ISSN 2616-5805 Vol. 4, Issue 5: 74-80, DOI: 10.25236/AJMS.2023.040510

An analysis of the solution of the integration of curves of type II

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Abstract: *The integration of curves of type II is an important part of integrals. This paper summarizes four methods of solving the integration of curves of type II, which are the transformation of the integration of curves of type I, definite integration, dual integration, and single connection, and explains in detail the intrinsic connection and scope of application between them. The transformation of the integration of curves of type I applies to all plane curves, Green's formula requires the construction of closed curves for solving. The definite integral method requires plane curves expressed as parametric equations to be solved. The single connectivity method solves plane curves that satisfy the condition that curve integrals are independent of paths. Finally, the four solution methods are applied with examples.*

Keywords: *Integrals of curves of type I, integrals of curves of type II, definite integrals, dual integrals*

The curve integral of type II is a very important branch of integrals, which has important applications in natural science and production technology. For example, using curve integrals to find the work done by variable forces: the correct result can be obtained by calculating the work done by the force on the object according to the curve integral algorithm, choosing the appropriate coordinate system and changing the curve integral to the definite integral[1], accurately calculating the distribution of complex pressure fields: based on the measured material parameters, the bending bearing capacity of concrete rectangular beams is calculated by the curve integral method[2], performing elastic analysis of complex structures: using the frequency curve integral, the low-frequency load shedding strategy is obtained. This strategy can automatically adjust the frequency of the first round of low-frequency load shedding and the cutting load of each round to ensure the safe and stable operation of the power system[3], describing complex fluid dynamics. It is important to understand the various methods of solving type II curve integrals, and the following four methods are introduced: the method of transforming type I curve integrals, the method of definite integrals, the method of dual integrals, and the method of single connectivity.

1. Background knowledge

1.1 Integration of curves of type I

Let the function $f(x,y)$ be a curve segment defined in the plane to find the length.If L is partitioned into n small curve segments, take a point (ξ_i,η_i) $(i=1,2,\cdots,n)$ on each small curve segment,where Δs_i is the arc length of the i small curve segment and T is the maximum value of the arc length Δs_i $(i=1,2,\cdots,n)$ of the small curve segment.

$$\lim_{\|T\|\to 0}\sum_{i=1}^{n} f(\xi_i,\eta_i)\Delta s_i \tag{1}$$

exists and the partition is independent of the way the point (ξ_i,η_i) is taken, then this limit is called the first type of curve integral of the function $f(x,y)$ along the curve L, denoted as

ISSN 2616-5805 Vol. 4, Issue 5: 74-80, DOI: 10.25236/AJMS.2023.040510

$$\int_L f(x,y)ds \tag{2}$$

where $f(x,y)$ is the product function and L is the integration arc[4].

1.2 Curve integrals of type II

Let the function $P(x,y),Q(x,y)$ be defined in the plane directed length curve. $L:\widehat{AB}$ On the plane-directed length curve, partition L into n small arc segments, take a point (ξ_i,η_i) $(i=1,2,\cdots,n)$ on each small arc segment, Δx_i is the arc length of the i small arc segment of the curve $P(x,y)$,and Δy_i is the arc length of the i small arc segment of the curve $Q(x,y)$. If the limit

$$\lim_{\|T\|\to 0}\sum_{i=1}^{n}P(\xi_i,\eta_i)\Delta x_i+\lim_{\|T\|\to 0}\sum_{i=1}^{n}Q(\xi_i,\eta_i)\Delta y_i \tag{3}$$

exists and the partition is independent of the way the point (ξ_i,η_i) is taken, then this limit is called the type II curve integral of the function $P(x,y),Q(x,y)$ along the directed curve L, denoted as

$$\int_L P(x,y)dx+Q(x,y)dy \tag{4}$$

where $P(x,y)$ and $Q(x,y)$ are the product functions and L is the integral arc segment[5].

After understanding the definition of the first type of curve integral and the second type of curve integral, it is necessary to know many solutions of the second type of curve integral, which is helpful for us to solve and calculate accurately.

2. A method for finding integrals of curves of type II

2.1 Integral transformations of curves of type I

The plane curve is $L\begin{cases}x=x(s),\\ y=y(s),\end{cases} 0\le s\le l,$ where the arc length s is the parameter, l is the full length of the curve L, and the direction of the curve is from 0 to l, with the tangent direction at each point point pointing to the side where the arc length increases. x The tangent direction is from $(t,x),(t,y)$ to, and the tangent direction of each point points to the side of the arc length increase. y axis of the positive angle, if $P(x,y),Q(x,y)$ is a continuous function on the curve, then

$$\int_L P(x,y)dx+Q(x,y)dy=\int_0^l[P(x(s)),y(s))cos(t,x)+Q(x(s)),y(s))cos(t,y)]ds=\int_L[P(x,y)cos(t,x)+Q(x,y)cos(t,y)]ds\,. \tag{5}$$

When the direction of the plane curve in the type II curve integral changes, the integral value also changes sign. This is because the direction of the plane curve changes, and the tangent direction at each point points in the opposite direction to the original. When the angle differs from the original angle by one π, then $cos(t,x),cos(t,y)$ differs from the original by one sign[6].

2.2 Definite integral method (math.)

The plane curve is $L\begin{cases}x=\phi(t),\\ y=\varphi(t),\end{cases} t\in[\alpha,\beta],$ where $\phi(t),\varphi(t)$ has a first-order continuous

ISSN 2616-5805 Vol. 4, Issue 5: 74-80, DOI: 10.25236/AJMS.2023.040510

derivative function on $[\alpha,\beta]$,the direction of the curve goes from α to β, and there is a continuous function $P(x,y),Q(x,y)$ on this plane curve L. Then the integral of the type II curve along L from α to β is

$$\int_L P(x,y)dx+Q(x,y)dy=\int_\alpha^\beta [P(\phi(t),\varphi(t))\phi'(t)+Q(\phi(t),\varphi(t))\varphi'(t)]dt \tag{6}$$

Based on the direction of the plane curve and the expression of the function, the parametric equation of the curve can be written directly and calculated using the above formula. In the process of writing parametric equations, pay particular attention to the direction of the plane curve.

A plane curve that cannot be generalized into a parametric equation, i.e., a closed curve, can be divided into n connected curves, and the curve integrals of these curves can be obtained by adding them together to obtain the required curve integral.

2.3 Dual integral method

If the function $\mathrm{P(x,y),Q(x,y)}$ is continuous over the closed region D and has first-order continuous partial derivatives, then we have

$$\iint_D(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y})d\sigma=\oint_L Pdx+Qdy\,. \tag{7}$$

Here L is the boundary curve of region D, segmented smooth and taken in the positive direction.

When the curve here is taken in a positive direction, i.e., when a person walks along the boundary, the region D is always to his left, and the above formula remains unchanged. If the direction is negative, add a negative sign to one end of the above formula.

If the curve is not a closed curve, a closed curve can be constructed by the construction method to satisfy the above conditions, and the above formula can be used for calculation.

2.4 Single connection method

If region D is a single-connected region, the following four conditions are equivalent if function $P(x,y),Q(x,y)$ is continuous within D and has first-order continuous partial derivatives:

(1) Along any smooth closed curve L in D by segments, there are

$$\oint_L Pdx+Qdy=0 \tag{8}$$

(2) For any smooth curve by segment in D L, the curve integral

$$\oint_L Pdx+Qdy \tag{9}$$

has nothing to do with paths, only with the start and end points of L.

(3) $\mathrm{Pdx+Qdy}$is a function within D $\mathrm{u(x,y)}$ that is fully differentiable, i.e., within D there are

$$du=Pdx+Qdy \tag{10}$$

(4) Established everywhere within D

$$\frac{\partial Q}{\partial x}=\frac{\partial P}{\partial y} \tag{11}$$

As long as any one of the above four conditions is satisfied, it can be obtained that the curve integral has nothing to do with the path but only with the start and end points. Therefore, it is possible to make the plane curve of type II curve integral simpler and easier to calculate.

The second type of curve integral has four solutions, respectively, the first type of curve integral transformation method, constant integral method, double integral method, simple connectivity method, the following describes the connection between these four solutions.

ISSN 2616-5805 Vol. 4, Issue 5: 74-80, DOI: 10.25236/AJMS.2023.040510

3. Intrinsic linkages among the four approaches

For all plane curves, the relationship between the type I plane curve integral and the type II plane curve integral can be utilized to solve the problem.

$(t,x),(t,y)$ denote the positive angle between the tangent direction t and the x axis and y axis, respectively: then,

$$\frac{dx}{ds} = \cos(t,x) \tag{12}$$

$$\frac{dy}{ds} = \cos(t,y) \tag{13}$$

$$dx(s) = x'(s)ds \tag{14}$$

$$dy(s) = y'(s)ds \tag{15}$$

$$\sqrt{x'(s)^2 + y'(s)^2} = (\cos\theta)^2 + (\sin\theta)^2 = 1\,, \tag{16}$$

Then, the type II plane curve integral can be expressed as:

$$\int_L Pdx + Qdy = \int_0^l [P(x(s),y(s))x'(s) + Q(x(s),y(s))y'(s)]\sqrt{x'(s)^2 + y'(s)^2}ds = \int_0^l [P(x(s),y(s))cos(t,x) + Q(x(s),y(s))cos(t,y)]ds = \int_L [P(x,y)cos(t,x) + Q(x,y)cos(t,y)]ds \tag{17}$$

In the method of definite integrals, for a plane curve L can be written as an expression for a parametric equation

$$L \ \begin{cases} x = \varphi(t), \\ y = \psi(t), \end{cases} \ t \in [\alpha, \beta] \tag{18}$$

Or the sum of n parametric equation expressions

$$A_1A_2 \ \begin{cases} x_1 = \varphi_1(t), \\ y_1 = \psi_1(t), \end{cases} \ t \in [\alpha_1, \beta_1] \tag{19}$$

$$A_2A_3 \ \begin{cases} x_2 = \varphi_2(t), \\ y_2 = \psi_2(t), \end{cases} \ t \in [\alpha_2, \beta_2] \tag{20}$$

$$\cdots\cdots$$

$$A_nA_1 \ \begin{cases} x_n = \varphi_n(t), \\ y_n = \psi_n(t), \end{cases} \ t \in [\alpha_n, \beta_n] \tag{21}$$

$$L = A_1A_2 + A_2A_3 + \cdots + A_nA_1 \tag{22}$$

Then, the type II plane curve integral can be expressed as:

$$\int_L P(x,y)dx + Q(x,y)dy = \int_\alpha^\beta [P\big(\varphi(t),\psi(t)\big)\varphi'(t) + Q(\big(\varphi(t),\psi(t)\big)\psi'(t)]dt \tag{23}$$

Or:

$$\int_L P(x,y)dx + Q(x,y)dy \ = \int_{\alpha_1}^{\beta_1} [P\big(\varphi_1(t),\psi_1(t)\big)\varphi_1{}'(t) + Q(\big(\varphi_1(t),\psi_1(t)\big)\psi_1{}'(t)]dt + \int_{\alpha_2}^{\beta_2} [P\big(\varphi_2(t),\psi_2(t)\big)\varphi_2{}'(t) + Q(\big(\varphi_2(t),\psi_2(t)\big)\psi_2{}'(t)]dt + \cdots + \int_{\alpha_n}^{\beta_n} [P\big(\varphi_n(t),\psi_n(t)\big)\varphi_n{}'(t) + Q(\big(\varphi_n(t),\psi_n(t)\big)\psi_n{}'(t) \tag{24}$$

Below is an example of one parametric equation, n and the parametric equations are similar.

For a type II plane curve integral with L a closed curve

$$\oint_L Pdx + Qdy \tag{25}$$

If we satisfy that the function $P(x,y),Q(x,y)$ is continuous on the closed region D with first-order continuous partial derivatives, which we can view as a region of type x , we have:

$$\oint_L Pdx + Qdy = \iint_D (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})d\sigma = \int_\alpha^\beta dx \int_{\varphi(x)}^{\psi(x)} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dy \tag{26}$$

ISSN 2616-5805 Vol. 4, Issue 5: 74-80, DOI: 10.25236/AJMS.2023.040510

We can also think of it as a y type region, then there is:

$$\oint_L Pdx + Qdy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) d\sigma = \int_\alpha^\beta dy \int_{\varphi(y)}^{\psi(y)} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx \tag{27}$$

If L is a semiclosed curve, it can be turned into a closed curve and solved using the construction method.

For type II plane curve integrals

$$\int_L Pdx + Qdy \tag{28}$$

If any one of the equivalence conditions is satisfied, then there are

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0 \tag{29}$$

That is, if the plane curve is independent of the path, then the simplest path can be determined from the start and end points of the plane curve for the calculation.

Suppose the starting point is (x_0, y_0) and the end point is (x, y) ; then, the simplest path is:

$$L_1 \begin{cases} x = x, \\ y = y_0, \end{cases} x \in [x_0, x] \text{ , } L_2 \begin{cases} x = x, \\ y = y, \end{cases} y \in [y_0, y] \tag{30}$$

Then there is:

$$\oint_L P(x,y)dx + Q(x,y)dy = \int_{x_0}^{x} P(x, y_0)dx + \int_{y_0}^{y} Q(x,y)dy \tag{31}$$

As long as the rules for solving the second type of curvilinear integral are strictly followed, the result can be calculated. The following example shows how to use these four solutions to solve the calculation problem of the second type of curve integral.

4. Example applications

4.1 Example 1

Calculate the curve integral $\int_L xdy + ydx$, where L is the $y = 2x, x \in [0,1]$[7].

Method 1: (definite integral method)

$y = 2x, x \in [0,1]$The parametric equations are:L: $\begin{cases} x = t, \\ y = 2t, \end{cases} t \in [0,1]$

According to the formula

$$\int_L P(x,y)dx + Q(x,y)dy = \int_\alpha^\beta [P(\varphi(t), \psi(t))\varphi'(t) + Q((\varphi(t), \psi(t))\psi'(t)]dt \tag{32}$$

the reason why

$$\int_L xdy + ydx = \int_0^1 (2t + 2t)dt = 2 \tag{33}$$

Method 2: (Dual Integration Method)

The region D is the $0 \le y \le x \le 1$, function $P(x,y) = y, Q(x,y) = x$ that is continuous on the closed region D and has first-order continuous partial derivatives, so that

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0 \tag{34}$$

assume

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) d\sigma = \iint_D 0 d\sigma = 0 = \oint_L Pdx + Qdy \tag{35}$$

$$\oint_L Pdx + Qdy = \int_0^1 xd0 + 0dx + \int_0^2 dy + yd1 - \int_L xdy + ydx \tag{36}$$

ISSN 2616-5805 Vol. 4, Issue 5: 74-80, DOI: 10.25236/AJMS.2023.040510

From the two equations above, we have

$$\int_L xdy + ydx = 2 \tag{37}$$

Method 3: (single connection method)

Let$A(0,0), B(1,0), C(1,2)$ be a single-connected region from$A \to B \to C \to A$ be a single connected region

The function$P(x,y) = y, Q(x,y) = x$ is continuous within D and has first-order continuous partial derivatives

$$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} = 1 \tag{38}$$

satisfies the conditions of the single-connection method, so

$$\int_L P(x,y)dx + Q(x,y)dy = \int_0^1 0dx + \int_0^2 dy = 2 \tag{39}$$

4.2 Example 2

Calculate $\oint_L ydx + zdy + xdz$ where L is $x^2 + y^2 + z^2 = 1$ and $x + y + z = 1$ and the intersection line of x is taken in the counterclockwise direction when viewed from the axis.

Analysis: The curve L is not good for writing the parametric equation directly, and the first type of surface integral transformation method is used.

Solution: Set $F(x,y,z) = x^2 + y^2 + z^2 - 1 \qquad G(x,y,z) = x + y + z - 1$,

Then, the line of intersection L is the outer circle of ΔABC $(A(1,0,0), B(0,1,0), C(0,0,1))$.

$$\frac{\partial F}{\partial x} = 2x \qquad \frac{\partial F}{\partial y} = 2y \qquad \frac{\partial F}{\partial z} = 2z \tag{40}$$

$$\frac{\partial G}{\partial x} = 2 \qquad \frac{\partial G}{\partial y} = 2 \qquad \frac{\partial G}{\partial z} = 2 \tag{41}$$

the reason why

$$\frac{\partial(F,G)}{\partial(y,z)} = 2(y-z) \qquad \frac{\partial(F,G)}{\partial(z,x)} = 2(z-x) \qquad \frac{\partial(F,G)}{\partial(x,y)} = 2(x-y) \tag{42}$$

The direction vector of the tangent line is:t $(2(y-z), 2(z-x), 2(x-y))$

The direction vector of the tangent line is modulo: $2\sqrt{2}$

The unit direction vector of the tangent line is: $\left(\frac{1}{\sqrt{2}}(y-z), \frac{1}{\sqrt{2}}(z-x), \frac{1}{\sqrt{2}}(x-y)\right)$

Bringing the C point in, the unit direction vector is: $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

From the meaning of the question, the positive view from the x axis takes the counterclockwise direction, so the unit direction vector has to change sign.

$$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz = 1 \tag{43}$$

the reason why

$$xy + yz + xz = 0. \tag{44}$$

$$\oint_L ydx + zdy + xdz = \oint_L \frac{1}{\sqrt{2}}[y(z-y) + z(x-z) + x(y-x)]ds = \oint_L -\frac{1}{\sqrt{2}}ds = -\frac{1}{\sqrt{2}}\oint_L ds \tag{45}$$

ISSN 2616-5805 Vol. 4, Issue 5: 74-80, DOI: 10.25236/AJMS.2023.040510

The radius of the outer circle is: $\frac{\sqrt{2}}{2} \times \frac{1}{\cos 30^\circ} = \sqrt{\frac{2}{3}}$

So:

$$-\frac{1}{\sqrt{2}} \oint_L ds = -\frac{1}{\sqrt{2}} \times 2\pi \times \sqrt{\frac{2}{3}} = -\frac{2\sqrt{3}\pi}{3} \tag{46}$$

assume

$$\oint_L ydx + zdy + xdz = -\frac{2\sqrt{3}\pi}{3} \tag{47}$$

5. Conclusions

The same second type of curve integral problem can have a variety of solutions: the first type of curve integral transformation method, definite integral method, double integral method and simply connected method, different solutions reflect the internal relationship between the integral, different integral can be transformed into each other. When solving a certain type II curve problem, the calculation method is selected according to the requirements of the problem, so as to reduce the difficulty of calculation and improve the efficiency of calculation. In teaching practice, it is very important for students to fully and accurately understand and master the definition and solution of the second type curve integral, and be able to apply it skillfully in calculation. Students should pay special attention to the direction of the curve, whether the curve is a closed curve, whether the curve can be expressed as a parametric equation, and whether the integral of the curve is related to the path.

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