

# The Extended TODIM Method and Its Applications

Juxiang Wang<sup>1,\*</sup>, Ying Zang<sup>1</sup>

<sup>1</sup>School of Mathematics and Physics, Anhui Jianzhu University, Hefei, 230601, China

**Abstract:** In multi-attribute group decision-making (MAGDM) problems, decision-makers often express their subjective preferences using linguistic terms, and their decision-making behavior is influenced by psychological factors. To address the limitations of existing methods in effectively handling the fuzziness of linguistic evaluation information and the psychological behavioral characteristics of decision-makers, this paper proposes an improved TODIM-MAGDM model based on probabilistic linguistic term sets (PLTSs) and prospect theory (PT). First, PLTSs are employed to represent the decision-makers' linguistic evaluation information, capturing the hesitation and randomness in the evaluation process through the introduction of probability distributions. Second, based on PT, a gain-loss relative dominance function is constructed to quantify the differences in decision-makers' loss aversion and risk attitudes during the alternative comparison process. Third, a mechanism combining subjective and objective weighting is introduced, which integrates decision-makers' subjective preferences with the objective characteristics of the evaluation data to comprehensively determine attribute weights. Finally, the combined weights are embedded into the dominance degree calculation of the TODIM method, enabling the ranking and optimization of multi-attribute alternatives in a probabilistic linguistic environment. Results from case analysis and comparative experiments demonstrate that the proposed method performs well in terms of ranking consistency, result stability, and parameter robustness. It can effectively process uncertain linguistic evaluation information and reflect the psychological behavioral characteristics of decision-makers. This study provides an analytical tool for complex MAGDM problems under uncertain environments, offering both behavioral interpretability and information expression capability.

**Keywords:** PLTSs, PT, TODIM method, MAGDM, Information fusion

## 1. Introduction

In the decision-making practice of complex socio-economic systems and engineering technology fields, how to scientifically and effectively integrate preference information from multiple evaluation subjects has become a core research issue in the field of MAGDM<sup>[1]</sup>. MAGDM aims to form a comprehensive ranking or optimization result of alternatives by aggregating the evaluation values of multiple decision-makers on multiple alternatives under different attributes, and it has broad application backgrounds in numerous fields such as risk management, project evaluation, and resource allocation<sup>[2]</sup>. Therefore, constructing a MAGDM model that can fully reflect the characteristics of evaluation information while taking into account the behavioral preferences of decision-makers holds significant theoretical value and practical significance<sup>[3]</sup>. In the actual operation process of MAGDM, when faced with complex decision-making problems, decision-makers often find it difficult to provide precise numerical judgments on the performance of alternatives under each attribute. Limited by cognitive capacity constraints, information fuzziness, and the complexity of the problems themselves, decision-makers tend to use natural language terms such as "good," "medium," and "poor" to express their subjective preferences<sup>[4]</sup>.

To more effectively characterize the aforementioned uncertain linguistic evaluation information, PLTSs have been proposed and gradually applied in multi-attribute decision-making research<sup>[5]</sup>. This method allows evaluators to assign corresponding probabilities to different linguistic terms while making linguistic judgments, thereby further describing hesitation and randomness while retaining the intuitiveness of linguistic evaluations. It is considered an information expression method that is more consistent with human cognitive habits<sup>[6]</sup>. In recent years, PLTSs have been successfully introduced into multi-attribute decision-making models to address alternative ranking and preference analysis problems under complex evaluation environments<sup>[7]</sup>.

Nevertheless, most existing multi-attribute decision-making methods based on probabilistic linguistic information primarily focus on the aggregation and calculation process of evaluation information, paying insufficient attention to the psychological behavioral characteristics exhibited by decision-makers during

the alternative comparison process<sup>[8]</sup>. PT points out that decision-makers have asymmetric psychological perception characteristics when facing gains and losses, particularly manifested as loss aversion and differences in risk attitudes, which limits the behavioral explanatory power of traditional decision models based on expected utility theory<sup>[9]</sup>.

The TODIM method is a multi-attribute decision-making approach founded on prospect theory. It quantifies the degree of preference between alternatives by constructing a relative dominance function under "gain-loss" scenarios, demonstrating significant advantages in reflecting the psychological behavior of decision-makers<sup>[10]</sup>. In recent years, the TODIM method has been extended to various uncertain information environments and has shown good application potential in probabilistic linguistic decision scenarios<sup>[11]</sup>. However, in terms of attribute weight determination and complex linguistic information processing, existing TODIM models still have room for further improvement, particularly requiring in-depth research on the integration of subjective preferences and objective data characteristics<sup>[12]</sup>.

Based on the above analysis, this paper introduces the TODIM method within the framework of prospect theory, employs PLTSs to model the evaluation information of college students' learning interest, determines attribute weights through a combined subjective-objective weighting mechanism, and constructs an improved TODIM decision model to achieve the ranking analysis of learning interest points. Through case analysis and comparative experiments with various classical multi-attribute decision-making methods, the robustness of the ranking results and the practical application feasibility of the proposed method are verified, providing a quantitative analytical tool with behavioral explanatory power for educational decision-making in colleges and universities.

## 2. Preliminaries

### 2.1. Basic Concepts

This paper employs probabilistic linguistic term sets to express evaluation information. The basic concepts are as follows.

**Definition 1** <sup>[13]</sup>: Let  $S_g = \{s_0, s_1, \dots, s_{g-1}\}$  be a linguistic term set with  $g$ -granularity for hesitant fuzzy evaluation. A probabilistic linguistic term set (PLTS) is defined as:

$$L(P) = \{L^{(k)}(P^{(k)}) \mid L^{(k)} \in S_g, P^{(k)} \geq 0, k = 1, 2, \dots, \#L(P), \sum_{k=1}^{\#L(P)} P^{(k)} \leq 1\}. \quad (1)$$

where  $L^{(k)}(P^{(k)})$  indicates that the probability associated with the linguistic term  $L^{(k)}$  is  $P^{(k)}$ , and  $\#L(P)$  denotes the number of all distinct linguistic terms in  $L(P)$ .

**Definition 2** <sup>[13]</sup>: Let  $L(P) = \{L^{(k)}(P^{(k)}) \mid L^{(k)} \in S_g, P^{(k)} \geq 0, k = 1, 2, \dots, \#L(P), \sum_{k=1}^{\#L(P)} P^{(k)} \leq 1\}$  be a PLTS, and let  $r^{(k)}$  be the subscript of the linguistic term  $L^{(k)}$ .  $L(P)$  is called an ordered PLTS if the elements are arranged in descending order according to the values of  $r^{(k)}P^{(k)}$  ( $k = 1, 2, \dots, \#L(P)$ ).

**Definition 3** <sup>[13]</sup>: Let  $L(P) = \{L^{(k)}(P^{(k)}) \mid L^{(k)} \in S_g, P^{(k)} \geq 0, k = 1, 2, \dots, \#L(P), \sum_{k=1}^{\#L(P)} P^{(k)} \leq 1\}$  be a PLTS. The normalized PLTS is defined as:

$$\dot{L}(P) = \{L^{(k)}(\dot{P}^{(k)}) \mid L^{(k)} \in S_g, k = 1, 2, \dots, \#L(P)\}. \quad (2)$$

where  $\dot{P}^{(k)} = P^{(k)} / \sum_{k=1}^{\#L(P)} P^{(k)}$ ,  $k = 1, 2, \dots, \#L(P)$ .

**Definition 4** <sup>[13]</sup>: Let  $L_1(P)$  and  $L_2(P)$  be two PLTSs, where:

$$L_1(P) = \{L_1^{(k)}(P_1^{(k)}) \mid L_1^{(k)} \in S_g, k = 1, 2, \dots, \#L_1(P)\}. \quad (3)$$

$$L_2(P) = \{L_2^{(k)}(P_2^{(k)}) \mid L_2^{(k)} \in S_g, k = 1, 2, \dots, \#L_2(P)\}. \quad (4)$$

Here,  $\#L_1(P)$  and  $\#L_2(P)$  represent the numbers of distinct linguistic terms in  $L_1(P)$  and  $L_2(P)$ , respectively. If  $\#L_1(P) > \#L_2(P)$ , then  $\#L_1(P) - \#L_2(P)$  elements are added to  $L_2(P)$  so that the numbers of distinct linguistic terms in  $L_1(P)$  and  $L_2(P)$  become equal. The added elements are the smallest linguistic term in  $L_2(P)$ , and their associated probabilities are set to 0.

**2.2. The Extended TODIM Algorithm**

To characterize the risk attitudes and psychological behavioral features of decision-makers in the multi-attribute decision-making process, this paper constructs a multi-attribute group decision-making model based on the TODIM method within a probabilistic linguistic term set environment. This method, grounded in prospect theory, characterizes the bounded rationality of decision-makers by distinguishing between "gain" and "loss" scenarios, and can effectively handle fuzzy linguistic evaluation information and uncertain preferences.

**2.2.1. Problem Description**

Consider a multi-attribute decision-making problem involving  $m$  alternative schemes, denoted as  $A = \{a_1, a_2, \dots, a_m\}$ , and  $j$  evaluation attributes, denoted as  $C = \{c_1, c_2, \dots, c_j\}$ . The evaluation of alternative  $a_i$  with respect to attribute  $c_j$  provided by decision-makers is expressed using probabilistic linguistic term sets, forming a probabilistic linguistic decision matrix  $PL = (pl_{ij})_{m \times n}$ , where  $pl_{ij}$  represents the probabilistic linguistic evaluation information of alternative  $a_i$  under attribute  $c_j$ . Let the attribute weight vector be  $w = (w_1, w_2, \dots, w_n)$ , satisfying  $\sum_{j=1}^n w_j = 1$  and  $w_j \geq 0$ .

**2.2.2. Determination of Attribute Weights**

To balance the subjective preferences of decision-makers with the objective information inherent in the evaluation data, this paper adopts a combined subjective-objective weighting method to determine attribute weights.

First, the subjective weight of each attribute is determined based on the decision-makers' preferences regarding the importance of each attribute. Let  $n_j$  represent the number of decision-makers who prefer attribute  $C_j$ . Then, the subjective weight  $w_j^{sub}$  of attribute  $C_j$  can be expressed as:

$$w_j^{sub} = \frac{n_j}{\sum_{j=1}^n n_j} \tag{5}$$

On this basis, to fully reflect the dispersion degree and information content of the attribute evaluation information, an objective weighting method based on the concept of entropy weight is introduced. The objective weight  $w_j^{obs}$  of attribute  $C_j$  is defined as:

$$w_j^{obs} = \frac{1-H_j}{n-\sum_{j=1}^n H_j} \tag{6}$$

Here,  $H_j$  is the information entropy of attribute  $C_j$ , calculated as:

$$H_j = -\frac{1}{\ln(m)} \sum_{i=1}^m \left( \frac{\sum_{k=1}^m r_{ij}^{(P(j))} P_{ij}^{(k)}}{\#L_{ij}(P(j))} \right) \ln \left( \frac{\sum_{k=1}^m r_{ij}^{(P(j))} P_{ij}^{(k)}}{\#L_{ij}(P(j))} \right) \tag{7}$$

In Equation (7),  $m$  is the number of alternative schemes,  $r_{ij}^{(P(j))}$  and  $P_{ij}^{(k)}$  are calculation parameters associated with the probabilistic linguistic term sets, and  $\#L_{ij}(P(j))$  is the number of linguistic terms.

Considering both subjective preferences and objective information comprehensively, this paper adopts a linear combination approach to construct the comprehensive weight  $w_j$  of attribute  $C_j$ , expressed as:

$$w_j = \mu w_j^{sub} + (1 - \mu) w_j^{obs}, 0 \leq \mu \leq 1. \tag{8}$$

The coefficient  $\mu$  reflects the trade-off between subjective preferences and objective information.

To facilitate subsequent alternative comparisons and decision information aggregation, the relative weight  $w_{jt}$  of attribute  $C_j$  with respect to the most important attribute  $C_t$  is further calculated. It is defined as:

$$w_{jt} = \frac{w_j}{w_t}, j, t = 1, 2, \dots, n. \tag{9}$$

Here,  $w_t = \max\{w_j \mid j = 1, 2, \dots, n\}$ , i.e., the maximum comprehensive weight.

**2.2.3. Dominance Degree Calculation Based on TODIM**

In the TODIM method, the relative performance of alternatives under different attributes is characterized by constructing a dominance function between alternatives. Let  $\phi_j(A_i, A_e)$  denote the dominance degree of alternative  $A_i$  over alternative  $A_e$  with respect to attribute  $C_j$ . It reflects the extent to which alternative  $A_i$  is better or worse than alternative  $A_e$  on this attribute. The calculation equation is as follows:

$$\phi_j(A_i, A_e) = \begin{cases} \sqrt{\frac{w_{jt} \cdot d(L_{ij}(p_{ij}), L_{ej}(p_{ej}))}{\sum_{j=1}^n w_{jt}}}, & \text{if } L_{ij}(p_{ij}) > L_{ej}(p_{ej}) \\ 0, & \text{if } L_{ij}(p_{ij}) \sim L_{ej}(p_{ej}) \\ -\frac{1}{\theta} \sqrt{\frac{(\sum_{j=1}^n w_{jt}) \cdot d(L_{ij}(p_{ij}), L_{ej}(p_{ej}))}{w_{jt}}}, & \text{if } L_{ij}(p_{ij}) < L_{ej}(p_{ej}) \end{cases} \quad (10)$$

where:

$$d(L_{ij}(p_{ij}), L_{ej}(p_{ej})) = \sqrt{\frac{1}{\#L_{ij}(p_{ij})} \sum_{l=1}^{\#L_{ij}(p_{ij})} (P_{ij}^{(l)} r_{ij}^{(l)} - P_{ej}^{(l)} r_{ej}^{(l)})^2}, i, e = 1, 2, \dots, 7. \quad (11)$$

Equation (11) represents the distance measure between probabilistic linguistic term sets.  $\theta > 0$  is the risk aversion coefficient, which characterizes the decision-maker's sensitivity to losses. When  $\theta$  is small, the decision-maker is more sensitive to losses; when  $\theta$  is large, the decision-maker's risk attitude tends to be neutral.

Based on this, the overall dominance degree of alternative  $A_i$  over alternative  $A_e$  is defined as  $\vartheta(A_i, A_e)$ , with the equation:

$$\vartheta(A_i, A_e) = \sum_{j=1}^6 \phi_j(A_i, A_e). \quad (12)$$

**2.2.4. Prospect Value Calculation and Ranking Rules**

To obtain the comprehensive performance of each alternative, the overall prospect value  $\delta(A_i)$  of alternative  $A_i$  is introduced, defined as:

$$\delta(A_i) = \frac{\sum_{e=1}^m \vartheta(A_i, A_e) - \min\{\sum_{e=1}^m \vartheta(A_i, A_e)\}}{\max\{\sum_{e=1}^m \vartheta(A_i, A_e)\} - \min\{\sum_{e=1}^m \vartheta(A_i, A_e)\}}. \quad (13)$$

Then, the alternatives are ranked according to the magnitude of their overall prospect values:

$$V(a_i) > V(a_k) \Rightarrow a_i > a_k. \quad (14)$$

The larger the prospect value, the more significant the advantage of the alternative in a comprehensive sense compared to other alternatives.

**3. Case Analysis**

To verify the feasibility and effectiveness of the proposed TODIM decision-making method under probabilistic linguistic environment in the problem of learning interest point mining, this paper constructs a specific case study for analysis and illustration, set against the background of identifying college students' interest points in advanced mathematics learning.

**3.1. Case Background**

In the teaching process of advanced mathematics, students exhibit significant differences in their level of interest toward various teaching contents and presentation formats. To systematically identify college students' interest points in advanced mathematics learning, this paper selects several representative learning topics as alternatives and conducts a comprehensive evaluation from a multi-attribute perspective. The alternative set is defined as  $A = \{a_1, a_2, \dots, a_7\}$ , where  $a_1$  to  $a_7$  represent seven learning interest points: philosophical thought, mathematical culture, mathematician stories, regional characteristics, campus culture, modern technology, and life examples, respectively. The evaluation attribute set is defined as  $C = \{c_1, c_2, \dots, c_6\}$ , where  $c_1$  to  $c_6$  represent six evaluation attributes: problem

context, mathematical concepts, mathematical properties, mathematical theorems, problem solving, and practical application, respectively.

The evaluation data were collected through a questionnaire survey, as shown in Table 1. The survey employed a 7-granularity linguistic evaluation set  $S_7 = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\} = \{\text{extremely poor, very poor, poor, medium, good, very good, extremely good}\}$ . The participating students were asked to evaluate the seven learning interest points based on the above six attributes. A total of 500 valid questionnaires were collected. The original linguistic evaluation information was statistically organized and transformed into the form of probabilistic linguistic term sets, providing the data foundation for subsequent model calculations.

### 3.2. Construction of Probabilistic Linguistic Decision Matrix

Based on the statistical results of the questionnaire, the probabilistic linguistic evaluation information of each interest point under different attributes was organized, and the probabilistic linguistic decision matrix  $PL = (pl_{ij})_{7 \times 6}$  was constructed, where  $pl_{ij}$  represents the probabilistic linguistic term set corresponding to interest point  $a_i$  under attribute  $c_j$ . Table 1 presents the complete probabilistic linguistic decision matrix.

Table 1: Probabilistic linguistic decision matrix

	$C_1$	$C_2$	$C_3$
$A_1$	$\{S_1(0.05), S_2(0.03), S_3(0.08), S_4(0.17)\}$ $\{S_5(0.24), S_6(0.17), S_7(0.26)\}$	$\{S_1(0.05), S_2(0.04), S_3(0.07), S_4(0.17)\}$ $\{S_5(0.20), S_6(0.19), S_7(0.28)\}$	$\{S_1(0.04), S_2(0.02), S_3(0.07), S_4(0.17)\}$ $\{S_5(0.22), S_6(0.20), S_7(0.28)\}$
$A_2$	$\{S_1(0.03), S_2(0.03), S_3(0.04), S_4(0.16)\}$ $\{S_5(0.23), S_6(0.22), S_7(0.29)\}$	$\{S_1(0.04), S_2(0.02), S_3(0.05), S_4(0.13)\}$ $\{S_5(0.23), S_6(0.22), S_7(0.31)\}$	$\{S_1(0.04), S_2(0.03), S_3(0.04), S_4(0.12)\}$ $\{S_5(0.24), S_6(0.22), S_7(0.31)\}$
$A_3$	$\{S_1(0.03), S_2(0.02), S_3(0.05), S_4(0.12)\}$ $\{S_5(0.24), S_6(0.19), S_7(0.35)\}$	$\{S_1(0.04), S_2(0.02), S_3(0.04), S_4(0.15)\}$ $\{S_5(0.21), S_6(0.22), S_7(0.32)\}$	$\{S_1(0.04), S_2(0.01), S_3(0.06), S_4(0.11)\}$ $\{S_5(0.24), S_6(0.20), S_7(0.34)\}$
$A_4$	$\{S_1(0.03), S_2(0.03), S_3(0.05), S_4(0.14)\}$ $\{S_5(0.23), S_6(0.20), S_7(0.32)\}$	$\{S_1(0.04), S_2(0.02), S_3(0.06), S_4(0.14)\}$ $\{S_5(0.24), S_6(0.20), S_7(0.30)\}$	$\{S_1(0.03), S_2(0.02), S_3(0.03), S_4(0.16)\}$ $\{S_5(0.24), S_6(0.21), S_7(0.28)\}$
$A_5$	$\{S_1(0.03), S_2(0.03), S_3(0.05), S_4(0.13)\}$ $\{S_5(0.25), S_6(0.23), S_7(0.28)\}$	$\{S_1(0.04), S_2(0.03), S_3(0.05), S_4(0.17)\}$ $\{S_5(0.24), S_6(0.19), S_7(0.28)\}$	$\{S_1(0.04), S_2(0.03), S_3(0.07), S_4(0.15)\}$ $\{S_5(0.25), S_6(0.19), S_7(0.27)\}$
$A_6$	$\{S_1(0.02), S_2(0.01), S_3(0.04), S_4(0.09)\}$ $\{S_5(0.20), S_6(0.29), S_7(0.35)\}$	$\{S_1(0.02), S_2(0.03), S_3(0.03), S_4(0.12)\}$ $\{S_5(0.23), S_6(0.22), S_7(0.35)\}$	$\{S_1(0.02), S_2(0.02), S_3(0.03), S_4(0.10)\}$ $\{S_5(0.27), S_6(0.23), S_7(0.33)\}$
$A_7$	$\{S_1(0.02), S_2(0.02), S_3(0.03), S_4(0.07)\}$ $\{S_5(0.23), S_6(0.24), S_7(0.39)\}$	$\{S_1(0.02), S_2(0.02), S_3(0.04), S_4(0.10)\}$ $\{S_5(0.23), S_6(0.22), S_7(0.37)\}$	$\{S_1(0.02), S_2(0.01), S_3(0.04), S_4(0.14)\}$ $\{S_5(0.24), S_6(0.20), S_7(0.35)\}$
	$C_4$	$C_5$	$C_6$
$A_1$	$\{S_1(0.04), S_2(0.03), S_3(0.06), S_4(0.16)\}$ $\{S_5(0.23), S_6(0.20), S_7(0.28)\}$	$\{S_1(0.03), S_2(0.04), S_3(0.07), S_4(0.13)\}$ $\{S_5(0.22), S_6(0.21), S_7(0.30)\}$	$\{S_1(0.04), S_2(0.02), S_3(0.07), S_4(0.13)\}$ $\{S_5(0.24), S_6(0.21), S_7(0.29)\}$
$A_2$	$\{S_1(0.03), S_2(0.02), S_3(0.05), S_4(0.15)\}$ $\{S_5(0.24), S_6(0.22), S_7(0.29)\}$	$\{S_1(0.03), S_2(0.03), S_3(0.05), S_4(0.11)\}$ $\{S_5(0.25), S_6(0.22), S_7(0.31)\}$	$\{S_1(0.03), S_2(0.02), S_3(0.05), S_4(0.13)\}$ $\{S_5(0.25), S_6(0.23), S_7(0.29)\}$
$A_3$	$\{S_1(0.03), S_2(0.01), S_3(0.05), S_4(0.13)\}$ $\{S_5(0.26), S_6(0.19), S_7(0.33)\}$	$\{S_1(0.03), S_2(0.02), S_3(0.05), S_4(0.13)\}$ $\{S_5(0.23), S_6(0.22), S_7(0.32)\}$	$\{S_1(0.03), S_2(0.01), S_3(0.07), S_4(0.12)\}$ $\{S_5(0.23), S_6(0.21), S_7(0.33)\}$
$A_4$	$\{S_1(0.04), S_2(0.03), S_3(0.07), S_4(0.15)\}$ $\{S_5(0.23), S_6(0.19), S_7(0.29)\}$	$\{S_1(0.03), S_2(0.02), S_3(0.06), S_4(0.14)\}$ $\{S_5(0.24), S_6(0.21), S_7(0.30)\}$	$\{S_1(0.03), S_2(0.01), S_3(0.06), S_4(0.15)\}$ $\{S_5(0.24), S_6(0.22), S_7(0.29)\}$
$A_5$	$\{S_1(0.04), S_2(0.03), S_3(0.06), S_4(0.17)\}$ $\{S_5(0.21), S_6(0.23), S_7(0.26)\}$	$\{S_1(0.03), S_2(0.03), S_3(0.04), S_4(0.16)\}$ $\{S_5(0.22), S_6(0.22), S_7(0.30)\}$	$\{S_1(0.03), S_2(0.02), S_3(0.05), S_4(0.17)\}$ $\{S_5(0.20), S_6(0.22), S_7(0.31)\}$
$A_6$	$\{S_1(0.02), S_2(0.02), S_3(0.04), S_4(0.14)\}$ $\{S_5(0.24), S_6(0.22), S_7(0.32)\}$	$\{S_1(0.01), S_2(0.01), S_3(0.04), S_4(0.11)\}$ $\{S_5(0.24), S_6(0.23), S_7(0.36)\}$	$\{S_1(0.01), S_2(0.01), S_3(0.03), S_4(0.12)\}$ $\{S_5(0.22), S_6(0.26), S_7(0.35)\}$
$A_7$	$\{S_1(0.02), S_2(0.01), S_3(0.03), S_4(0.13)\}$ $\{S_5(0.23), S_6(0.25), S_7(0.33)\}$	$\{S_1(0.02), S_2(0.00), S_3(0.03), S_4(0.13)\}$ $\{S_5(0.23), S_6(0.21), S_7(0.38)\}$	$\{S_1(0.01), S_2(0.01), S_3(0.02), S_4(0.11)\}$ $\{S_5(0.22), S_6(0.23), S_7(0.40)\}$

### 3.3. Analysis of Attribute Weight Calculation Results

According to the combined weighting method proposed in Section 2.2, the subjective weights and objective weights of the six evaluation attributes were calculated separately, and on this basis, the comprehensive weight vector of the attributes was obtained.

First, based on the statistical results of evaluators' preferences for different learning interest points, the subjective weight vector of each attribute was calculated using Equation (5), with the results as follows:

$$w_1^{sub} = 0.1870, w_2^{sub} = 0.1314, w_3^{sub} = 0.1307,$$

$$w_4^{sub} = 0.1328, w_5^{sub} = 0.1974, w_6^{sub} = 0.2207.$$

These results reflect the differences in evaluators' subjective attention to different attributes, among which attribute  $C_6$  has a relatively high subjective weight, indicating that it receives more attention in the evaluation of learning interest.

Subsequently, based on the constructed decision matrix, the objective weights of each attribute were calculated using the entropy weight method and Equations (6) and (7), with the results as follows:

$$w_1^{obs} = 0.1711, w_2^{obs} = 0.1572, w_3^{obs} = 0.1588,$$

$$w_4^{obs} = 0.1551, w_5^{obs} = 0.1764, w_6^{obs} = 0.1814.$$

On the basis of comprehensively considering subjective preference information and objective data characteristics, taking the trade-off coefficient  $\mu = 0.5$ , the subjective weights and objective weights were linearly combined using Equation (8) to obtain the comprehensive weight vector of the attributes:

$$w_1 = 0.1790, w_2 = 0.1443, w_3 = 0.1447,$$

$$w_4 = 0.1440, w_5 = 0.1869, w_6 = 0.2011.$$

Furthermore, to facilitate subsequent alternative comparison and dominance degree calculation, attribute  $C_6$  with the largest comprehensive weight was selected as the reference attribute, i.e.,  $w_t = w_6 = 0.2011$ , and the relative weights of the remaining attributes with respect to  $C_6$  were calculated according to Equation (9). The calculation results are as follows:

$$w_{1t} = 0.8901, w_{2t} = 0.7176, w_{3t} = 0.7195,$$

$$w_{4t} = 0.7161, w_{5t} = 0.9294, w_{6t} = 1.0000.$$

### 3.4. Analysis of Interest Point Ranking Results Based on TODIM

Based on the obtained comprehensive attribute weights, following the TODIM decision-making process described in Section 2.2, the dominance degree  $\phi_j(A_i, A_e)$  for any two alternatives under each attribute  $C_j$  was calculated sequentially according to Equations (10) and (7), forming a  $7 \times 7$  dominance matrix. Here, the parameter  $\theta$  represents the decision-maker's risk aversion coefficient. Setting  $\theta = 2.5$  [14], the calculation using Equation (10) yields:

$$\phi_1(A_i, A_e) = \begin{bmatrix} 0 & -0.1209 & -0.1809 & -0.1498 & -0.1341 & -0.2083 & -0.2243 \\ 0.0541 & 0 & -0.1501 & -0.1113 & 0.0422 & -0.1805 & -0.1961 \\ 0.0810 & 0.0672 & 0 & 0.0467 & 0.0719 & -0.1661 & -0.1431 \\ 0.0670 & 0.0498 & -0.1044 & 0 & 0.0581 & -0.1748 & -0.1725 \\ 0.0600 & -0.0942 & -0.1607 & -0.1299 & 0 & -0.1810 & -0.1986 \\ 0.0932 & 0.0808 & 0.0743 & 0.0782 & 0.0810 & 0 & -0.1467 \\ -0.1004 & 0.0878 & 0.0640 & 0.0772 & 0.0889 & 0.0656 & 0 \end{bmatrix}$$

$$\phi_2(A_i, A_e) = \begin{bmatrix} 0 & -0.1469 & -0.1468 & -0.1302 & -0.0995 & -0.1883 & -0.2098 \\ 0.0530 & 0 & -0.0944 & 0.0349 & 0.0476 & -0.1319 & -0.1620 \\ 0.0530 & 0.0341 & 0 & 0.0440 & 0.0507 & -0.1259 & -0.1578 \\ 0.0470 & -0.0966 & -0.1219 & 0 & 0.0392 & -0.1535 & -0.1790 \\ 0.0359 & -0.1320 & -0.1405 & -0.1087 & 0 & -0.1809 & -0.2049 \\ 0.0679 & 0.0476 & 0.0454 & 0.0554 & 0.0653 & 0 & -0.0995 \\ -0.0757 & 0.0584 & 0.0569 & 0.0646 & 0.0739 & 0.0359 & 0 \end{bmatrix}$$

$$\phi_3(A_i, A_e) = \begin{bmatrix} 0 & -0.1429 & -0.1720 & -0.0872 & 0.0313 & -0.1824 & -0.1767 \\ 0.0517 & 0 & -0.1231 & 0.0466 & 0.0508 & -0.1177 & -0.1378 \\ 0.0622 & 0.0445 & 0 & 0.0605 & 0.0639 & -0.1255 & -0.0955 \\ 0.0315 & -0.1289 & -0.1674 & 0 & 0.0290 & -0.1683 & -0.1735 \\ -0.0866 & -0.1405 & -0.1767 & -0.0802 & 0 & -0.1797 & -0.1854 \\ 0.0660 & 0.0426 & 0.0454 & 0.0609 & 0.0650 & 0 & 0.0499 \\ 0.0639 & 0.0499 & 0.0345 & 0.0628 & 0.0671 & -0.1379 & 0 \end{bmatrix}$$

$$\phi_4(A_i, A_e) = \begin{bmatrix} 0 & -0.0972 & -0.1522 & -0.0775 & 0.0443 & -0.1398 & -0.1709 \\ 0.0350 & 0 & -0.1338 & 0.0386 & 0.0466 & -0.1139 & -0.1453 \\ 0.0548 & 0.0482 & 0 & 0.0510 & 0.0643 & -0.0802 & -0.1121 \\ 0.0279 & -0.1071 & -0.1418 & 0 & 0.0511 & -0.1331 & -0.1685 \\ -0.1230 & -0.1294 & -0.1786 & -0.1418 & 0 & -0.1680 & -0.1813 \\ 0.0503 & 0.0410 & 0.0289 & 0.0479 & 0.0605 & 0 & -0.1113 \\ 0.0615 & 0.0523 & 0.0403 & 0.0606 & 0.0653 & 0.0401 & 0 \end{bmatrix}$$

$$\phi_5(A_i, A_e) = \begin{bmatrix} 0 & -0.0970 & -0.0901 & -0.0741 & -0.0867 & -0.1471 & -0.1636 \\ 0.0453 & 0 & -0.0825 & 0.0406 & 0.0514 & -0.1295 & -0.1544 \\ 0.0421 & 0.0386 & 0 & 0.0412 & 0.0443 & -0.1186 & -0.1410 \\ 0.0346 & -0.0869 & -0.0881 & 0 & -0.0846 & -0.1455 & -0.1626 \\ 0.0405 & -0.1099 & -0.0948 & 0.0395 & 0 & -0.1492 & -0.1639 \\ 0.0687 & 0.0605 & 0.0554 & 0.0680 & 0.0697 & 0 & -0.0984 \\ -0.0764 & 0.0722 & 0.0659 & 0.0760 & 0.0766 & 0.0656 & 0 \end{bmatrix}$$

$$\phi_6(A_i, A_e) = \begin{bmatrix} 0 & -0.0785 & -0.1112 & -0.0677 & -0.1143 & -0.1526 & -0.1859 \\ 0.0395 & 0 & -0.1189 & 0.0356 & 0.0602 & -0.1447 & -0.1847 \\ 0.0559 & 0.0598 & 0 & 0.0586 & 0.0570 & -0.1237 & -0.1509 \\ 0.0340 & -0.0708 & -0.1165 & 0 & -0.1055 & -0.1489 & -0.1859 \\ 0.0575 & -0.1197 & -0.1134 & 0.0530 & 0 & -0.1369 & -0.1723 \\ 0.0767 & 0.0727 & 0.0622 & 0.0749 & 0.0688 & 0 & -0.1306 \\ -0.0935 & 0.0929 & 0.0759 & 0.0530 & 0.0866 & 0.0656 & 0 \end{bmatrix}$$

The overall dominance degrees and overall prospect values are further obtained, with the results as follows:

$$\vartheta(A_i, A_e) = \begin{bmatrix} 0 & -0.6834 & -0.8532 & -0.5865 & -0.3590 & -1.0185 & -1.1312 \\ 0.2786 & 0 & -0.7028 & 0.0850 & 0.2988 & -0.8182 & -0.9803 \\ 0.3490 & 0.2924 & 0 & 0.3020 & 0.3521 & -0.7400 & -0.8004 \\ 0.2420 & -0.4405 & -0.7401 & 0 & -0.0127 & -0.9241 & -1.0420 \\ -0.0157 & -0.7257 & -0.8647 & -0.3681 & 0 & -0.9957 & -1.1064 \\ 0.4228 & 0.3452 & 0.3116 & 0.3853 & 0.4103 & 0 & -0.5366 \\ 0.4714 & 0.4135 & 0.3375 & 0.3942 & 0.4584 & 0.1153 & 0 \end{bmatrix}$$

Considering the psychological differences of decision-makers in gain and loss situations, the ranking results of alternatives  $A_i$  under different parameter  $\theta$  values were calculated using Equation (13), as shown in Table 2 and Figure 1.

Table 2: Ranking results of  $A_i$  under different parameter  $\theta$  values

	$\theta = 0.5$	$\theta = 1$	$\theta = 1.5$	$\theta = 2$	$\theta = 2.5$	$\theta = 3$
	$\delta$	$\delta$	$\delta$	$\delta$	$\delta$	$\delta$
$A_1$	0	0	0	0	0	0
$A_2$	0.4451	0.4340	0.4246	0.4165	0.4094	0.4032
$A_3$	0.6795	0.6681	0.6585	0.6503	0.6430	0.6368
$A_4$	0.2788	0.2702	0.2630	0.2568	0.2513	0.2465
$A_5$	0.0771	0.0784	0.0796	0.0807	0.0814	0.0879
$A_6$	0.8946	0.8885	0.8834	0.8790	0.8752	0.8718
$A_7$	1	1	1	1	1	1

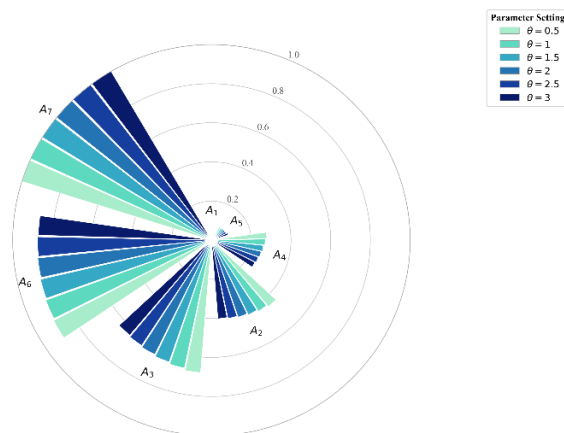


Figure 1: Ranking of  $A_i$  under different parameter  $\theta$  values

The above case analysis reveals the following (as shown in Figure 1): (1) The overall prospect value  $\delta(A_i)$  of the alternatives is influenced by the parameter  $\theta$ . As  $\theta$  varies from 0.5 to 3.0, the  $\delta$  values of each topic exhibit a certain degree of change, indicating that decision-makers' sensitivity to risk affects the evaluation results of interest points. (2) Despite the variations in the parameter  $\theta$ , the comprehensive ranking results of each learning topic remain consistent across all parameter values:  $A_7 > A_6 > A_3 > A_2 > A_5 > A_4 > A_1$ . That is, "Life Examples ( $A_7$ )" and "Modern Technology ( $A_6$ )" consistently rank in the top two positions, while "Philosophical Thought ( $A_1$ )" ranks the lowest. This consistency in ranking results demonstrates that the evaluation model based on probabilistic linguistic hesitant fuzzy sets and the TODIM method possesses good robustness and scientific validity, can effectively handle uncertain linguistic information, and supports the stable identification and ranking of students' learning interests.

**4. Comparative Experiment**

To comprehensively compare the performance of different multi-attribute decision-making methods in the problem of learning interest point ranking, this study compares the proposed method with the Weighted Sum Model (WSM), the TOPSIS method<sup>[15]</sup>, and the VIKOR method<sup>[16]</sup>. The calculation results are shown in Table 3, and the ranking results are illustrated in Figure 2.

Table 3: Calculation results of different decision-making methods

Method	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$
Proposed Method ( $\delta(A_i)$ )	0.0000	0.4094	0.6430	0.2513	0.0814	0.8752	1.0000
WSM ( $S(A_i)$ )	0.0000	0.4568	0.6541	0.2341	0.2389	0.8123	1.0000
TOPSIS ( $C(A_i)$ )	0.0000	0.2874	0.6017	0.4236	0.2655	0.7451	0.8932
VIKOR ( $Q(A_i)$ )	1.0000	0.4638	0.2742	0.7824	0.3846	0.0842	0.0000

In this table, the results of the proposed method, WSM, TOPSIS, and VIKOR are represented by the overall prospect value  $\delta(A_i)$ , comprehensive score  $S(A_i)$ , closeness coefficient  $C(A_i)$ , and compromise value  $Q(A_i)$ , respectively.

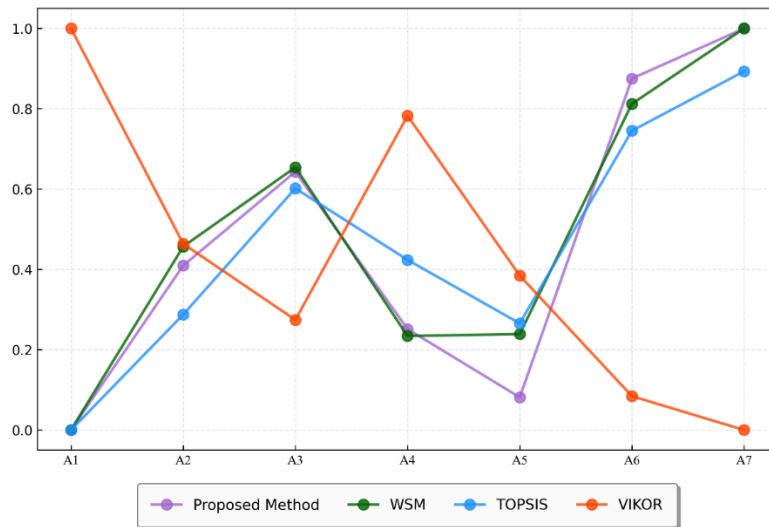


Figure 2: Ranking results of different decision-making methods

From the overall ranking results, it can be observed that the four methods exhibit high consistency in identifying the top and bottom interest points. Specifically,  $A_7$  and  $A_6$  are stably ranked in the top two positions across all methods, while  $A_1$  consistently ranks at the bottom. This indicates that different decision-making methods, when processing the same evaluation data, share a consistent understanding of the overall preference structure of students' learning interests, suggesting that the decision problem itself possesses relatively clear preference characteristics.

Further comparison reveals that the proposed method is highly similar to the WSM and TOPSIS methods in terms of ranking structure, with the top three alternatives remaining consistent across these methods. Only slight differences are observed in the ranking positions of the middle alternatives. For instance, regarding the relative ranking of  $A_4$  and  $A_5$ , different methods yield different results due to variations in their evaluation mechanisms. The WSM method, based on linear weighted summation,

emphasizes the overall average level of attribute evaluations; the TOPSIS method focuses on the distance relationship between alternatives and the ideal solution; and the proposed method further distinguishes between "gain" and "loss" scenarios in alternative comparisons, thereby providing a more behaviorally interpretable characterization of subtle differences among interest points.

Comparison with the VIKOR method shows that although VIKOR exhibits some differences in the ranking of middle alternatives compared to other methods, its rankings at both ends remain consistent with the proposed method. This demonstrates that the core ranking results of learning interest points possess strong stability under different theoretical frameworks and ranking mechanisms. Meanwhile, while the VIKOR method emphasizes the concept of compromise solutions, the proposed method, while maintaining ranking consistency, can intuitively reflect the relative advantage degrees of different interest points through prospect values, which is more conducive to interpretation and application in actual teaching decision-making.

The comprehensive comparison results indicate that the proposed method can achieve ranking conclusions highly consistent with classical decision models under different benchmark methods, and it provides more detailed differentiation among middle alternatives. This suggests that the method not only possesses good robustness at the result level but also has strong practical value in explaining differences in students' learning interests and supporting optimization decisions for teaching content.

## 5. Conclusion

This paper addresses common issues in multi-attribute group decision-making processes, such as the fuzziness of linguistic information, uncertainty in evaluators' preferences, and the difficulty in characterizing decision-makers' psychological behavioral features. An improved TODIM group decision-making model based on probabilistic linguistic term sets and prospect theory is constructed. This model utilizes probabilistic linguistic term sets to represent decision-makers' linguistic evaluation information, effectively capturing the hesitation and randomness distribution characteristics inherent in evaluation results while preserving the intuitiveness of linguistic evaluations. Within the framework of prospect theory, a gain-loss relative dominance function is constructed to quantify decision-makers' psychological behavioral characteristics during the alternative comparison process, thereby enhancing the model's explanatory power for actual decision-making behavior.

In terms of methodological construction, this paper introduces a combined subjective-objective weighting mechanism to comprehensively determine attribute weights. While balancing decision-makers' subjective preference information, this mechanism effectively utilizes the objective information characteristics embedded in the evaluation data itself, avoiding the information bias that may arise from single weighting approaches. On this basis, the combined weights are embedded into the dominance degree calculation process of the TODIM method, enabling the ranking and optimization of multi-attribute alternatives in a probabilistic linguistic environment. Results from case analysis demonstrate that the proposed method can obtain clear and stable ranking results in complex and uncertain evaluation contexts, exhibiting good discriminability and operability.

Through comparative experimental analysis with various classical multi-attribute decision-making methods, it can be observed that the proposed method performs well in terms of ranking consistency and result stability, demonstrates certain robustness to parameter variations, and exhibits stronger adaptability and interpretability when processing probabilistic linguistic evaluation information. This indicates that the improved TODIM model, combining prospect theory with probabilistic linguistic information, possesses significant theoretical value and broad application potential in handling complex group decision-making problems characterized by strong subjectivity, fuzziness, and uncertainty.

## Acknowledgement

This work was supported by the Projects of Natural Science Research in China (12301664), the Projects of Natural Science Research in Anhui Colleges and Universities (2024jyxm0255, HYB20230132, KJ2021JD20, HYB20220179, 2023AH050178, 2023kcszsf094, 2023QDZ25, 2023jyxm0437, 2024dzxkc047).

**References**

- [1] Liu P, Shen J, Zhang P. A dual-driven MAGDM method based on single-valued neutrosophic credibility numbers Einstein variable extended power geometric aggregation operator and SPAMARCOS[J]. *Artificial Intelligence Review*, 2025, 58(10): 330.
- [2] Li W, Ye J. MAGDM model using an intuitionistic fuzzy matrix energy method and its application in the selection issue of hospital locations[J]. *Axioms*, 2023, 12(8): 766.
- [3] Shi Z, Ji W. Advanced technique for intuitionistic fuzzy MAGDM and applications to park landscape planning and design schemes evaluation[J]. *International Journal of Agricultural and Environmental Information Systems (IJAEIS)*, 2025, 16(1): 1-19.
- [4] Akram M, Ali U, Santos-García G, et al. 2-tuple linguistic Fermatean fuzzy MAGDM based on the WASPAS method for selection of solid waste disposal location[J]. *Math Biosci Eng*, 2023, 20(2): 3811-3837.
- [5] Ma X, Han X, Xu Z, et al. Fusion of probabilistic linguistic term sets for enhanced group decision-making: Foundations, survey and challenges[J]. *Information Fusion*, 2025, 116: 102802.
- [6] Wang Y, Zhan J, Zhang C, et al. A group consensus model with prospect theory under probabilistic linguistic term sets[J]. *Information Sciences*, 2024, 653: 119800.
- [7] Liu Z, Liao H, Li M, et al. A deep learning-based sentiment analysis approach for online product ranking with probabilistic linguistic term sets[J]. *IEEE Transactions on Engineering Management*, 2023, 71: 6677-6694.
- [8] Lin G, Lin M S, Song H. An assessment of prospect theory in tourism decision-making research[J]. *Journal of Travel Research*, 2024, 63(2): 275-297.
- [9] Zhang H, Wang H, Wei G. Spherical fuzzy TODIM method for MAGDM integrating cumulative prospect theory and CRITIC method and its application to commercial insurance selection[J]. *Artificial Intelligence Review*, 2023, 56(9): 10275-10296.
- [10] Sun H, Yang Z, Cai Q, et al. An extended Exp-TODIM method for multiple attribute decision making based on the Z-Wasserstein distance[J]. *Expert Systems with Applications*, 2023, 214: 119114.
- [11] Wu W. Probabilistic Linguistic TODIM Method with Probabilistic Linguistic Entropy Weight and Hamming Distance for Teaching Reform Plan Evaluation[J]. *Mathematics*, 2024, 12(22): 3520.
- [12] Tang J, Liu X, Wang W. Consensus-based generalized TODIM approach for occupational health and safety risk analysis with opinion interactions[J]. *Applied Soft Computing*, 2024, 150: 111093.
- [13] Pang Q, Wang H, Xu Z S. Probabilistic linguistic term sets in multi-attribute group decision making[J]. *Information sciences*, 2016(369):128-143.
- [14] Xu D S, Wei X L, Ding H, Bin H Q. A New Method Based on PROMETHEE and TODIM for Multi-Attribute Decision-Making with Single-Valued Neutrosophic Sets[J]. *Mathematics*, 2020, 8, 1816.
- [15] Chaube S, Pant S, Kumar A, et al. An overview of multi-criteria decision analysis and the applications of AHP and TOPSIS methods[J]. *International Journal of Mathematical, Engineering and Management Sciences*, 2024, 9(3): 581.
- [16] Mallick R, Pramanik S, Giri B C. TOPSIS and VIKOR strategies for COVID-19 vaccine selection in QNN environment[J]. *OPSEARCH*, 2024, 61(4): 2072-2094.