Academic Journal of Engineering and Technology Science
ISSN 2616-5767 Vol.7, Issue 3: 64-71, DOI: 10.25236/AJETS.2024.070309

Submarine Search and Rescue Study Based on Digital Elevation Model (DEM) and Probabilistic Grids

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Abstract: *This article explores a comprehensive strategy to enhance safety measures for undersea tourism and exploration, particularly focusing on addressing submarine safety concerns in emergencies or malfunctions. The research aims to gain regulatory approval and trust by proposing strategies to ensure the safety of both tourists and submarine operators within various constraints. The study begins by predicting the submarine's position after power loss through an analysis of seawater density, ocean currents, and the creation of a digital elevation model of the Ionian Sea to pinpoint where the submarine could potentially strand on the seabed. Efforts are also dedicated to optimizing search and rescue equipment selection by synthesizing relevant parameters, developing utility formulas via the entropy weight method, and determining cost-effectiveness ratios to maximize utility within set budget limits. Moreover, a gridded probabilistic search model is introduced that leverages predictions of the submarine's potential stranding location in the ocean. This model assigns probabilities to possible submarine sightings within each grid, facilitating efficient search operations by continuously updating the search and rescue location with the highest cumulative probability. The approach extends its reach by adapting the model to different oceanic regions and incorporating multiple submarine interaction dynamics to significantly enhance search and rescue efforts.*

Keywords: *Gridded probability; entropy weighting method; DEM; route planning*

1. Introduction

MCMS, a Greek submarine company, aims for underwater tourism in the Ionian seabed. They need a precise tracking model due to underwater complexities. The project involves prediction modeling, equipment recommendations, and efficient search and rescue strategies. Analysis of factors like topography informs 3D seabed modeling. Tailored rescue plans are essential for multiple submarines. In this study, we aim to develop a model that predicts the position of a submarine over time while considering various uncertainties that may influence the results. Additionally, we will assess whether it is advisable for the company in question to carry life-saving equipment, taking into account factors such as availability, maintenance, preparation, and cost implications of different equipment types. Recommendations on the most effective search and rescue methods and deployment locations will be provided based on our modeling outcomes. Our approach involves predicting and evaluating the potential location of a submersible and the associated probabilities under specific conditions. We will offer practical search and rescue solutions, extend our analysis to diverse scenarios, and address the following key aspects [1].

2. Predictive Modeling of Submarine Motion and Seabed Stranding in the Ionian Sea

2.1 Physical State of the Submarine

For the designated oceanic region, we establish a coordinate system based on latitude, longitude, and geographic conversion to describe the position of each point. Within this system, the coordinates of the submarine at the time it was lost can be expressed as (x_0, y_0, z_0).

where (x_0, y_0) represents the coordinates of this point projected on a horizontal plane parallel to the sea level, and y_0 represents the vertical distance of this point from the sea level, and the y value corresponding to any point on the sea level is 0 in this coordinate system.

Because the speed of the submarine is slow, we ignore the deceleration process after the submarine loses power, and immediately after the loss of power, the magnitude and direction of the speed tends to be the same as the sea current.

According to relevant information and literature references, the relevant parameters of a common powered submarine are: a height of 1.5m, a cross-sectional area of $2.25m^2$, and a mass of 1125kg when not actively storing or releasing water to control uplift or sinking, and these quantities do not change over short periods of time.

2.2 Prediction of submarine positions

Before the submarine loses power, it is subjected to four main forces: its own power and the resistance of the current, its own gravity and the buoyancy of the sea on the submarine.

The resistance of the sea water to the submarine can be decomposed by the combined force into component forces in the horizontal plane and in the vertical plane, in the horizontal plane:

$$\frac{dv}{dt} = \frac{-F_f + F_{xy} \cdot cos\theta}{M+m} \tag{1}$$

Where θ is the angle between the submarine's running direction and the line of latitude, and F resistance is a physical quantity related to the submarine's speed and current speed:

$$F_{drag} = 0.5c\left(v_{xy} - v_{currents}\right)^2 \cdot s \tag{2}$$

Where c is a constant for calculating fluid resistance. We can conclude that at the moment of time t, the horizontal plane acceleration of the submarine is at, and $M + m$ is the net self-weight of the submarine plus the mass of the water in the water storage bin.

Now consider the motion of the submarine in the vertical direction:

$$\frac{dv}{dt} = \frac{\rho_{seawater} \cdot V \cdot g - F_{drag}}{M+m} - g \tag{3}$$

Derive the magnitude of the acceleration of the submarine in the vertical direction at a given moment. The motion of the submarine in the horizontal plane when power is present can be derived:

$$v_z = v_0 + 0.5(a_0 + a_1) \cdot dt \tag{4}$$

After a tiny moment dt, a_0 is the magnitude of the vertical acceleration at the previous moment, a_1 is the magnitude of the vertical acceleration at the next moment, and v_0 is the submarine's partial velocity in the vertical direction at the previous moment (disregarding the partial velocity of the current in the vertical direction).

Based on Eqs. (4) and (5), we can calculate the displacement of the horizontal plane produced by the submarine after a certain tiny moment before it loses power:

$$x_{xy} = v_0 dt + 0.5(a_0 + a_1) \cdot (dt)^2 \tag{5}$$

It also yields the displacement in the vertical direction that occurs after a tiny moment of experience:

$$x_z = v_0 dt + 0.5(a_0 + a_1) \cdot (dt)^2 \tag{6}$$

Vector summing x_{xy} and x_z gives the total displacement of the submarine running in dt.

After the loss of power, the submarine's communication link is interrupted to the power off, at this time, it is assumed that the submarine's deceleration process in the horizontal plane is instantaneous (i.e., it soon reaches a common speed with the current), the submarine is only subject to the force of seawater in the horizontal plane, and only subject to the gravity and buoyancy in the vertical direction, and v_x,v_y are used to represent the partial velocity of the current in the X-axis and Y-axis directions, respectively, and x_x, x_y, x_z are used to represent the displacement in the X, Y, and Z directions after the loss of power. Use x_x,x_y,x_z to denote the displacement of the submarine in the X,Y,Z directions after loss of power.

The displacement in the horizontal plane, i.e. in the X and Y axes:

$$x_x = \int v_x dt \tag{7}$$

$$x_y = \int v_y dt \tag{8}$$

Vector summing x_x and x_y gives the horizontal plane displacement of the submarine operating in dt.

The partial velocity on the z-axis is affected by only 3 forces: current resistance, buoyancy, and gravity, yielding the process of changing partial velocity on the z-axis:

$$\frac{dv}{dt} = \frac{\rho_{currents} \cdot V \cdot g}{M+m} - g \tag{9}$$

Where the initial velocity v0 in the Z-axis gives the submarine's partial velocity v_z in the Z-axis at a given moment:

$$v_z = v_0 + 0.5(a_0 + a_1) \cdot dt \tag{10}$$

Where a_0 is the acceleration at the previous moment and a_1 is the acceleration at that moment and therefore the displacement in the vertical direction:

$$x_z = (v_0 + v_0 + a \cdot dt) \cdot dt \tag{11}$$

Total displacement in dt: vector summing x_{xy} and x_z gives the total displacement of the submarine running in dt.

We will use the above model to obtain the final model result based on the displacement of the submarine: the exact latitude and longitude of the submarine:

We assume the initial latitude and longitude of the submarine when it was just lost: $lat1, lon1$

The final latitude and longitude that can be deduced is: $lat2, lon2$

Submarine changes in longitude and latitude, respectively:

$$dlat = lat2 - lat1 \tag{12}$$

$$dlon = lon2 - lon1 \tag{13}$$

According to the haversine formula:

$$d = 2r \cdot arcsin\sqrt{\left(sin^2\left(\frac{lat2-lat1}{2}\right) + cos(lat1) \cdot cos(lat2) \cdot sin^2\left(\frac{lon2-lon1}{2}\right)\right)} \tag{14}$$

Thus, we can get the final longitude and latitude:

$$lat = pre - lat + \frac{North-south\ displacement}{haversine(lat,lon,lat+1,lon+1)} \tag{15}$$

$$\mathrm{l}on = pre - lon + \frac{East-west\ displacement}{haversine(lat,lon,lat+1,lon+1)} \tag{16}$$

Combined with Digital Elevation (DEM) Data Map and the model's calculation of the submarine's z-axis coordinates [2], we can determine when:

$$z_0 + x_z = h_{lat,lon} \tag{17}$$

$h_{lat,lon}$ is the depth of the seabed from sea level at a certain place, i.e., the submarine ran aground on the seabed at a certain moment and stopped moving and floating.

2.3 Results

The density of seawater ρseawater at different latitudes, longitudes and depths in the Ionian Sea is represented in visual form, which is then brought into Eq. (10), Eq. (11) and Eq. (12) and solved by integrating it to derive the displacement of the submarine in the vertical direction. A visual representation of the magnitude and direction of the velocity of the currents at different latitudes and longitudes in the Ionian Sea, which can be brought into Eqs. (8) and (9) to solve for the displacement of the submarine in the horizontal plane by integrating. According to the model, when the submarine is lost at the coordinates (x_0, y_0, z_0), the displacement of its motion in time t can be calculated, as well as the latitude and longitude coordinates at which it stops moving (sinks).It can be seen that the actual trajectory of the submarine operation is basically fitted, while the actual, measured and drifting trajectories of the submarine for another given parameter. It can be concluded that the calculations of the model basically fit the actual trajectory [3].

3. Search and Rescue Equipment

3.1 Types, Utility and Cost of Search and Rescue Equipment

From the China Knowledge Network and the public information of some underwater rescue companies, we can access the relevant information to determine the categories, utility and cost of the SAR equipment we can use [4].

3.2 Benefit Analysis Model for Search and Sescue Equipment

Different equipment has different accuracy, search and rescue range, and reliability, and we use a benefit analysis model of the equipment to represent the combined benefits of the relevant SAR equipment:

$$E = w_1 R + w_2 A + w_3 D \tag{18}$$

where E is the combined benefit of a particular device, R is the SAR range of the device, A is its SAR accuracy, and D denotes its reliability. It is possible to derive the combined benefits of any SAR equipment when the weighting is (w_1, w_2, w_3).

3.3 Cost-benefit Analysis Model for Multiple Search and Rescue Equipment

After knowing the combined benefits of each device, we have to model the cost-benefit of multiple devices. We use the entropy weighting method to carry out a weighted analysis of the benefits and costs of multiple devices, with the CER denoting the difference between costs and benefits:

$$CER = aE - bC \tag{19}$$

In the above equation a and b are the weighting ratios for calculating the total costs to the total benefits.

3.4 Results

In Eq. (20), we use the entropy weighting method to carry out a weighted analysis of the benefits and costs of multiple devices, which can be used to derive the value of (a, b) by the entropy weighting method: for the two influences of costs and benefits [5], we analyse the information entropy values of these two factors:

$$H(X) = -\sum \left(p(xi) log2\big(p(xi)\big) \right) \tag{20}$$

Where $p(xi)$ denotes the probability that the factor takes the value of xi, and then its weight is derived from the information entropy value:

$$w1 = 0.633419$$

$$w2 = 0.206794$$

$$w3 = 0.159787$$

From the above data, it is possible to derive the combined benefits of either SAR equipment.

In Eq. (19), we use the benefit analysis model of the equipment to represent the comprehensive benefits of the relevant SAR equipment, and by reviewing the literature and analysing and collating relevant data, we take the values of the weight share (a, b) respectively:

$$a = 0.95$$

$$b = 0.05$$

Based on the above model, we can derive the calculation result: the scheme of the maximum benefit of the set of SAR equipment obtained by the model when the budget given by the SAR company is $W: \sum E \le W$ when the value of CERmax is sought, and at this time it is the optimal scheme for carrying equipment.

4. A Probabilistic Grid-based Multidimensional Device Search Scheme

4.1 Gridded Probability of SAR

For search and rescue over a stretch of water, our proposed model divides the water surface area into grids and assigns initial search probabilities to each grid. Adoption of constraints on submarine motion [6], we build the initial lattice probability model:

$$(x-x_0)^2+(y-y_0)^2 \le [(t-t_0)\cdot v_m]^2 \tag{21}$$

$$(x-x_1)^2+(y-y_1)^2 \le [(t-t_1)\cdot v_m]^2 \tag{22}$$

In equations (23) and (24):$\mathrm{t}_0 \le \mathrm{t} \le \mathrm{t}_1$.

With each part of the waters gridded, we derive a grid probability formula for the sea area:

$$\sigma_x = \begin{cases} \frac{(v_m-\bar{v})(t-t_1)}{3} & t \in \left(t_1, \frac{t_2-t_1}{2}\right) \\ \frac{(v_m-\bar{v})(t_2-t_1)}{3} & t \in \left(\frac{t_2-t_1}{2}, t_2\right) \end{cases} \tag{23}$$

$$\sigma_y = \begin{cases} \frac{v_m(t-t_1)}{3} & t \in (t_1, t_2) \\ \frac{\sqrt{(v_m t_1)^2-(k+c)^2}}{3} & \\ \frac{v_m(t_2-t)}{3} & t \in (t_1, t_2) \end{cases} \tag{24}$$

Derive the final sea grid probability formula:

$$f(x,y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}[\frac{(x-\mu_1)^2}{\sigma_1{}^2}+\frac{(y-\mu_2)^2}{\sigma_2{}^2}]} \tag{25}$$

Since x, y are independent of each other in this scenario, $\rho = 0$, the probability distribution of the sea grid is obtained as shown in Fig. 1:

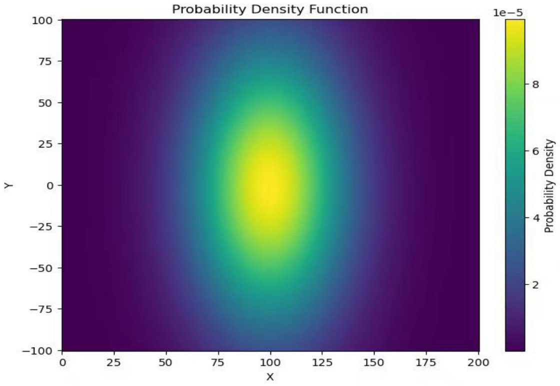

Figure 1: Lattice Probability Distribution

4.2 Optimal SAR Programme

While determining the probability of search and rescue for each grid, determine the search range for each SAR device, and the speed at which the device will move. Adjust the depth and speed of search by considering the equipment performance and SAR mission requirements. Information about the SAR equipment at t=0, i.e., the initial state, the SAR equipment is deployed in the grid with the largest probability (in order of probability from the largest to the smallest), and at each time step, the first few grids with the largest cumulative sum of SAR probability are selected as the target area of the SAR equipment. Optimal Search and Rescue Scheme: find the idle devices, and among the idle grids, search for an optimal grid, i.e., the one that maximises the ratio of the probability of satisfying that grid to the time T (according to the data given in 6.2.1.) that the current device has travelled to that grid:

$$OR = (\tfrac{p_{next}}{T})_{max} \tag{26}$$

Based on the search results, the probability of the grid is updated. Noting that the time for the SAR equipment to move to the current grid is 0 and the ratio is infinite, the "optimal grid" is set to be the grid with the largest ratio after excluding the current grid.

For each device, the time loss to move to the target grid is calculated. Consider the complexity of the underwater environment for path planning: when the seabed height is higher than the depth at which the SAR device is located, the SAR device automatically avoids obstacles around the edge of the terrain.

4.3 All Lattice Probability Update Process

The search probability for the grid is updated based on the search results of the device. The probability that a submarine happens to be present at the time of searching the grid with device accuracy $P0$ is P, then:

$$p(\text{"}seems\ in\text{"}|\text{"}actually\ in\text{"}) = p \cdot p_0 \tag{27}$$

That is, taking into account the errors present in the search and rescue equipment, a probabilistic analysis of the post-search grid is required. The probability that a submarine exists on this grid after getting a search and rescue:

$$p_T = \frac{p(1-p_0)}{pp_0+p(1-p_0)+p'^{p_1}+p'^{p_2}} \tag{28}$$

Consider a simplified model, assuming that the judgement derived by the search equipment is accurate when the submarine is not in fact in the grid and $p' = 1 - p$, the probability of the factual update p_T at this point simplifies to $p_T' = p(1 - p0)$, and based on the updated probability distribution, the search and rescue instrumentation can proceed to the next step of the search and rescue operation. At each time step, the search probability of the current grid is adjusted in real time according to the search results of the SAR device, and then with the adjusted probability, the device is allowed to go to the next optimal grid to search.

4.4 Results

Assuming that there are i search and rescue devices under the depth of each search and rescue device is hj respectively, then the search and rescue process and results displayed after several iterations are shown in Fig. 2:

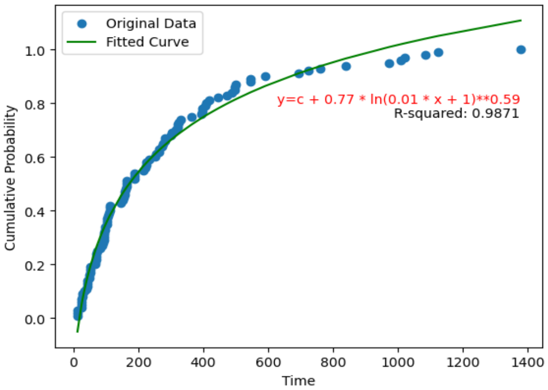

Figure 2: Correlation of Results

We observe that the data image is close to the curve image similar to the second-order derivative decreasing gradually, so we assume that the fitting function of the data is a nonlinear function $F(x) = aln(bx + d) + c$ after the relevant statistical processing of the data, through the relevant fitting procedure, and in the deployment of the model to deal with the search and rescue program as well as the path planning, the fitting result reaches 98.71% correlation.

5. Broadening Models

5.1 Broadening Point I: New Sea Areas

In new ocean areas (e.g., the Caribbean Sea), the ocean environment, such as current speed, current direction, seafloor geomorphology, and seawater density distribution, are different from the ocean parameters assumed in the original model, so in order to broaden the model to work in new ocean areas, we have to modify the parameters related to the ocean in the original model. It was also extended to include the temperature distribution in the Caribbean Sea.

5.2 Expansion Point 2: Modelling Multiple Submarines

When there is only one submarine, the signal strength from that submarine to the primary vessel will gradually diminish, which is not conducive to search and rescue efforts. The signal strength from the submarine to the communication station:

$$SS = Pt - n \cdot log_{10}\left(\frac{distance}{d0}\right) + N(0,1) + N(0,2) \tag{29}$$

SS is Signal Strength, is the signal strength, n is the signal attenuation rate, and $d0$ is the signal attenuation reference distance. Given that the signal of a single submarine will keep degrading, in a sea with multiple submarines, we can set up all submarines in the sea as signal relay stations to analyse the joint communication strength:

$$SS = \{Yi - nlog_{10}\left[\frac{(distance)i}{d0}\right] + N(0,1) + N(0,2)\}_{max} \tag{30}$$

Yi is the communication strength between the communication relay submarine and this submarine, and the movement trajectory. It can be seen that compared to the original single submarine communication, after a threshold, the decline is slower than before.

There are n lost submarines, the probability of the existence of the ith lost submarine in a certain grid is Pi,then the probability of the existence of a submarine in this grid is $P = 1 - (1 - P1)(1 - P2)\ldots(1 - Pn)$.The visualisation of the search process shows that two submarines are assumed to be lost and two submarines involved in the rescue.

6. Conclusions

In this article, we have successfully addressed a series of critical problems related to submarine rescue operations by employing innovative methodologies and predictive models. Our Submarine Position Prediction Model (SPPM) has demonstrated its efficacy in accurately forecasting submersible positions over time while facilitating continuous data transmission for enhanced situational awareness.

Furthermore, our analysis has led to the identification of essential equipment requirements for efficient rescue operations, including autonomous underwater drones, sonar systems, and communication buoys, among others. These recommendations, backed by comprehensive cost-benefit analyses, ensure preparedness for unforeseen circumstances.

By leveraging probabilistic grid-based search models and optimizing the Search and Rescue (SAR) process for multiple submersibles, we have significantly elevated operational efficiency and responsiveness. Incorporating communication synergy and advanced search patterns, our model accounts for various factors influencing successful rescue missions.

Looking ahead, potential enhancements include refining Model I to incorporate deceleration processes post-power loss and further fine-tuning communications factors in Model IV for increased effectiveness. Additionally, exploring the integration of mind mapping techniques can offer visual clarity and aid in problem-solving processes.

In conclusion, our comprehensive approach not only addresses current challenges but also paves the way for continuous improvement and innovation in submarine rescue operations, ensuring heightened safety and success rates in maritime emergencies.

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