Research on the Static WTA of Terminal Cooperative Air Defense Impacted by Coupling Factors

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Abstract: The factors impacting the effect of terminal cooperative air defense were analyzed and classified from the coupling mechanism perspective. Air defense scenery as a key point of weapon target assignment (WTA) algorithm research was set considering both the reality of the terminal air defense and the demand of algorithm comparison. We design suitable particle coding structure for the problem about WTA of cooperative air defense based on the characteristics of soft and hard weapon. Two methods are designed based on Hungarian algorithm and particle swarm optimization (PSO) algorithm separately. Design a terminal cooperative air defense scenery based on coupling factors, in which we can demonstrate and compare the effect of two method of static WTA problem. It argues the advantage and foresight of application of artificial intelligence (AI) algorithm in static WTA based on numeric calculation.

Keywords: PCA, terminal cooperative air defense, guided air defense missile, electronic countermeasure weapon, WTA, AI algorithm

1. Introduction

Guided air defense missiles and electronic countermeasure equipment are typical terminal air defense weapons[1]. Guided missiles include short range Infrared guided missile and short range radar guided missile [2]. Other longer range guided missiles may be used in some instances[3]. Electronic countermeasure weapons include active optic electric jamming, passive optic jamming, and high power microwave weapons etc[4].

Weapon target assignment (WTA), which is also known as Weapon Allocation or Weapon Assignment (WA), refers to the reactive assignment of defensive weapons to counter identified threats\(^5\). With the development of advanced weapons and combat theory, it is difficult for human decision-makers to counter the fire allocation problem effectively in the complex operational environment\(^6\). WTA is studied as a critical problem in an intelligent decision support system in order to reduce the decision pressure of human decision-makers or replace them\(^7\). Originally introduced into the field of operations research by Manne, the Weapon Target Assignment (WTA) Problem, or Missile Allocation Problem (MAP) as it is sometimes known, seeks to assign available interceptors to incoming missiles so as to minimize the probability of a missile destroying a protected asset\(^8\).

Cooperative air defense is engaged in developing the advantage of different weapons to the most degree so as to achieve the effect of “1+1>2”, among which weapon target assignment (WTA) is very important\(^9\). Sun Weidong presented an aid decision making method of cooperate ship air defense with terminal hard and soft weapons[10]. Consider how hard and soft weapons can be properly configured so that they can work together to provide the best air defense. WANG Zhaohui proposed an optimal allocation algorithm for cooperative air defense targets of warship formation based on distance factor and angle factor, which are the key factors affecting the allocation of air defense targets\(^11\), the Hungarian algorithm is adopted, which mainly considers target distance and Angle factors to determine the optimal air defense task assignment for each ship. Wang Lei presented an aerial target threat assessment model of warship cooperative air defense combat[12].

On the other hand, researchers have been using AI algorithm in static WTA research in recent years. Previous studies, especially regarding static WTA algorithms, have extensively focused on resolving the
computational complexity of general WTA problems. Sophisticated search methods and heuristic methods for solving general WTA problems have been developed over the past decades, such as Lagrangian relaxation[13], exact and heuristic or ant colony optimization[14], [15], particle swarm optimization[16], genetic algorithms[17], permutation and tabu search[18], variable neighborhood search[19], harmony search[20], hybrid discrete grey wolf optimization[21]. Of these methods, particle swarm optimization have received considerable attention due to their time efficiency advantages.

The design of static WTA method using AI algorithm should utilize the advantage of algorithm, and should be modified against the Coupling Mechanism as well. This paper proposes a new scenery of terminal air defense in static WTA, which considering complex firepower compatibility constraints of soft and hard weapons, and uses PSO to obtain allocation result, comparing it with a method based on Hungary algorithm.

2. Coupling Factors and Coupling Mechanism Analysis

Due to the working requirements and principles of the terminal air defense weapons, three factors, time, space, and magnetism impact on the air defense effect and task completion[22]. They can be divided into two kinds of coupling mechanism according to the coupling complexity and the coupling factors.

2.1. Single Coupling

Single coupling time factor is whether a target can be assigned to more than one kind of air defense weapons, such as one air defense missile and one type of electronic countermeasure weapons. It usually depends on the amount of remaining air defense missile and possible attacking target.

Single coupling spatial factor is ballistic conflict of different missiles attacking different targets. The ballistic conflict can be checked by ballistic calculation after assigning targets to the missiles. The assignment result should be adjusted if there is ballistic conflict.

Single coupling magnetic factor is mainly the magnetic compatibility of the radar guided air defense missiles. It can be resolved by proper frequency allocation before WTA in most circumstances.

2.2. Interweaving Coupling

Due to the different combination of the time factor, spatial factor, and magnetic factor, there are four possible kinds of interweaving coupling cases. Interweaving coupling related to the time factor is neglected in this static WTA research.

The interweaving coupling caused by spatial factor and magnetic factor is that the electric countermeasure weapons can impact the effect of guided missile or impact each other under some spatial conditions. The passive optic electric countermeasures will affect the active optic electric countermeasure or the infrared guided air defense missile. If the position of the smoke produced by the passive optic electric countermeasure is in the path of the active optic electric countermeasure or the infrared guided air defense missile, the effect of cooperative air defense will be reduced. If the radial direction and the frequency interval between the high power weapon and the radar guided missile is near and small, the target tracking and the guidance of the missile will be impacted, even missile miss will occur. The degree of interference is also related to the jamming pattern of the high power weapons.

3. Principles and Restrictions of Cooperative Air Defense

Considering the characteristics of different type of weapons, principles and restrictions of cooperative air defense are set as below:

a) The number of assigned target to the hard weapon cannot exceed the firepower passage of hard weapon.

b) One active optic electric countermeasure equipment can only be assigned to one optic electric guided air-to-ground missile.

c) The passive optic electric countermeasure equipment can jam all the optic electric guided air-to-ground missiles within three solid angle of the jamming direction. The jamming direction is the connection line between the protected target and the center of the smoke forged by the passive optic
electric countermeasure.

d) The passive optic electric countermeasure equipment will cause the effectiveness loss of the active optic electric countermeasure and the infrared guided missile whose effect optic path cross the smoke produced by the passive optic electric countermeasure.

e) If the passive optic electric countermeasure equipment cause the effectiveness loss of the infrared guided missile, adjust the infrared guided missile to radar guided missile if possible. Aiming to research static WTA methods under the influence of the coupling factors, the method of setting proper time interval between the firing of the passive optic electric countermeasure equipment and the infrared guided missile will be considered in the dynamic WTA and is neglected here.

f) If the passive optic electric countermeasure equipment cause the effectiveness loss of the active optic electric countermeasure, cease target assignment of the passive optic electric countermeasure equipment.

4. Air Defense Scenery Setting

4.1. Considerations

a) The attacking targets do not shoot down by the outer air defense weapons. The total number of the attacking targets is nearby the total target passage of the air defense weapons.

b) One main attacking direction and two or more assistant attacking direction are set according to the typical air-to-ground attacking tactic.

c) We should set the scenery at an extent as larger as possible to demonstrate the process of the static WTA method and compare the performance of different method.

d) Different type of attacking targets such as optic electric guided air-to-ground missile and anti-radiation air-to-ground missile are set in three solid angle in order to create enough decision space for different static WTA method to choose different cooperative anti defense weapons.

4.2. Protected Targets and Equipment Arrangement

One control tower and one ground hangar are protected targets in the setting of airport defense. The air defense weapons include two middle range radar guided missiles, two short range radar guided missiles, two short range infrared guided missiles, two active optic electric jamming equipment, two passive optic electric jamming equipment. Each weapon has only on firepower passage.

4.3. Attacking Fighters and Missiles

Two fighters and eight air-to-ground missiles coming from one main attacking direction and two assistant directions are set as attacking targets. The distribution and the motion parameters are shown in Table 1. To be more specific, two infrared guided missiles and one anti-radiation missile are set in Table 1 solid angle. In this scenery, different process and effect of static WTA method can be demonstrated.

Table 1 Attacking target parameters.

<table>
<thead>
<tr>
<th>Number</th>
<th>Type</th>
<th>Target distance(km)</th>
<th>Target course short(km)</th>
<th>Target velocity(m/s)</th>
<th>Target altitude (km)</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>T₁</td>
<td>A</td>
<td>8</td>
<td>1.39</td>
<td>300</td>
<td>0.025</td>
<td>East by north 50º</td>
</tr>
<tr>
<td>T₂</td>
<td>A</td>
<td>20</td>
<td>1.74</td>
<td>400</td>
<td>3</td>
<td>East by north 10º</td>
</tr>
<tr>
<td>T₃</td>
<td>B</td>
<td>5</td>
<td>0.02</td>
<td>800</td>
<td>1.3</td>
<td>East by south 50º</td>
</tr>
<tr>
<td>T₄</td>
<td>B</td>
<td>5</td>
<td>0.02</td>
<td>300</td>
<td>1.5</td>
<td>Southeast</td>
</tr>
<tr>
<td>T₅</td>
<td>B</td>
<td>11</td>
<td>0.01</td>
<td>300</td>
<td>1.2</td>
<td>Northeast</td>
</tr>
<tr>
<td>T₆</td>
<td>B</td>
<td>6</td>
<td>0.04</td>
<td>700</td>
<td>1.2</td>
<td>East in the angle of 3 solid angle</td>
</tr>
<tr>
<td>T₇</td>
<td>C</td>
<td>10.5</td>
<td>0.05</td>
<td>600</td>
<td>1</td>
<td>East in the angle of 3 solid angle</td>
</tr>
<tr>
<td>T₈</td>
<td>C</td>
<td>7</td>
<td>0.03</td>
<td>550</td>
<td>1.1</td>
<td>East in the angle of 3 solid angle</td>
</tr>
<tr>
<td>T₉</td>
<td>C</td>
<td>6</td>
<td>0.01</td>
<td>650</td>
<td>1.3</td>
<td>East by north 15º</td>
</tr>
<tr>
<td>T₁₀</td>
<td>C</td>
<td>6</td>
<td>0.02</td>
<td>700</td>
<td>1.8</td>
<td>East by north 20º</td>
</tr>
</tbody>
</table>
5. Solution based on Hungary Algorithm

5.1. Static WTA from the point view of Hungary Algorithm

Static WTA is an assignment problem in essence. It can be transferred to 0-1 Integer Programming under the constraint conditions. “1” represents that the target whose number is the number of the column is allocated to the weapon whose number is the number of the row.

According to characteristics of weapons and the intention of the air defense commander, we can choose different object functions to judge whether an assignment result is good enough to reach the fight effect that we want. In the passage, we choose the fight effect with maximum damage value, therefore the object function is

\[
E = \max \sum_{j=1}^{n} w_j [1 - \prod_{i=1}^{m} (1 - q_{ij} x_{ij})]
\]  

where \(w_j\) is the threat value of 10 attacking target, \(q_{ij}\) is the damage probability of Weapon \(W_i\) to the attacking target \(T_j\), \(x_{ij}\) is element of assignment matrix, if weapon \(W_i\) is assigned to hit target \(T_j\), then \(x_{ij} = 1\), otherwise \(x_{ij} = 0\).

5.2. Design of Static WTA Method Based on Max Damage-value Rule

The method is divided by three steps as below:

a) Initial Allocation

The matrix we used in this method is damage-value matrix, in which the value of each element is the damage-probability of each weapon to each target times the threat value of each target correspondingly.

The damage-probability matrix of different weapons to different attacking targets is Table 2, which weapon is set in row and the target is set in column.

<table>
<thead>
<tr>
<th></th>
<th>(T_1)</th>
<th>(T_2)</th>
<th>(T_3)</th>
<th>(T_4)</th>
<th>(T_5)</th>
<th>(T_6)</th>
<th>(T_7)</th>
<th>(T_8)</th>
<th>(T_9)</th>
<th>(T_{10})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W_1)</td>
<td>0.9</td>
<td>0.9</td>
<td>0.85</td>
<td>0.83</td>
<td>0.82</td>
<td>0.8</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>(W_2)</td>
<td>0.9</td>
<td>0.9</td>
<td>0.85</td>
<td>0.83</td>
<td>0.82</td>
<td>0.8</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>(W_3)</td>
<td>0.8</td>
<td>0.78</td>
<td>0.76</td>
<td>0.76</td>
<td>0.75</td>
<td>0.75</td>
<td>0.74</td>
<td>0.74</td>
<td>0.74</td>
<td>0.74</td>
</tr>
<tr>
<td>(W_4)</td>
<td>0.8</td>
<td>0.78</td>
<td>0.76</td>
<td>0.76</td>
<td>0.75</td>
<td>0.75</td>
<td>0.74</td>
<td>0.74</td>
<td>0.74</td>
<td>0.74</td>
</tr>
<tr>
<td>(W_5)</td>
<td>0.76</td>
<td>0.75</td>
<td>0.75</td>
<td>0.72</td>
<td>0.72</td>
<td>0.72</td>
<td>0.71</td>
<td>0.71</td>
<td>0.71</td>
<td>0.71</td>
</tr>
<tr>
<td>(W_6)</td>
<td>0.76</td>
<td>0.75</td>
<td>0.75</td>
<td>0.72</td>
<td>0.72</td>
<td>0.72</td>
<td>0.71</td>
<td>0.71</td>
<td>0.71</td>
<td>0.71</td>
</tr>
<tr>
<td>(W_7)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.7</td>
<td>0.68</td>
<td>0.67</td>
<td>0.65</td>
</tr>
<tr>
<td>(W_8)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.7</td>
<td>0.68</td>
<td>0.67</td>
</tr>
<tr>
<td>(W_9)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.65</td>
<td>0.62</td>
<td>0.62</td>
<td>0.6</td>
</tr>
<tr>
<td>(W_{10})</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.65</td>
<td>0.62</td>
<td>0.62</td>
<td>0.6</td>
</tr>
</tbody>
</table>

We calculate the threat value of 10 attacking target based on velocity, course short and distance [4], the value is shown in Table 3.

<table>
<thead>
<tr>
<th>Target</th>
<th>(T_1)</th>
<th>(T_2)</th>
<th>(T_3)</th>
<th>(T_4)</th>
<th>(T_5)</th>
<th>(T_6)</th>
<th>(T_7)</th>
<th>(T_8)</th>
<th>(T_9)</th>
<th>(T_{10})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threat value</td>
<td>0.071</td>
<td>0.079</td>
<td>0.112</td>
<td>0.098</td>
<td>0.098</td>
<td>0.109</td>
<td>0.111</td>
<td>0.104</td>
<td>0.109</td>
<td>0.109</td>
</tr>
</tbody>
</table>

In our air defense scenery, because passive optic jamming will jam two attacking target at most, we use the method of adapted Hungary Algorithm [23], the exact damage-value matrix is as following:
we obtain the assignment result as following:

\[
X = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\] (3)

b) Confliction Detection

Confliction detection is carried out according to b-d of principles and restrictions of cooperative air defense in section 2.

Assignment result (10-7) which using passive optic jamming jam will affect assignment(6-6) Infrared guided missile attack anti-radiation missile.

c) Confliction Resolution

Confliction Resolution is carried out according to e and f of principles and restrictions of cooperative air defense in section 2.

After confliction resolution, the maximum damage-value of final assignment from Hungary Algorithm is 0.750074.

6. Solution based on AI Method

6.1. AI Method Choice

Along with the uprising of the air defense countering complexity, the resolution space is enlarged, it’s more difficult to build models according to the countering rules and common optimization algorithms [24]. Thus the possibility of finding the best resolution is downsized. AI method utilizing the characteristic of biography evolution is used in static WTA [25]. Considering the countering scale and the cooperative level of constraint conditions that we research are comparatively simple and we ignore the restrictions of zone of hard weapon responsibility, the PSO algorithm is selected ultimately[26].

6.2. Method based on PSO Algorithm

The key to solving allocation problem using PSO is to establish an effective particle coding structure, each particle represents a possible allocation scheme of a weapon to the target. This paper improves the discrete particle swarm coding for the many-to-many soft and hard weapon target allocation problem, and sets the particle coding to an integer matrix. The position of each particle is a binary matrix of size 12*10, where each element indicates whether a weapon is assigned to a target. For example, if particle position \([3, 5] = 1\), it means that the 4th weapon is assigned to the 6th target. If particle position \([7, 2] = 0\), it means that the 8th weapon is not assigned to the 3rd target.

The method is divided by five steps as below:

a) set the number of the particles and the iterations, and initialize all the particles randomly.
b) refresh the optimal position of the particles and the particle swarm according to the calculation of the fitting function.

c) refresh the velocity of the particles.

d) refresh the position of the particles

e) go to b to iterate.

The fitting function is same as the one we used in Hungary Algorithm as (1), meanwhile the damage-probability matrix and threat values of attacking targets are same as those we used in Hungary Algorithm, shown in Table 2 and Table 3, in order to compare the allocation effect of two methods.

6.3. Numeric Computing and Analysis

Aiming to demonstrate PSO can fit this complex scenery well, we set different number of particles with 20, 40, 60 and 100 separately. The outcomes of four scenarios are shown in figure 1 and figure 2.

![Fig. 1 Fitness value with particle number are 20 and 40](image1)

![Fig. 2 Fitness value with particle number are 60 and 80](image2)
It can be seen from figure 3 and figure 4 that the algorithm converges rapidly with the increase of the number of iterations, and tends to stabilize after a certain number of iterations, which means PSO find the optimal allocation results. The optimal adaptation value of PSO is lower than the value solved by the Hungarian algorithm when the number of particles equal to 20, 40 and 60, but exceed it when the number of particles reach at 80.

In order to increase the value of fitting function further, we improving the learning factor, adjusting the local search ability and global search ability, the adjusted results as iteration increasing are shown in figure 3 and figure 4.

![Fig. 3 Fitness value with particle number are 20 and 40 after improving learning factor](image)

![Fig. 4 Fitness value with particle number are 60 and 80 after improving learning factor](image)

It can be seen that the adaptation value after the increase of the number of particles is significantly higher than the optimal value solved by the Hungarian algorithm. It is also proved that the adaptability of PSO to this scenery under the coupling condition of multiple constraints also satisfies the target allocation problem under the condition of coordinated scenario of soft and hard weapons in multiple incoming directions. At the same time, due to the decrease of the number of particles at n=20, the solution falls into local optimum, and the damage situation is lower than that of the Hungarian algorithm, but with the increase of the number of particles, the number of iterations decreases and the damage probability increases, which verifies the efficiency of the algorithm for the coupling scenario of soft and hard...
Equation (4) is the allocation result with n=80 and improving learning factor.

At the same time, this paper considers the existence of coupling factors in weapons. Hungarian algorithm cannot constrain coupling factors itself, and can only obtain a maximum at 0.75. The allocation results are shown in the matrix (2), which cannot satisfy constrain conditions of soft and hard compatibility, there exists confliction in the assignment result. While the discrete PSO considering the coupling of soft and hard weapons and the constraints on limit of soft weapons on multiple targets. It can be seen from the Equation (4), scheme, targets 7 and 8 are assigned to soft weapon 9 and target 6 are allocated to weapon 3, avoiding the influence of passive optic electric jamming equipment on infrared guided missiles and active optic electric jamming equipment, meeting the constraints of soft weapons on firepower compatibility under the design of this scenario.

7. Conclusions

In this paper, static WTA problem under the coupling factors of soft and hard weapons is proposed and a specific scenery is designed. PSO is introduced and the particle coding form is improved to adapt to the problem of soft and hard weapon fire compatibility. At the same time, the numeric simulation verifies the advantage of PSO in solving the static WTA of terminal cooperative air defense at certain countering scale. Compare with the Hungary Algorithm, PSO is good at solving static WTA with complex constrain condition. Hungary Algorithm only can obtain initial allocation result, confliction detection and confliction resolution must done by ourselves, while PSO can obtain results by constrain condition in the code. Besides, PSO can achieve a good interception effect, with effectiveness and adaptability.

References

2008.