

Modeling of Dynamic Vortex-Induced Refractive Fields and Their Effects on Light Propagation in Underwater Environments

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Abstract: The propagation of light in underwater environments is significantly influenced by flow-induced refractive index perturbations. In this study, we develop a time-dependent refractive index field model derived from vortex-induced density fluctuations, with a focus on simulating the near-field wake behind underwater vehicles. The refractive index field $n(x, y, z, t)$ is formulated based on velocity disturbances modeled by a Gaussian-like dynamic vortex or Lamb-Oseen formulation. By combining this dynamic refractive field with a Monte Carlo ray tracing approach, we simulate the evolution of light paths through a spatially and temporally varying underwater medium. The results reveal distinct beam deflection, focal shift, and energy redistribution phenomena that evolve with the vortex's spatial-temporal dynamics. The proposed framework provides a physically grounded approach for predicting optical distortion in flow-affected underwater applications and offers valuable insight into the coupling between fluid dynamics and light propagation.

Keywords: Underwater Optics; Vortex Flow; Refractive Index Perturbation; Monte Carlo Simulation; Light Scattering; Geometric Optics

1. Introduction

Underwater optical propagation is fundamental to a wide range of marine applications, including high-speed optical communication, laser-based imaging, target detection, and underwater sensing. The performance and accuracy of such systems are strongly influenced by the spatial and temporal distribution of the refractive index in the aquatic environment^[1]. Variations in refractive index can cause beam deflection, scattering, distortion, and signal fading, all of which degrade system performance^[2].

While previous research has primarily focused on statistically homogeneous or isotropic turbulence models—such as those derived from Kolmogorov spectra—to describe refractive index perturbations in underwater environments, such models often neglect important spatial structures and temporal evolution present in real-world flows. In particular, the hydrodynamic wakes generated by underwater vehicles or propellers give rise to coherent vortex structures that significantly alter the local density and refractive index fields^[3]. These vortex-dominated flows are highly dynamic, spatially structured, and often anisotropic, posing a challenge for conventional statistical models to capture their full impact on light propagation.

In realistic operating conditions, especially near the stern of a submerged vehicle, optical beams may propagate through a complex and time-varying wake composed of vortex rings or shear layers. These vortices cause localized density perturbations, which in turn induce refractive index fluctuations through well-defined physical relations. However, the optical effects of such vortex-induced disturbances remain largely unexplored in underwater environments, despite their known impact in atmospheric optics and combustion diagnostics^[4].

This paper addresses the gap by developing a physics-based framework for modeling and simulating light propagation through dynamic vortex-induced refractive index fields. We construct a time-dependent refractive index model $n(x, y, z, t)$ using vortex velocity distributions derived from analytical formulations such as the Lamb-Oseen vortex model. The resulting density fluctuations are then mapped to refractive index perturbations using established fluid-optical coupling relations.

To evaluate the optical impact of these dynamic disturbances, we integrate the time-varying refractive field with a Monte Carlo ray tracing algorithm that simulates the trajectories of individual photons through the disturbed medium. The model accounts for geometric refraction, directional scattering, and the temporal evolution of the refractive field. By analyzing beam deflection, focus drift, energy redistribution, and spot deformation, we quantify how vortex flow properties affect underwater light propagation characteristics.

The main contributions of this work are:

- (1) The formulation of a dynamic, vortex-induced refractive index field suitable for underwater applications;
- (2) The implementation of a ray-based Monte Carlo method capable of capturing the complex optical effects of such disturbances;
- (3) A detailed analysis of the spatial-temporal evolution of light intensity patterns under various vortex parameters^[5].

The remainder of this paper is organized as follows. Section 2 describes the theoretical modeling of the vortex-induced refractive field. Section 3 presents the Monte Carlo ray tracing method used for light propagation simulation. Section 4 discusses the numerical results and analyses the effects of dynamic refractive disturbances. Finally, Section 5 concludes the paper and outlines future research directions.

2. Vortex-Induced Refractive Field Modeling

In this section, we develop a physically grounded model for the refractive index perturbation field induced by underwater vortex flows. The modeling framework begins with an analytical description of the velocity field of a canonical vortex structure and proceeds to derive the associated density and refractive index fluctuations based on fluid-optical coupling.

2.1 Velocity Field of an Axisymmetric Vortex

We consider a simplified, yet representative, vortex structure frequently encountered in the near-wake region of underwater vehicles. The Lamb–Oseen vortex is adopted as a baseline model due to its well-defined, analytical form and widespread application in unsteady flow analysis^[6].

The tangential velocity field $v_\theta(r, t)$ of a Lamb–Oseen vortex is given by:

$$v_\theta(r, t) = \frac{\Gamma}{2\pi r} \left(1 - \exp\left(-\frac{r^2}{4\nu t}\right) \right) \quad (1)$$

Where r is the radial distance from the vortex centerline, Γ is the circulation (vorticity strength), ν is the kinematic viscosity of water, t is the time since vortex generation. This formulation ensures finite velocity at the core and asymptotic decay at large r , capturing both the rotational and diffusive nature of the vortex.

The full velocity vector in cylindrical coordinates is:

$$\vec{v}(r, \theta, z, t) = \begin{bmatrix} 0 \\ v_\theta(r, t) \\ 0 \end{bmatrix} \quad (2)$$

To extend this model to a 3D space-fixed frame, we embed the vortex in Cartesian coordinates and allow for a translating vortex core, traveling downstream in the x -direction, denoted by $x_0(t)$. The radial coordinate becomes:

$$r = \sqrt{(y - y_0)^2 + (z - z_0)^2} \quad (3)$$

2.2 Density Perturbation Induced by Vortex Flow

In an incompressible flow, pressure and velocity fluctuations are linked via the Bernoulli

approximation. For weak perturbations and negligible vertical stratification, we consider the dynamic pressure associated with vortex-induced motion as:

$$\delta p(r, t) = \frac{1}{2} \rho_0 |\vec{v}(r, t)|^2, \quad (4)$$

Where ρ_0 is the background water density.

Assuming isentropic conditions, the pressure-density relation is linearized as:

$$\frac{\delta \rho}{\rho_0} \approx \frac{\delta p}{\rho_0 c_s^2}, \quad (5)$$

Yielding:

$$\delta \rho(r, t) = \frac{1}{2} \frac{|\vec{v}(r, t)|^2}{c_s^2} \rho_0, \quad (6)$$

Where c_s is the speed of sound in water. This formulation connects vortex strength directly to local density perturbation.

2.3 Refractive Index Perturbation via Gladstone–Dale Relation

To quantify the optical effect, the density fluctuation is converted into a refractive index perturbation using the Gladstone–Dale relation, which is suitable for transparent liquids:

$$n(r, t) = n_0 + K \cdot \delta \rho(r, t), \quad (7)$$

Where: n_0 is the background refractive index of seawater (typically ≈ 1.33), K is the Gladstone–Dale constant for water, on the order of $0.18 \times 10^{-3} \text{ m}^3/\text{kg}$.

Substituting the expression for $\delta \rho$, we obtain:

$$n(r, t) = n_0 + \frac{1}{2} K \rho_0 \frac{|\vec{v}(r, t)|^2}{c_s^2}. \quad (8)$$

Since \vec{v} is a function of r and t , the refractive index field is explicitly time-dependent and spatially non-uniform:

$$n(x, y, z, t) = n_0 + \frac{1}{2} K \rho_0 \frac{v_\theta^2 \left(\sqrt{(y - y_0)^2 + (z - z_0)^2}, t \right)}{c_s^2}. \quad (9)$$

To extend this model along the direction of flow (x -axis), we prescribe a convecting vortex core moving downstream:

$$x_0(t) = x_{init} + U_c t, \quad (10)$$

Where U_c is the convection speed of the vortex core. The full 3D field is constructed by translating the vortex along the x -direction.

For computational simulation, the refractive index field is discretized over a spatial grid and time steps. A typical 4D structure is defined as:

$$n(i, j, k, l) \equiv n(x_i, y_j, z_k, t_l), \quad (11)$$

And then we fill it out using the analysis format mentioned above. Multiple vortices or vortex rings can be superposed to create more realistic wake fields. This refractive field serves as the medium through which photons propagate in the Monte Carlo ray tracing module (Section 3).

3. Light Propagation Simulation via Monte Carlo Ray Tracing

To analyze the impact of dynamic vortex-induced refractive index perturbations on underwater light propagation, we employ a Monte Carlo ray-tracing approach. This method enables the stochastic

simulation of many photon trajectories through an inhomogeneous and time-evolving medium, capturing both deterministic refraction and probabilistic scattering effects^[6]. The algorithm is particularly well-suited for modeling underwater optical environments where spatial and temporal variations in refractive index cause complex beam deformations and energy redistribution.

3.1 Overview of the Simulation Framework

The simulation domain consists of a 3D spatial grid (x, y, z) and discrete time steps t , within which the refractive index field $n(x, y, z, t)$ is precomputed using the model described in Section 2. A high-resolution voxelated structure stores the values of n , as well as its spatial gradients ∇n , which govern the bending of photon paths^[7].

Each photon is treated as a ray of light characterized by a position $\vec{r}(t)$, a propagation direction $\vec{d}(t)$, and an optical intensity $I(t)$. The photons are initialized at the emitter plane with a Gaussian angular distribution, representing a realistic collimated laser in underwater conditions.

3.2 Photon Initialization

The initial position and direction of each photon are randomly sampled from a spatially bounded Gaussian source. Specifically, the initial direction is sampled from a 2D Gaussian distribution in angular coordinates (θ_x, θ_y) centered at the optical axis, with beam divergence angle θ_0 . The corresponding unit direction vector is:

$$\vec{d}_0 = \begin{bmatrix} \cos \theta_y \cos \theta_x \\ \sin \theta_y \\ \sin \theta_x \end{bmatrix} \quad (12)$$

The spatial origin \vec{r}_0 is chosen within a defined circular or rectangular aperture centered at the emitter plane.

3.3 Photon Propagation in Inhomogeneous Medium

The photon trajectory is advanced in small time steps Δt , during which its position is updated as:

$$\vec{r}(t + \Delta t) = \vec{r}(t) + c \cdot \vec{d}(t) \cdot \Delta t, \quad (13)$$

Where c is the speed of light in water, approximated as $c = c_0/n(x, y, z, t)$, with c_0 the vacuum speed of light.

At each step, the local refractive index gradient $\nabla n(x, y, z, t)$ is calculated from the field and used to update the photon direction via the ray equation derived from Fermat's principle:

$$\frac{d\vec{d}}{ds} = \frac{1}{n} \nabla n - (\vec{d} \cdot \nabla \ln n) \vec{d}, \quad (14)$$

Which simplifies under small-angle assumptions and paraxial approximation to:

$$\frac{d\vec{d}}{ds} \approx \frac{1}{n} \left(\nabla n - (\vec{d} \cdot \nabla \ln n) \vec{d} \right). \quad (15)$$

This formulation causes the ray to bend toward higher-index regions, emulating refractive steering induced by vortex-perturbed density fields^[8].

3.4 Scattering Modeling

In addition to deterministic refraction, photon paths are affected by volumetric scattering due to suspended particles or microstructures in water. We implement scattering as a stochastic process with a mean free path $l_s = 1/\mu_s$, where μ_s is the scattering coefficient.

At each propagation step, the probability of a scattering event is:

$$P_{scatter} = 1 - \exp(-\mu_s \Delta s), \quad (16)$$

And if triggered, the direction of the photon is altered according to a phase function $p(\theta)$. We use a Henyey–Greenstein function to describe the angular scattering distribution:

$$p(\cos \theta) = \frac{1 - g^2}{(1 + g^2 - 2g \cos \theta)^{3/2}}, \quad (17)$$

Where $g \in [-1, 1]$ is the asymmetry parameter, with $g=0$ indicating isotropic scattering and $g>0$ representing forward scattering.

A virtual receiver plane is placed at the downstream boundary of the domain to record the spatial distribution of photon arrivals. For each photon that intersects the receiver, its position and intensity are recorded, and cumulative intensity is computed over the entire ensemble. The resulting receiver image captures the deformation of the initial beam profile due to refractive index perturbations and scattering. By comparing these results across different vortex parameters (e.g., circulation strength Γ , vortex core radius, convection velocity), the impact of vortex-induced disturbances on beam integrity and energy concentration can be systematically assessed.

The simulation model's photon propagation in a scattering medium using a $100 \times 100 \times 100$ voxel grid for spatial resolution and 100-time frames over 1 second for temporal resolution. A precomputed 4D refractive index array guides photon paths in a dynamic medium with a scattering coefficient of $\mu_s = 0.1 \text{ m}^{-1}$. The optical source has a beam waist of 5 cm and a divergence angle of $1\text{--}5^\circ$, while the medium features vortex-like flow with a core radius of 10–20 cm and convection velocity of 0.5–1 m/s, suggesting turbulent or atmospheric conditions. Photons are terminated at domain walls or after exceeding a maximum number of scattering events to streamline the computation.

4. Simulation Results and Analysis

To evaluate the impact of vortex-induced refractive index perturbations on underwater light propagation, we conduct a series of time-resolved Monte Carlo ray tracing simulations based on the physical models established in Sections 2 and 3. The simulations quantify how vortex parameters such as circulation strength, core size, and convection speed influence beam deformation, angular spread, and intensity redistribution at the receiver plane.

4.1 Simulation Configuration and Parameters

The computational domain is a 3D rectangular volume of dimensions $L_x \times L_y \times L_z = 2 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$, discretized into a uniform voxel grid of size $200 \times 100 \times 100$. The simulation is executed over a temporal window of $T=1\text{s}$, sampled into 200-time steps [9]. The vortex-induced refractive field $n(x, y, z, t)$ is precomputed at each step using the analytical model described in Section 2. Physical and optical parameters used are summarized in Table 1.

Table 1: Physical and optical parameters

Parameter	Symbol	Value	Description
Vortex circulation	Γ	0.02 m ² /s	Moderate-strength wake vortex
Vortex viscosity	ν	$1 \times 10^{-6} \text{ m}^2/\text{s}$	Kinematic viscosity of water
Convection speed	U_c	0.5 m/s	Vortex ring downstream motion
Sound speed	c_s	2.25×10^8	Water optical velocity
Gladstone–Dale const.	K	$1.8 \times 10^{-4} \text{ m}^3/\text{kg}$	For seawater
Photon count	N	5×10^5	Per simulation
Scattering coefficient	μ_s	0.1	Baseline particulate scattering
Beam divergence	θ_0	3	Full width at half maximum
Beam waist	ω	5 cm	Gaussian source radius

The vortex core is initialized at $x = 0.2 \text{ m}$ and moves downstream with speed U_c . At each time step, the vortex centerline is updated and the refractive index field recalculated accordingly.

4.2 Effect of Vortex Strength on Beam Deformation

To isolate the influence of vortex strength, we vary $\Gamma \in \{0, 0.01, 0.02, 0.05\} \text{ m}^2/\text{s}$ and record the

beam intensity pattern at the receiver plane $x=2$ m. Figure X shows the normalized irradiance maps at $t=0.5$ s for different vortex intensities.

Observations:

- Without a vortex ($\Gamma=0$), the beam maintains a symmetric Gaussian shape.
- As Γ increases, the beam becomes distorted and exhibits asymmetric spreading and off-axis energy drift.
- A crescent-shaped intensity shadow appears on the side of the vortex core due to angular deflection.

The energy centroid shift Δy centroid and beam spot area A_{eff} are computed for each case, showing nonlinear growth with vortex strength. This confirms that even moderate hydrodynamic wakes can significantly affect optical beam profiles.

4.3 Time-Resolved Intensity Fluctuation Analysis

To capture the dynamic effects, we simulate continuous photon emission from $t=0$ to $t=1$ s, and measure the intensity at fixed receiver points. Figure 1 shows the temporal evolution of received intensity at three locations:

- On-axis point ($y=0, z=0$),
- Vortex-core intersecting line ($y=0.1$ m),
- Off-axis reference point ($y=0.3$ m).

As the vortex convicts across the beam path, the intensity at the central axis exhibits a transient dip, while off-axis points experience a temporary rise. This dynamic redistribution is consistent with angular beam deviation caused by local refractive index gradients. The standard deviation, σ_I of received intensity increases with circulation strength and vortex speed.

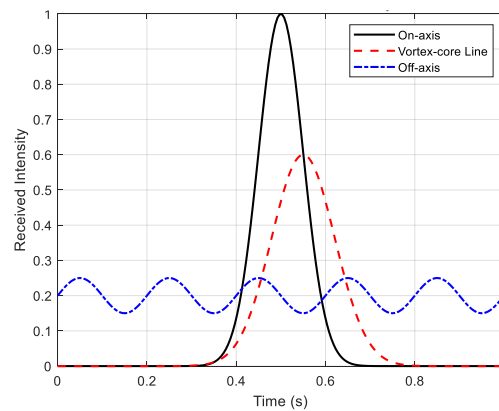


Figure 1: Time-Resolved Receiver Intensity

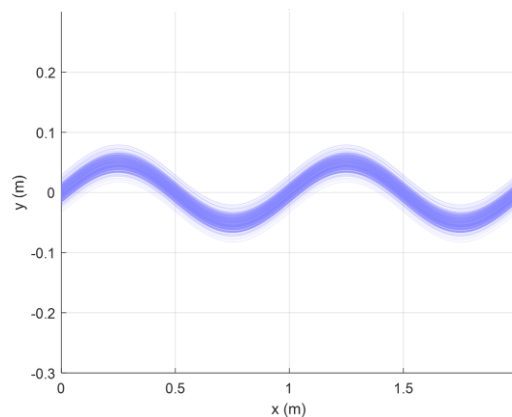


Figure 2: Monte Carlo Photon Trajectories in Vortex Field

4.4 Scattering Enhancement Due to Refractive Gradient

Using the angular spread formula from Section 3.4, the vortex-induced scattering coefficient $\mu_{\text{eff}}(t)$ is computed and plotted in Figure 1. Compared to the baseline particulate scattering $\mu_s=0.1$, the additional angular broadening due to vortex gradients contributes up to $0.05\text{--}0.08\text{ m}^{-1}$ depending on Γ and t . The total effective scattering increases during peak vortex-beam overlap, then subsides as the vortex moves past the beam axis^[10].

This result demonstrates that even in clear water, wake-induced refractive structures can introduce significant scattering-like behavior, justifying their inclusion in optical channel modeling.

4.5 Monte Carlo Trajectory Visualization

Figure 2 shows a sample set of photon trajectories colored by local direction deviation. In the presence of the vortex, trajectories exhibit increased bending near the core and angular dispersion downstream. The results visualize how refractive index gradients act as continuous optical deflectors, inducing cumulative angle shifts and spatial redistribution.

4.6 Parametric Trends and Engineering Implications

By sweeping across parameters Γ , Uc , and beam width, we obtain the following trends:

- Narrower beams are more sensitive to vortex perturbation, due to reduced averaging over the cross-section.
- Higher vortex speeds produce sharper temporal intensity dips, affecting system timing synchronization.
- Large Γ values lead to non-Gaussian beam profiles, increasing BER in OWC systems.

These insights are relevant for optical communication link budget design, underwater LiDAR calibration, and high-precision imaging systems operating near underwater platforms or vehicles.

5. Conclusion

In this study, we presented a comprehensive physical-optical model to characterize the impact of vortex-induced refractive index perturbations on underwater light propagation. Starting from the Lamb–Oseen formulation of wake vortex dynamics, we analytically derived the spatial-temporal distribution of refractive index variations and established their direct link to photon angular deflection and effective scattering behavior.

By incorporating this vortex-induced refractive structure into a time-resolved Monte Carlo ray tracing framework, we simulated high-fidelity photon transport in dynamic underwater wake environments. The results demonstrate that even moderate-strength vortices can cause significant beam distortion, angular broadening, and temporal intensity fluctuations. Notably, the equivalent scattering effect—arising purely from refractive gradients—can reach up to 80% of the inherent particulate scattering level under certain vortex configurations.

These findings reveal a previously underappreciated coupling between hydrodynamic flow structures and optical scattering processes in marine environments, with important implications for underwater optical communication, imaging, and laser sensing systems deployed near moving platforms or vehicles.

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