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Complexity Characteristics and Robustness Analysis of Chengdu Metro Network

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Abstract: *The study of robustness of the metro rail network in Chengdu, a central city in western China, contributes to the stability as well as safe operation of the metro network. Based on the complex network theory, the Space-L topological structure model of the complex network constituted by the first 9 metro lines in Chengdu is constructed, and its complex network characteristics are analyzed by topological properties such as degree and degree distribution, average path length, average clustering coefficient and scale-free degree. Through its topological properties, a robustness evaluation index system applicable to the Chengdu metro network is constructed, and simulations of deliberate attacks and random attacks are performed. The data of both simulations are counted and the trends and reasons of each data are analyzed and compared. Among the many stations, three stations, namely, Chengdu University of TCM & Sichuan Provincial People's Hospital Station, Science and Technology University Station and the Culture Palace Station, have higher vulnerability. When they are attacked, the subway network connectivity reliability decreases by 1/3, indicating the need to strengthen the security check of these three stations with higher degree values.*

Keywords: *Chengdu metro network; topology; Simulation Attack; robustness*

1. Introduction

Chengdu Urban Housing Authority released the list of "14th Five-Year" Urban Construction Plan of Chengdu, which further improves the metro network of Chengdu with the development strategy of "Double Core Co-Center". As of August 11, 2022, Chengdu has put into operation 23 subway lines with 490 stations, the overall scale of which is catching up with cities in North China, Shanghai and Guangzhou.

A large number of scholars have studied transportation networks based on complex network theory. Cai Hui[1] studied the index evaluation system of the complex characteristics of metro networks without considering the intermediate common stations. Chongnan Li[2] et al. analyzed the complex characteristics of metro networks in several cities based on Person correlation coefficients. J. Gan et al.[3] studied the complex characteristics and robustness of the Wuhan subway network, and quantified the global impact of different station failures on the subway network by formulating different ways of random failures and deliberate attacks. Zheng Sujiang[4] draws the topology of Shanghai metro network and analyzes the interrelationships among the parameters by calculating the relevant topological parameters of metro stations, and also briefly analyzes the small-worldness and scale-free properties. Liping Lai[5,6] The complexity analysis of the metro network and its robustness are performed by using the robustness index circle number. Bao Deng[7] The robustness of transportation networks is evaluated comprehensively by four metrics: connectivity probability, average path length, network diameter and network performance parameters. Junyan Sun[8] A comparative analysis of static network characteristics, correlation between degree and mesoscope, and network robustness of three types of networks: bus, subway, and bus-subway composite. Albert Réka et al.[9] used metrics such as average path length to measure the degree of damage to the network after an attack.

Chengdu, as the central city in central and western China, has developed metro lines rapidly. However, most of the current studies on urban metro networks in China focus on cities such as Beijing and Shanghai, and there is a lack of studies on metro networks in Chengdu. Therefore, based on the Space-L model, this paper obtains the unweighted adjacency matrix of the first 9 metro lines in Chengdu, calculates the relevant topological parameters of the network of 257 metro stations, and basically understands the network characteristics of the metro network in L-space in Chengdu.

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Meanwhile, this paper investigates the robustness and vulnerability of the Chengdu metro network through two strategies of deliberate attack and random attack. This research result can provide realistic significance for the daily operation as well as the security of metro in Chengdu city in the future.

2. Chengdu Metro Network's Statistical Parameters

The Space-L method treats all subway stations as nodes, and each station is only connected to its neighboring stations, i.e., each connected edge is a line owned by 2 neighboring stations, and the neighboring stations do not affect their relationship with other neighboring stations.

2.1 Degree and degree distribution of nodes

The degree of a node is the branch of a node. The degree ki of node i of the unprivileged network is equal to the sum of its branches, denoted ki =∑aij as (aij= 1 when node i is connected to node j, otherwise aij = 0). The degree distribution of a node is the probability p(k) that a node with degree k accounts for all nodes. In complex networks, if the degree distribution obeys a power-law distribution (also called scale-free distribution), the network is called scale-free network.

2.2 Clustering coefficients of nodes

The clustering coefficient of a node is a measure of the average probability that two neighboring nodes of a node are also neighbors of each other, denoted as

$$C_i = \frac{2E_i}{k\cdot(k_i-1)} \tag{1}$$

From a geometric point of view, the equivalence is Ci = number of triangles connected to point i / number of triads connected to point i. In a metro network, the clustering coefficient reflects the density of the overall network nodes.

2.3 Shortest path length, average path length and efficiency between nodes

The shortest path length between nodes dij is the minimum number of connected edges between node i and node j. The average path length between nodes is the average of the shortest path lengths of all node pairs, denoted as

$$L = \frac{1}{N\cdot(N-1)} \cdot \sum_{i=1}^{N} \sum_{j=1}^{N} d_{ij} \tag{2}$$

In a subway network, the shortest route length reflects the shortest distance required to move between stations.

Network efficiency is the average of the reciprocal of the shortest distance between all pairs of nodes in the network, where the efficiency of isolated nodes and other nodes is 0. The network efficiency expression is.

$$E = \frac{1}{N\cdot(N-1)} \cdot \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{d_{ij}} \tag{3}$$

3. Complexity Analysis of Chengdu Metro Network

According to the metro rail network map of Chengdu in early 2022, a 257×257 real symmetric adjacency matrix is obtained after processing the data using MATLAB for a total of 257 stations of 9 metro lines.

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3.1 Degree and degree distribution

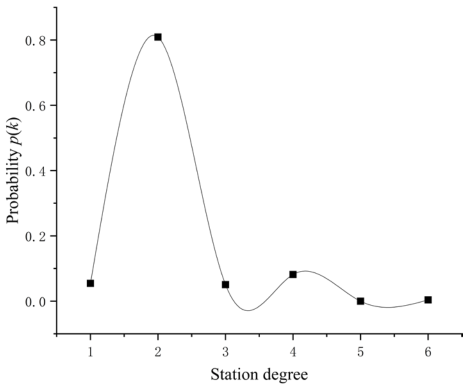

Figure 1: Distribution of node degrees in Chengdu metro

The data are processed and plotted to obtain the degree and degree distribution of metro network nodes in Chengdu City for the Space-L network topology model. It can be seen that the maximum value of degree of nodes in Chengdu metro network is 6, the minimum value is 1, and the average degree is 2.18, which reflects that each metro station can directly reach about 2~3 stations, indicating that it is good and convenient to use metro as a transportation mode to travel in Chengdu. From Fig 1, the probability of nodes with degree 2 is 80.9%, while only 3.4% of the nodes with degree value greater than 2, reflecting that metro in Chengdu mostly relies on small stations with degree 2 rather than large station interchanges to provide convenient travel for passengers.

3.2 Clustering coefficients

The average clustering coefficient of the Chengdu metro network was found to be 0. Based on the definition of the clustering coefficient combined with the fact that the degree of most of the nodes is known to be 2 in the degree distribution analysis, i.e., most of the nodes of the Chengdu metro network have only 2 contiguous edges, it is concluded that "the Chengdu metro network has its weak clustering characteristics in Space-L space The conclusion of "Chengdu metro network has its weak clustering characteristics in Space-L space".

3.3 Shortest path length, average path length and efficiency

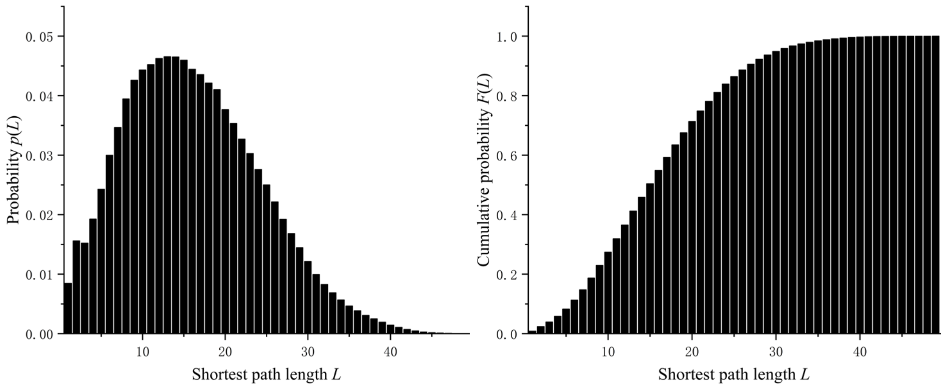

Figure 2: Probability of shortest path length between nodes of Chengdu metro network and its cumulative probability distribution

Using MATLAB to find out the shortest path length between nodes of Chengdu metro network, the average shortest path length is 16.2, the maximum value of shortest path length is 49, and the efficiency is 0.0943. From Fig 2, we can see that the shortest path length between most nodes is 13, which indicates that the distance between most stations is 13 stations, which is more consistent with the result of average shortest path length. About 50% of the shortest path lengths between nodes are less than 15, and about 80% of the shortest path lengths between nodes are less than 22, reflecting that Chengdu Metro has good travel efficiency.

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3.4 Scale-free analysis

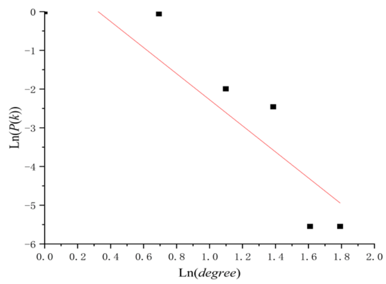

Figure 3: Cumulative degree distribution in double logarithmic coordinates

The cumulative degree distribution function $Pk = \sum P(k')$ is plotted to represent the probability distribution of degree not less than k nodes. If the degree distribution is a power-law distribution, i.e., $P(k) \propto k\text{-}\gamma$, then the cumulative degree distribution function conforms to the power law with power exponent γ-1: $Pk \propto \sum k'\text{-}\gamma \propto k\text{-}(\gamma\text{-}1)$. From Fig 3, it can be seen that it basically conforms to the power-law property, i.e., the metropolitan subway network in Space-L space can be roughly described by The power-law distribution roughly describes the station degree distribution.

4. Robustness Analysis of Chengdu Metro Network

4.1 Robustness analysis metrics

4.1.1 Relative size of the maximum connected subgraph of the network

The relative size S of the maximum connected subgraph indicates the ratio of the number of nodes of the maximum connected subgraph to the total number of nodes of the original network after the network receives an attack, where the relative size of the maximum connected subgraph of the original network is 1. The mathematical expression is

$$S = \frac{N'}{N} \tag{4}$$

where N' denotes the number of nodes of the maximum connectivity subgraph after the network is attacked, and N denotes the initial number of nodes of the network. The relative size of the maximum connectivity subgraph can reflect the change of the complex characteristics of the network before and after the attack, especially showing the degree of network damage.

4.1.2 Average path length of the maximum connected subgraph of the network

For the network after the attack, isolated nodes and other subgraphs are not considered, and only the maximum connected subgraph is considered to obtain its average shortest path length and visualize its change.

The average path length reflects the connectivity reliability of the network. The smaller the average path length, the greater the reliability; conversely, the smaller the reliability. After a certain level of attack, the average path length of the original network is replaced by the average path length of the maximum connectivity subgraph.

4.1.3 Network efficiency

When a network is attacked, its connectivity reliability is impaired, and isolated nodes in the network are disconnected from the maximum connectivity subgraph. The average path length of the maximum connectivity subgraph does not take into account the influence of isolated nodes, i.e., the connectivity reliability evaluation using this metric alone is lacking, while the network efficiency can reflect the connectivity reliability change of the complex network before and after the attack very accurately and directly represents the connectivity reliability, as shown in Table 1.

Table 1: Topological properties of Chengdu metro network

	C	N	S	L	E
Chengdu	0	257	1	16.2	0.0943

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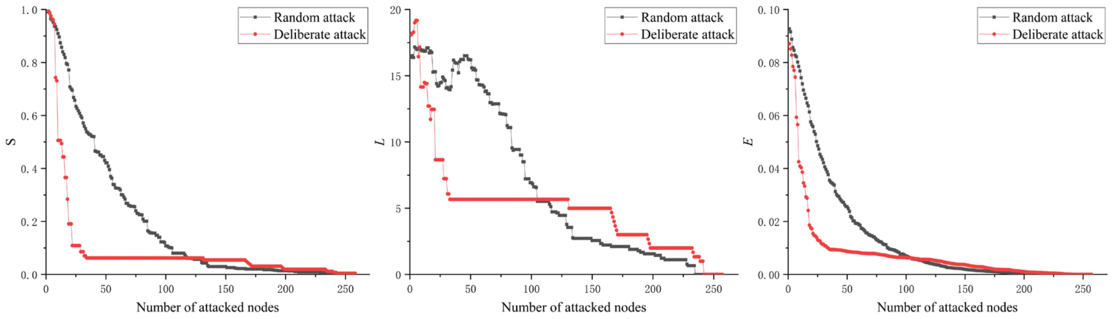
4.2 Simulation Attack



(a) (b) (c)

Figure 4: Changes in S, L and E under metro network attack in Chengdu

4.2.1 Relative size change of the maximum connected subgraph of the network

As can be seen in Fig 4(a), the network S-value decreases rapidly in the deliberate attack mode, and when attacking only about 25 nodes (9.7% of the total), the S-value decreases rapidly to 0.109, which is basically close to paralysis; when attacking about 35 nodes (13.6% of the total), the S-value becomes 0.062, which is basically completely paralyzed. In the random attack mode, the network S-value decreases more slowly, and when attacking about 125 nodes (accounting for 48.6%), the S-value is 0.057, and the original metro network disintegrates into many small-scale networks with encouraged nodes, which is basically completely paralyzed.

4.2.2 Average path length variation of the maximum connected subgraph of the network

As can be seen in Fig 4(b), under the deliberate attack mode, when the number of attacking nodes is less than 10, the network L-value has a tendency to increase instead; when attacking 10~33 nodes (accounting for 3.9%~12.8%), the network L-value decreases sharply until 5.667, and when attacking about 125 nodes (accounting for 48.6%), the L-value starts to decrease slowly in a step-like manner, from 5.667 to 0.667 slowly decreasing. In the random attack mode, the network L-value decreases rapidly when the attack nodes are 50 to 130 (19.5% to 50.6% of the total), and then more slowly.

4.2.3 Network efficiency changes

As can be seen in Fig 4(c), in the deliberate attack mode, the network E value plummets rapidly with the increase in the number of attack nodes, and when 20 nodes are attacked, the network E value is 0.017, which is very small, indicating that the connectivity of the metro network is very low, i.e., the network is down; while in the random attack mode, the network E value decreases more slowly, and when 100 nodes are attacked (38.9% of the total) 0, the E value is 0.007 and the network is down.

4.3 Analysis of important nodes

In the selection of deliberate attacks, the preferred targets are the nodes with high degree values, which are called important nodes, and the top 10 nodes in terms of finishing degree values are shown in Table 2. 2 and 3 important nodes are attacked respectively, and the robustness indexes of the metro network before and after the attacks are analyzed, which can analyze the reflection status of the metro network to the unexpected security crisis, and the results are shown in Table 3.

Table 2: Top 10 nodes of Chengdu metro network degree

Order	Station Name	Degree	Order	Station Name	Degree
1	Chengdu University of TCM & Sichuan Provincial People's Hospital	6	6	Tianfu Plaza	4
2	Science and Technology University	4	7	Ox King Temple	4
3	Cultural Palace	4	8	Liulichang	4
4	Chengdu East Railway Station	4	9	South Railway	4
5	Team Bridge	4	10	Wuqing South Road	4

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Table 3: Pre- and post-attack metrics for critical nodes

Number of attacks	Order of attacked nodes	Pre-attack indicators				Post-attack indicators			
		N	S	L	E	N'	S'	L'	E'
2	1,2	257	1	16.2	0.0943	254	0.9883	18.2	0.0851
	3,4					231	0.8988	16.2	0.0799
3	1,3,5					175	0.6809	14	0.0609
	2,4,6					229	0.8911	17.1	0.0767

As can be seen from Table 3, when attacking 2 important nodes, although the degree value of the Chinese Medicine University Hospital station is 6, the indicators are close to each other after suffering an attack relative to the two nodes with a degree of 4. When attacking 3 important nodes, when attacking the node with the hospital station of TCM University, the indicators are significantly reduced after suffering the attack, indicating that when attacking only 3 nodes, attacking the node with a greater degree is more destructive to the network connectivity.

5. Conclusion

1) The complex characteristics of the Space-L network topology model of the Chengdu metro network were analyzed. The results show that the network has moderate average path length and very small clustering coefficients, indicating good convenience of metropolitan subway travel; 80.9% of the node degrees are 2, indicating that most stations are connected to only two stations; plotting the cumulative degree distribution in double logarithmic coordinates, it is found that the metropolitan subway network basically conforms to the power law property.

2) The robustness of the metro network is analyzed by simulating both deliberate and random attacks on the metro network. The results show that the metro network in Chengdu is vulnerable to deliberate attacks, and the metro network decreases rapidly when fewer nodes are attacked, while random attacks have better robustness. It indicates that the metro network should be inspected more frequently and carefully at the stations with higher degree and higher security measures during daily operation to prevent the metro network from being paralyzed.

3) The analysis of the important nodes of the Chengdu metro network is carried out. The results show that when the number of nodes selected for attack is 2, the difference in which important nodes are selected for attack has a small impact on network robustness; while when the number of nodes is 3, attacking nodes with a larger degree of the Chengdu metropolitan network will cause a large blow to the network robustness, and the indicators are significantly reduced to about 60% of the original. Among them, the three stations of Chengdu University of TCM & Sichuan Provincial People's Hospital Station, Science and Technology University Station and the Culture Palace Station have higher vulnerability. When they are attacked, the reliability of metro network connectivity decreases by 1/3, indicating the need to strengthen the security checks on these three stations with larger degree values.

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