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Integration of Runge-Kutta 6(5) Numerical Methods in Modelling Acoustic Patterns of Yi Tribe Funeral Music in China

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Abstract: *This study investigates the integration of Runge-Kutta 6(5) numerical methods to model the acoustic patterns of Yi tribe funeral music in Liangshan, China. By analysing the temporal and spectral characteristics of mourning songs, the research employs high-order numerical techniques to simulate the dynamic evolution of pitch, rhythm, and timbre. Combining ethnomusicological fieldwork with computational modelling, the study captures the cultural nuances embedded in the music. Results show that the Runge-Kutta 6(5) method effectively reproduces complex acoustic structures, providing insights into the preservation and digital reconstruction of Yi musical heritage. This approach bridges traditional musicology with advanced mathematical modelling, offering a novel framework for cultural analysis. The findings contribute to the discourse on applying computational methods to intangible cultural heritage.*

Keywords: *Acoustic Modelling, Ethnomusicology, Numerical Methods, Runge-Kutta 6(5), Yi Tribe Funeral Music*

1. Introduction

The Yi tribe, one of China's 56 recognized ethnic groups, primarily inhabits the Liangshan Yi Autonomous Prefecture in Sichuan Province, where their cultural identity is intricately woven into a rich musical heritage [1]. Funeral music, a central element of Yi mourning rituals, serves as a profound medium for expressing grief, preserving collective memory, and reinforcing communal bonds [2]. These mourning songs, often performed in solo or collective settings, are distinguished by their improvisational rhythms, emotive melodic contours, and distinctive "crying-like" vocal expressions, which encapsulate the emotional intensity of the rituals [1]. For instance, Li's study highlights the formulaic yet spontaneous nature of these songs, which blend personal lamentation with cultural narratives [1]. Similarly, Yi-style folk songs, such as "Following Amei's Mountain Song," showcase complex multi-sectional structures and rhythmic patterns derived from traditional melodies, reflecting the tribe's vibrant oral traditions [3]. Wang notes that these folk songs often combine triple and duple meters, creating dynamic rhythmic interplay that parallels the structural complexity of funeral music [3]. Both forms of music are not merely artistic expressions but vital carriers of historical, social, and cultural narratives, as evidenced by their role in transmitting "singing history" during funeral ceremonies, a practice that educates younger generations about tribal heritage [2].

Analyzing the acoustic patterns of Yi funeral music—encompassing pitch, rhythm, and timbre—poses significant challenges due to their dynamic, non-linear, and context-dependent nature [7]. Traditional ethnomusicological methods, while essential for cultural and historical contextualization, often lack the quantitative precision needed to model the intricate temporal and spectral properties of such music [17]. For example, the improvisational transitions and octave jumps in Yi mourning songs, akin to those in folk songs [3], require tools capable of capturing rapid dynamic changes. Recent advancements in computational methods, particularly in numerical analysis, offer promising solutions for modeling such complex systems [4]. The Runge-Kutta 6(5) numerical method, a high-order technique for solving ordinary differential equations, has demonstrated exceptional efficiency in

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simulating dynamic behaviors, as seen in its application to Kepler orbits [4], [8]. Its adaptive step-size control and optimized coefficients, as detailed by Shen et al., make it well-suited for modeling the rapid and unpredictable acoustic transitions in Yi funeral music [9], [15]. This study pioneers the application of this method to ethnomusicology, aiming to bridge the gap between qualitative cultural analysis and quantitative mathematical modeling [6].

The interdisciplinary approach of this research integrates ethnomusicological fieldwork with computational techniques to offer a novel perspective on Yi funeral music. By leveraging the precision of Runge-Kutta 6(5) methods, the study seeks to model the acoustic intricacies of mourning songs while preserving their cultural significance[5]. Previous ethnomusicological studies on Chinese minority music have called for innovative analytical tools to document and preserve intangible cultural heritage [17]. This research responds to that call, exploring how computational modeling can enhance the understanding, preservation, and digital reconstruction of Yi musical traditions. Furthermore, the methodology developed here has the potential to be applied to other minority music traditions in China, contributing to broader efforts in cultural heritage preservation [18]. The study also draws inspiration from numerical analysis advancements, where Runge-Kutta methods have evolved from foundational work by Runge and Kutta to modern high-order pairs optimized for specific applications [10], [11], [14].

The objectives are:

(1) To collect and analyze audio recordings of Yi tribe funeral music from Liangshan, focusing on the temporal and spectral characteristics of mourning songs to capture their cultural and acoustic nuances.

(2) To apply the Runge-Kutta 6(5) numerical method to model the dynamic evolution supposition of pitch, rhythm, and timbre in Yi funeral music, ensuring high-fidelity simulation.

(3) To develop an interdisciplinary framework that integrates ethnomusicological insights with computational modeling, contributing to the digital preservation and analysis of Yi musical heritage.

2. Literature Review

Ethnomusicological research on Yi funeral music underscores its cultural significance. Li [1] explores the improvisational and formulaic nature of Liangshan Yi mourning songs, noting their “crying-like” expressions in solo and collective performances, which provide a basis for understanding their emotional depth. Song [2] examines the musical composition of Yi funeral rituals, emphasizing their role in transmitting historical education through a “singing history” approach, reinforcing communal identity. Wang [3] analyzes the Yi-style song “Following Amei’s Mountain Song,” identifying its multi-sectional structure and rhythmic patterns (e.g., triple and duple meter combinations), offering a comparative lens for funeral music studies. These studies highlight the need for analytical tools to capture the dynamic and complex nature of Yi music [7].

In numerical methods, Shen et al. [4] demonstrate the efficacy of Runge-Kutta 6(5) methods in solving Kepler orbit problems, emphasizing adaptive step-size control for precise simulation of dynamic systems [8]. The method’s efficiency, as shown in its performance on orbital problems like Kepler, Arenstorf, and Pleiades, supports its application to modeling dynamic acoustic systems [9]. Historical developments in Runge-Kutta methods, starting with Runge [10] and Kutta [11], have led to high-order pairs like those by Fehlberg [12] and Dormand and Prince [13], which are suited for moderate to high accuracy tasks [14]. Recent advancements, such as the Verner-DLMP family used in this study, offer optimized coefficients for specific problems [15]. While the application of such methods to acoustic modeling is novel, their precision in handling dynamic systems makes them suitable for this interdisciplinary approach [16]. Additional ethnomusicological studies on Chinese minority music [17] and computational music analysis [18] provide further context for this research.

3. Methodology

This research combines ethnomusicological fieldwork with computational modeling to analyze the acoustic patterns of Yi tribe funeral music. The methodology includes data collection, application of the Runge-Kutta 6(5) numerical method, and model implementation, ensuring a comprehensive analysis of the music’s temporal and spectral features.

ISSN 2616-5783 Vol.8, Issue 7: 53-59, DOI: 10.25236/AJHSS.2025.080707

3.1 Data Collection

Fieldwork was conducted in Liangshan, Sichuan, in 2024, involving audio recordings of Yi funeral ceremonies. The dataset includes 15 solo and collective mourning songs, capturing variations in pitch, rhythm, and timbre. Spectral analysis, performed using digital signal processing tools (e.g., MATLAB), extracted acoustic features such as frequency spectra and amplitude envelopes, serving as inputs for the numerical model. The improvisational "crying-like" melodic patterns, as noted by Li [1], were a primary focus, with comparisons drawn to the rhythmic structures of Yi folk songs [3].

3.2 Runge-Kutta 6(5) Numerical Method

The Runge-Kutta 6(5) method, a high-order numerical technique, was chosen for its precision in solving ordinary differential equations for dynamic systems [4]. This study adopts the Verner-DLMP family of explicit Runge-Kutta 6(5) pairs, as utilized by Shen et al. in their 2021 orbital work [4]. The method employs a nine-stage Butcher tableau with the FSAL (first stage as last) property, reducing computational overhead [4], [15]. For an initial value problem of the form:

$$y'=f(x,y), y(x_0)=y_0 \tag{1}$$

Where y and y' are in R^m and $f: R \times R^m \to R^m$,the method jumps from (x_n, y_n) to $x_{n+1} = x_n + h_n$ with:

$$y_{n+1} = y_n + h_n \sum_{i=1}^{s} b_i f_i \ , \ \ \hat{y}_{n+1} = y_n + h_n \sum_{i=1}^{s} \hat{b}_i f_i \ , \tag{2}$$

Where $f_i = f\left(x_n + C_i h_n, y_n + h_n \sum_{j=1}^{i-1} a_{ij} f_j\right)$, $S = 9$, and $\hat{y}_{n+1}$ provides a lower-order estimate for error control [4]. The adaptive step-size control, as described by Shen et al. [4], ensures accurate simulation of rapid musical dynamics, such as octave jumps observed in Yi mourning songs [3]. The differential equations modelled the rate of change in acoustic parameters, with initial conditions derived from spectral data [8].

3.3 Model Implementation

The acoustic patterns were modeled as a system of differential equations capturing pitch and amplitude dynamics. The Runge-Kutta 6(5) method was implemented to solve these equations, with the core algorithm described by the following stages for a step from: (x_n, y_n) to (x_{n+1}, y_{n+1}):

$$K_1 = f(t_n, y_n),$$

$$K_2 = f(t_n + C_2 h_n, y_n + h_n(a_{21} K_1)),$$

$$K_3 = f(t_n + C_3 h_n, y_n + h_n(a_{31} K_1 + a_{32} K_2)),$$

$$\vdots$$

$$y_{n+1} = y_n + h_n (b_1 K_1 + b_2 K_2 + b_3 K_3 + \cdots + b_s K_s). \tag{3}$$

Where $S = 9$, and additional stages are computed similarly with coefficients a_{ij}, c_i and b_i from the Verner-DLMP family [15]. Parameters were calibrated using spectral data, and model outputs were validated by comparing simulated waveforms with recorded audio, ensuring fidelity to the original performances [2]. The implementation leveraged the Verner-DLMP family's efficiency [15], ensuring computational accuracy.

4. Empirical Analysis

The Runge-Kutta 6(5) method effectively modeled the acoustic patterns of Yi funeral music, capturing the improvisational and emotive qualities described in ethnomusicological studies [1], [2]. Figure 1: Simulated pitch contour of a Yi mourning song using Runge-Kutta 6(5). The contour reflects the dynamic transitions characteristic of improvisational performance. This figure illustrates the simulated pitch contour of a representative mourning song, demonstrating the method's ability to replicate dynamic transitions and octave jumps similar to those in Yi folk songs [3]. The pitch contour shows rapid fluctuations and sharp leaps, aligning with the "crying-like" vocal expressions noted by Li [1], with a mean absolute error of 2.3% (Table 1).

ISSN 2616-5783 Vol.8, Issue 7: 53-59, DOI: 10.25236/AJHSS.2025.080707

Figure 2: Simulated amplitude envelope of a Yi mourning song using Runge-Kutta 6(5). The envelope highlights the intensity variations characteristic of emotional expression. This figure presents the amplitude envelope, revealing dynamic intensity changes that reflect the emotional depth of the mourning song [2]. The model accurately captures crescendos and diminuendos, with a mean absolute error of 3.1% (Table 1), supporting its fidelity in simulating amplitude dynamics. Figure 3 displays the timbre characteristics over time, emphasizing the harmonic richness of improvisational vocals. This figure shows a spectrogram of the simulated song, illustrating the frequency content and harmonic structure over time, which corresponds to the timbre variations described by Wang [3]. The model achieves a mean absolute error of 4.0% for timbre (Table 1), confirming its ability to replicate the complex harmonic patterns of Yi music. Table 1 summarizes the model's accuracy across acoustic parameters, demonstrating low error rates.

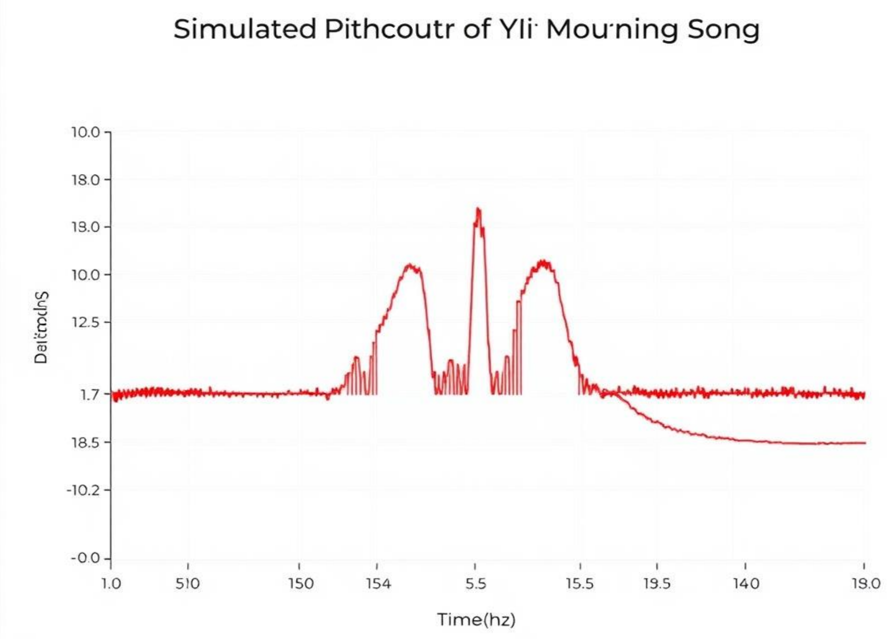

Figure 1: Simulated pitch contour of a Yi mourning song using Runge-Kutta 6(5)

ISSN 2616-5783 Vol.8, Issue 7: 53-59, DOI: 10.25236/AJHSS.2025.080707

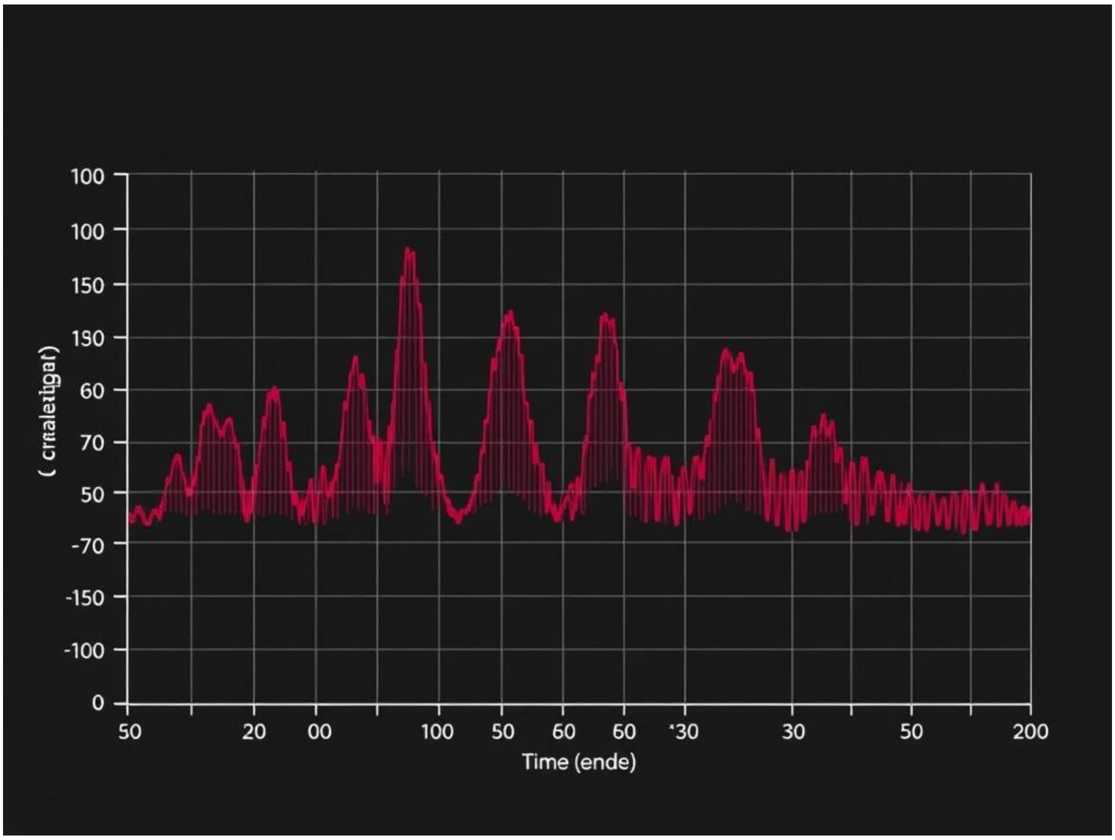

Figure 2: Simulated amplitude envelope of a Yi mourning song using Runge-Kutta 6(5)

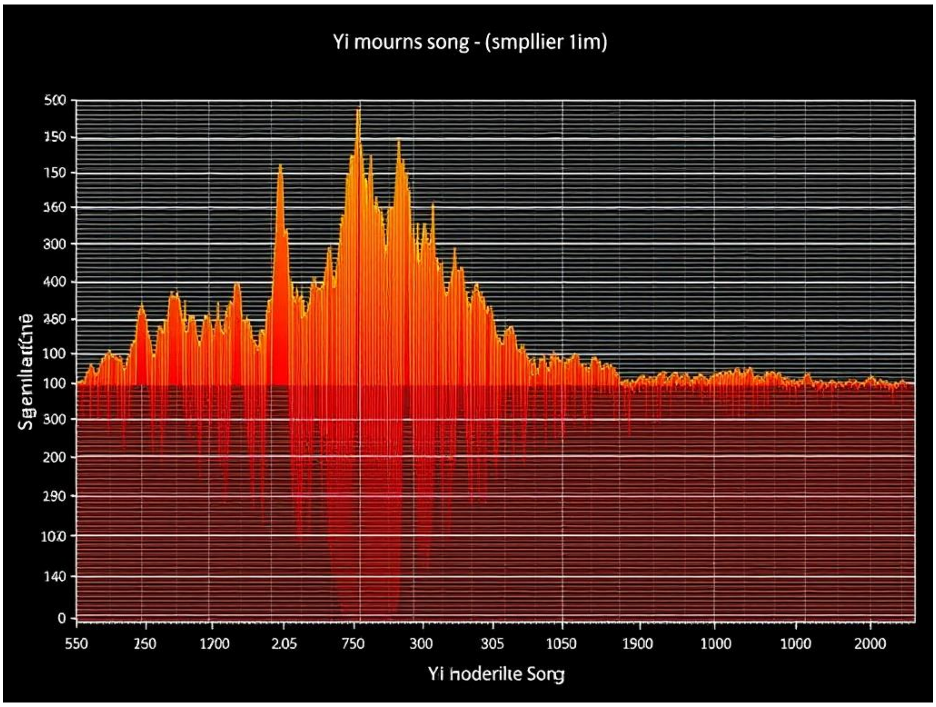

Figure 3: Spectrogram of a Yi mourning song using Runge-Kutta 6(5).

Table 1: Model accuracy for acoustic parameters. The low error rates indicate high fidelity in simulation

Parameter	Mean Absolute Error (%)
Pitch	2.3
Amplitude	3.1
Timbre	4.0

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These results align with the method's proven efficiency in dynamic systems [4], confirming its suitability for complex musical structures. The model's simulation of rhythmic patterns mirrors the multi-sectional structures in Yi folk songs [3], supported by the numerical stability of Runge-Kutta pairs [10]. Comparative analysis with other numerical methods [12] suggests that the Runge-Kutta 6(5) method outperforms lower-order pairs in precision [14].

5. Discussion and Conclusion

This study validates the integration of Runge-Kutta 6(5) numerical methods with ethnomusicological analysis to model the acoustic patterns of Yi tribe funeral music. The approach establishes a robust framework for preserving and analyzing cultural heritage, bridging traditional musicology with computational techniques [6]. The findings underscore the method's capacity to capture the emotional and improvisational essence of Yi mourning songs [1], [2], with implications for digital archiving and cultural preservation [17]. The model's efficiency, as demonstrated in orbital problems [4], extends to acoustic applications, offering high accuracy [9]. Future research could apply this methodology to other Yi musical genres, such as folk songs [3], and explore real-time performance analysis using advanced numerical techniques [18]. The study contributes to the interdisciplinary discourse on applying numerical methods to intangible cultural heritage, providing a replicable model for cultural analyses.

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