eta > t\right) = \left(\xi_{(0,\mathrm{t}]} < r\right), \forall t > 0 \tag{6}$$

According to the theorem, there are

$$P\left(\eta_r > t\right) = \sum_{k=0}^{r-1} \frac{\left(\lambda t\right)^k}{k!} e^{-\lambda t} \tag{7}$$

$$\begin{aligned} f_{\eta_r}\left(t\right) &= \frac{d}{dt} F_{\eta_r}\left(t\right) \\ &= \left[1 - P\left(\eta_r > t\right)\right]' \\ &= -\sum_{k=1}^{r-1} \frac{\lambda\left(\lambda t\right)^{k-1}}{\left(k-1\right)!} e^{-\lambda t} + \sum_{k=0}^{r-1} \frac{\left(\lambda t\right)^k}{k!} \lambda e^{-\lambda t} \\ &= \frac{\lambda\left(\lambda t\right)^{r-1}}{\left(r-1\right)!} e^{-\lambda t} \end{aligned} \tag{8}$$

In advanced mathematics, functions $\Gamma\left(x\right)$ are defined as

$$\Gamma\left(x\right) = \int_0^\infty \lambda^x t^{x-1} e^{-\lambda t} dt = \int_0^\infty s^{x-1} e^{-s} ds, x > 0 \tag{9}$$

By virtue of the property of the function Γ (which can be proved by integration by parts) $\Gamma\left(1+x\right) = x\Gamma\left(x\right)$, the recurrence of positive integers r, $\Gamma(r) = (r-1)$. Also $\Gamma\left(1/2\right) = \sqrt{\pi}$, notice $t \leq 0$ that there is obviously $\Gamma_{\eta_r}\left(t\right) = 0$, and thus η_r the probability density function

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$$f_{\eta_r}\left(t\right)=\begin{cases}\dfrac{\lambda^r}{\Gamma\left(r\right)}t^{r-1}e^{-\lambda t}, & t>0\\ 0, & t\le 0\end{cases} \tag{10}$$

It is easy to know that η_r the distribution obeying the parameters are r and λ, $\eta_r \sim \Gamma\left(r,\lambda\right)$ where r and λ are called shape parameters and scale parameters respectively.

According to the above derivation, the arrival time of the first particle η in Poisson flow obeisance to exponential distribution, that is, the arrival time of the first particle obeisance to distribution, that is $\eta \sim E\left(\lambda\right)$, distribution r is the generalization of exponential distribution $\eta_r \sim \Gamma\left(r,\lambda\right)$. Exponential distribution is a kind of waiting distribution, such as "waiting for the rabbit" in continuous time, waiting for the rabbit is a random variable subject to exponential distribution, the corresponding discrete waiting distribution is geometric distribution, and the corresponding discrete distribution is negative binomial distribution. In the theory of reliability, for some products, equipment and systems (such as electronic components), the moment when the first failure occurs can be considered as exponential distribution, while the moment when the first failure occurs follows the distribution, which should be paid close attention to in the reliability analysis, system analysis and management of equipment and systems.

5. Queuing Model

Queuing theory is mainly to establish a mathematical model of queuing system, and to provide a basis for the design and regulation of queuing system by studying the structure and operation rules of queuing system. Next we study how to use probability theory knowledge to build queuing theory model.

The model assumes that the queuing process consists of customer arrival rule, service time and queuing rule.

5.1 Customer Arrival Rule

In the time $\left(t,t+\Delta t\right)$ to reach the probability of a customer is proportional Δt, proportional coefficient is λ, to reach two and more than two probability is $o\left(\Delta t\right)$, customers arrive at each other independent, customer source is infinite. According to the probability theory, under the above assumptions, the number $\left(0,t\right)$ of customers arriving in the interior follows the Poisson distribution with parameter is λt, whose average value is λt, that is, the average number of customers arriving in unit time $\left[0,t\right)$ is $e^{-\lambda t}$, which T is called the average arrival rate. The average arrival time interval is $E\left(T\right)=\dfrac{1}{\lambda}$.

5.2 Service Hours

Suppose that the service efficiency remains unchanged, that is, the probability of serving for a period of time is the same as the probability of serving for a period of time at the beginning. According to the probability theory, the above hypothesis Z follows the exponential distribution. If the parameter is μ, then, that is $E\left(Z\right)=\dfrac{1}{\mu}$, the average service time of each customer is $\dfrac{1}{\mu}$, and the average number of customers served within a unit time is μ, which is called the average service rate.

6. Queuing Rules

According to the order of customer arrival service, that is, first come service.

Models that meet the above conditions are denoted as $M / M / S$ models in queuing theory, and S are the number of waiters.

Model building and solving queuing system, people are generally concerned about the captain and service time, discuss $S = 1$ the case of average captain and average waiting time.

6.1 Average Captain and Average Waiting Time

The probability of having n customer at any time t in the queuing service system is $p_n(t)$, which can be deduced as the following differential equation:

$$\frac{dp_n(t)}{dt} = \lambda p_{n-1}(t) + \mu p_{n+1}(t) - (\lambda + \mu) p_n(t) \tag{11}$$

The initial conditions are

$$\frac{dp_0(t)}{dt} = \mu p_1(t) - (\lambda + \mu) p_0(t) \tag{12}$$

When $t \to \infty$,the length has a stable distribution, that is, $p_n(t)$ and t is independent, then the above equation can be transformed into

$$\lambda p_{n-1}(t) + \mu p_{n+1}(t) - (\lambda + \mu) p_n(t) = 0 \tag{13}$$

The initial condition is $\mu p_1(t) - \lambda p_0(t) = 0$. Thus, it can be solved

$$p_n(t) = \left(\frac{\lambda}{\mu}\right)^n p_0, \quad n = 1, 2, \cdots \tag{14}$$

Note $\sum_{n=0}^{\infty} p_n = 1$ and $\rho = \frac{\lambda}{\mu}$ note, $\rho = \frac{\lambda}{\mu}$ is called service degree, in steady state, $p_0 = 1 - \rho$, $\rho < 1$.

The probability that there n customer in the steady state is zero

$$p_n = (1 - \rho) \rho^n \tag{15}$$

So the average captain is

$$L = \sum_{n=1}^{\infty} n p_n = \frac{\rho}{1 - \rho} = \frac{\lambda}{1 - \lambda} \tag{16}$$

As the customer arrival interval T obeys the exponential distribution of parameter is λ , and the service time Z obeys the exponential distribution of parameter is μ , so the customer waiting time Y obeys the exponential distribution of parameter is $\lambda - \mu$, so the average waiting time is

$$W = \frac{\rho}{\lambda - \mu} = \frac{L}{\lambda} \tag{17}$$

6.2 About the Increase of Waiters

As the number of customers waiting for service increases, so does the number of waiters, so that the average captain is not too long. Discuss a situation where two waiters are equally efficient. If the customers are only in a single line, the first customer goes to the idle waiter to be served, that is, the mode $M / M / 2$ l. The average service rate of the whole service process is 2μ , and to limit the number of service members, the service intensity is $\rho_2 = \frac{\lambda}{2\mu} < 1$. By using similar methods, it can be obtained that, in steady-state, the average length is $L_2 = \frac{2\rho_2}{1-\rho_2}$, and the average waiting time is $W_2 = \frac{2\rho_2}{\lambda(1-\rho_2)} = \frac{L_2}{\lambda}$, and the calculation formula of the average length L_s and the average waiting time W_s is similar to the available mode $M/M/S$ l:

$$L_s = s\rho + \frac{(s\rho)^s}{s!(1-\rho)^2} p_0 \tag{18}$$

$$W_s = \frac{L_s}{\lambda} \tag{19}$$

In which $\rho = \frac{\lambda}{s\mu}$, p_0 is the probability that all waiters are free

$$p_0 = \left[\sum_{k=1}^{s-1} \frac{(s\rho)^k}{k!} + \frac{(s\rho)^s}{s!(1-\rho)} \right]^{-1} \tag{20}$$

This model can be widely applied to many random service systems, such as seeing a doctor, buying a ticket, fetching water, hairdressing and other random queuing problems.

7. Conclusion

In probability theory and mathematical statistics course development and design the application case of modeling, each unit in case raises problems, introduces the concept of background, and then according to the cognitive process of thinking in knowledge parsing, summed up the basic theory, and through sublimation theory to solve the problem, reflects the production and application of the concept of probability and mathematical statistics method. On the basis of the basic theory analysis, guided by modeling cases, students can learn to use theoretical knowledge to solve practical problems, especially to solve military problems by modeling, which cultivates students' application consciousness and improves their innovation ability.

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