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Research on Instructional Design for High School Mathematics Based on the STEAM Education Philosophy—A Case Study of "Ellipses and Their Standard Equations" with Reduced Complexity

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Abstract: *This study delves into the instructional design framework and strategies for high school mathematics based on the STEAM (Science, Technology, Engineering, Arts, Mathematics) education concept. By meticulously analyzing the high school mathematics curriculum standards, integrating student learning profiles, and mathematical content, we have developed a framework that integrates STEAM elements. This framework applies STEAM literacy in high school mathematics teaching through strategies such as creating authentic situations, posing problem sequences, fostering student collaborative inquiry, encouraging self-directed summaries, and assigning differentiated assignments. The research focuses on: principles and strategies for designing mathematics curricula based on the STEAM concept, enhancing students' comprehensive literacy by creating authentic situations and integrating STEAM competencies; utilizing information technology and experimental collaborative inquiry to strengthen students' intuitive understanding and application of mathematical concepts; and implementing individualized teaching strategies to cater to diverse learning needs. Taking "Ellipses and Their Standard Equations" as an example, this paper explores and practices the instructional design of high school mathematics under the STEAM education concept.*

Keywords: *STEAM Education philosophy; High school mathematics; Conic curve; Instructional design*

1. Introduction

The newly revised "General High School Mathematics Curriculum Standards (2017 Edition, Revised in 2020)" (hereinafter referred to as the "Curriculum Standards") aims to equip students with the "four basics" and "four abilities" necessary for further learning and future development. It advocates conducting mathematical activities centered around developing students' six core competencies, thereby enhancing their innovative consciousness and practical skills [1]. In 1986, the National Science Foundation (NSF) of the United States released the report "Undergraduate Science, Mathematics, and Engineering Education," marking the inception of STEM education. In 2006, American scholars introduced Arts into STEM education, giving rise to STEAM education. STEAM education has gradually evolved into a global education strategy [2], integrating science, technology, engineering, arts, and mathematics. It emphasizes a mathematics-based approach, interpreting science and technology from engineering and artistic perspectives [3]. STEAM education integrates science, technology, engineering, arts, and mathematics, transcending disciplinary boundaries and embodying a comprehensive curriculum concept that closely links mathematics with other disciplines. It aims to cultivate innovative talents for the future. Integrating STEAM competencies in teaching positively impacts students' critical thinking, practical innovation, collaboration, and problem-solving abilities, fostering the development of versatile, creative, and applied talents. This aligns with the requirements of the "Curriculum Standards" [4]. In 2007, Professor Lederman first proposed the essence and significance of STEM literacy. He believed that STEM literacy refers to an individual's ability to understand the world using knowledge from STEM fields, and integrating scientific, technological, engineering, and mathematical literacy helps students explore and comprehend the world. Renowned Chinese scholar Professor Song argues that students' STEAM literacy differs from STEM literacy, emphasizing the full integration of artistic literacy and sociocultural factors. Developing students' STEAM literacy contributes to cultivating comprehensive, practical, and innovative talents [5].

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Based on this, this paper chooses to construct an instructional design framework rooted in the STEAM education concept. It integrates STEAM literacy into teaching objectives in alignment with curriculum standards, content, and student profiles. The STEAM education concept is applied in high school mathematics teaching through strategies such as creating authentic situations, posing problem sequences, fostering student collaborative inquiry, encouraging self-directed summaries, and assigning differentiated assignments. This fusion of scientific, technological, engineering, artistic, and mathematical literacy transforms sterile, monotonous textbook knowledge into engaging, relevant, and concrete learning experiences. It enables students to fully appreciate the beauty of mathematics and flexibly apply mathematical knowledge to other domains, fostering their innovative spirit and consciousness. Next, a detailed demonstration of research and practice in high school mathematics instructional design under the STEAM education concept is presented, using "Ellipses and Their Standard Equations" as an example.

2. Instructional Design Process Based on the STEAM Education Concept

Curriculum standard analysis, student situation analysis, and content analysis are essential preparatory steps in instructional design, providing a solid foundation for setting teaching objectives and designing teaching activities. Through thorough analysis, teachers can clarify the teaching direction, ensuring that activities align with curriculum standards and are close to students' realities, thereby effectively promoting knowledge absorption and skill enhancement. This paper outlines a framework for instructional design based on the STEAM education concept, as illustrated in figure 1. Initially, teachers should comprehensively analyze curriculum standards, student situations, and content within the STEAM framework, designing teaching objectives that not only meet class requirements but also integrate STEAM literacy. Subsequently, based on these objectives, teachers should apply teaching strategies such as creating authentic situations, posing problem sequences, fostering student collaborative inquiry, encouraging self-directed summaries, and assigning differentiated assignments in high school mathematics practice.

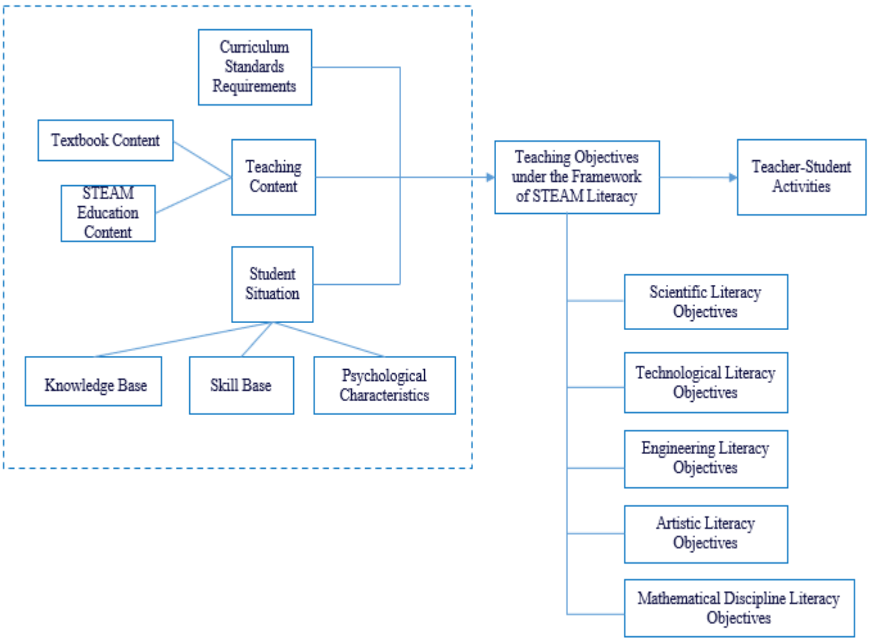

Figure 1: Instructional design framework diagram

2.1. Analysis of Curriculum Standards

Curriculum standards embody the national requirements for subject teaching. As educators, it is essential to understand these national requirements for relevant subjects in order to clarify the direction of teaching and teaching research, and enhance classroom effectiveness. Therefore, before designing a lesson plan or organizing classroom activities, teachers should carefully study the corresponding curriculum standards, grasp the precise learning direction, and clarify what students should know and what they can do. Furthermore, teachers should integrate STEAM literacy goals with the core competencies of mathematics, facilitating the design and implementation of teaching activities.

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2.2. Analysis of STEAM Interdisciplinary Teaching Content

When preparing instructional design, teachers must thoroughly understand and master textbook content, accurately identify and integrate multidisciplinary knowledge to ensure the integrity of the knowledge system and content coherence. This requires not only deep expertise in a single discipline but also interdisciplinary integration skills, skillfully blending knowledge from science, technology, engineering, arts, and mathematics.

2.3. Student Learning Situation Analysis

Analyzing student learning situations is crucial for designing high-quality courses, especially in classrooms integrating the STEAM education philosophy. In preparing instructional design, teachers must fundamentally understand students' age characteristics, current cognitive levels, and overall class learning tendencies and attitudes. Additionally, considering students' diversity in STEAM literacy, creativity, and individual differences is key to enhancing teaching effectiveness. This analysis should capture both students' common characteristics and individual differences, including their interests and proficiency in science, technology, engineering, arts, and mathematics, as well as their ability to innovate with these knowledge areas. Through such analysis, teachers can more accurately identify the starting point of teaching and design activities that align with students' actual abilities while stimulating their potential, ensuring each student grows at their own pace, enjoys learning, and achieves comprehensive development through STEAM education.

Below is a detailed analysis using the example of "Ellipses and Their Standard Equations":

(1) Knowledge Base: Prior to this lesson, students have already acquired familiarity with the shape of an ellipse and learned one method of drawing it during their physics studies in the previous semester, gaining an understanding of the formation process of an ellipse. Furthermore, students have studied "Equations of Lines and Circles" in the previous chapter, laying a solid foundation for learning about curve equations. They now possess basic knowledge for exploring trajectory problems and have a preliminary understanding of using the coordinate method to study geometric issues.

(2) Capability Foundation: Building on the previous chapter, students have initially grasped the relevant applications of the coordinate method and possess strong problem-analyzing skills, analogical learning abilities, and summarization capabilities. They have a preliminary understanding of and can simply apply the idea of combining numerical and graphical methods. However, their ability to simplify two radical equations, or in other words, their computational skills, is relatively weak.

(3) Psychological Characteristics: After undergoing the systematic learning process of the middle school entrance examination and the first year of high school, students have accumulated extensive classroom learning experience and are gradually maturing psychologically. They demonstrate initiative and self-consciousness in learning and can maintain a high level of concentration for extended periods. However, teachers still need to reasonably design teaching sessions to alleviate students' sense of boredom. Students have active minds and are readily accepting of new things. They have an interest and desire to learn the interdisciplinary, authentic, and engaging content emphasized in STEAM courses. However, when it comes to related content from other disciplines, students may face difficulties due to forgetfulness or the high difficulty of switching between different ways of thinking.

3. Analysis of High School Mathematics Teaching Objectives Based on the STEAM Education Concept

Teaching objectives serve as the compass for planning and executing instructional activities, clearly outlining the expected learning outcomes for students upon completion of the teaching process. The design and implementation of the entire instructional process are tightly centered around these objectives to ensure their effective achievement. When analyzing high school mathematics teaching objectives infused with the STEAM education concept, we first need to thoroughly study the curriculum standards to accurately grasp the specific requirements of the knowledge content for that section. Then, based on the three dimensions emphasized in general curricula: knowledge and skills, processes and methods, as well as emotional attitudes and values, and closely integrating the five core elements of STEAM—science, technology, engineering, arts, and mathematics—we meticulously craft comprehensive teaching objectives that not only align with the characteristics of the mathematics discipline but also embody the principles of STEAM education. Below is an analysis of teaching objectives using "Ellipses and Their

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Standard Equations" as an example. The teaching objectives as shown in table 1.

Table 1: Teaching objectives grounded in the STEAM education concept

Category of literacy	Content of objectives
Scientific literacy	Through dynamic IT demonstrations and real-life examples, students recognize the existence and characteristics of conical sections, particularly ellipses, and understand their definitions and geometric conditions. Through experimental observation and abstraction, they deeply grasp the ellipse definition, distinguish it from similar shapes, and refine their understanding of its features. By exploring Kepler's First Law and the optical properties of ellipses, students appreciate the application of mathematics in physics, enhancing their interdisciplinary integration skills.
Technological literacy	By observing and recording experimental data, students use IT to analyze and visualize results, enhancing their data processing and information extraction skills. In deriving the standard equation of an ellipse, they try different coordinate systems, use tools for trial and error, and comparison, fostering innovative thinking and technical application abilities.
Engineering literacy	In designing a pool table activity, students apply elliptical properties to physical design, experiencing the transition from theory to practice and cultivating engineering design and implementation skills. Faced with practical problems (e.g., determining ellipse equations, designing prototypes), they use acquired knowledge for analysis and resolution, enhancing engineering thinking and problem-solving abilities.
Artistic literacy	By observing and appreciating elliptic aesthetic features (e.g., symmetry, simplicity), students cultivate aesthetic taste and creativity, integrating mathematical beauty into daily life and design. They present learning outcomes artistically (e.g., through graphics, charts, models), enhancing expression skills and self-confidence.
Mathematical literacy	Curriculum standards clearly outline the content requirements, teaching suggestions, and academic expectations for conic sections, including ellipses. They stipulate an understanding of the practical backgrounds of conic sections and experiencing the process of abstracting ellipses from specific contexts. Students are required to grasp the definition and standard equations of ellipses. With the help of the Cartesian coordinate system, based on the characteristics of geometric problems and shapes, students should be able to convert geometric problems into algebraic ones using algebraic language and solve geometric problems using algebraic methods.

4. Application of High School Mathematics Teaching Strategies Based on the STEAM Education Concept

4.1. Create authentic situations

Teachers should closely focus on the learning theme and create real-world, positively valued situations closely linked to reality. These situations not only effectively stimulate students' curiosity and enthusiasm for learning but also guide them to actively explore and develop problem-solving skills. Teachers can create teaching situations by integrating historical, current, and interdisciplinary knowledge. They should leverage IT for visual demonstrations of knowledge formation [6], using software like Geometer's Sketchpad and GeoGebra to dynamically showcase curves, and tell the story of knowledge discovery against historical backgrounds to spark students' curiosity. Teachers should introduce real-life examples of ellipses, guiding students to observe and ponder their commonalities, enhancing the relevance of learning. Teachers should also emphasize interdisciplinary connections, prompting students to reflect on the mathematical essence of mathematical knowledge. Such contextual settings enrich learning content while cultivating students' interdisciplinary integration skills and innovative thinking.

Here is an illustration using an excerpt of teacher-student activities for "Ellipses and Their Standard Equations":

Scenario 1: Introduce the origin of conic sections to students by presenting the history of conic curves and the research conducted by great ancient mathematicians. Apollonius gave the original static and

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geometric intuitive definition of conic curves: When a plane intersects a cone at different angles relative to the cone's axis, different types of curves are formed at the intersection. These curves are collectively known as conic curves. Utilize GeoGebra software to demonstrate the dynamic generation of conic curves to students, as shown in the screenshot of figure 2, and inform them of the names of each type of conic curve that will be studied in this chapter.

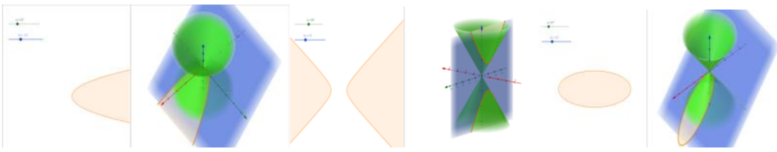

Figure 2: Diagram of conic section generation using information technology software

Scenario 2: Introduction through real-life situations. The teacher guides students to observe images and identify common features. Students observe the images and notice that they all contain "ellipses."

Scenario 3: Reviewing Kepler's First Law (the Law of Ellipses) learned in physics during the first year of high school, students are prompted to ponder what ellipses really are, leading into the introduction of new knowledge.

4.2. Design a sequence of problems

Teachers should design driving, progressively challenging problem chains closely tied to real-life situations to guide students in deep exploration and conceptual understanding, enhancing their analytical and synthetic abilities in the process. This teaching approach fosters students' comprehensive literacy and sharpens their analytical and summarizing skills. Below is an illustration using an excerpt of teacher-student activities for the topic "Ellipses and Their Standard Equations":

The teacher guides students to recall Kepler's three laws of planetary motion, specifically the first law related to ellipses and the method of drawing ellipses, which they learned in physics during the previous semester. Using geometric sketchpad software, the teacher demonstrates the formation process of an ellipse, followed by hands-on activities for students.

Experiment: Prepare a piece of inelastic string, two thumbtacks, and a pencil. Fix both ends of the string on the same thumbtack, put the pencil through the string and stretch it tight, then move the pencil point M. The trajectory drawn is a circle. Now, stretch the string apart and fix its ends at two points F1 and F2, put the pencil through the string and stretch it tight again, then move the pencil point. What kind of curve is the trajectory drawn?

Question 1: We have already studied circles. What is the definition of a circle?

Follow-up question: Using a piece of inelastic string, one thumbtack, and a pencil, how can you draw a circle?

Question 2: During the movement of the pencil tip, what remains constant and what changes?

Follow-up question: Can you provide a definition of an ellipse by analogy with the definition of a circle?

Question 3: How do we understand the phrase "stretching the string apart to a certain distance"? Are there any requirements for this distance?

Follow-up question: Can you analyze the reasons behind this?

Question 4: Please provide a definition of an ellipse.

Through experimental exploration and comparison with the circle's definition, students deduce the definition and geometric attributes of an ellipse. Specifically, an ellipse is formed by the set of points in a plane that maintain a constant sum of distances (exceeding the distance between F_1 and F_2) from two stationary points, F_1 and F_2, known as foci. The distance spanning these foci is called the focal length, and half of this length is labeled the semi-focal length.

4.3. Collaborative Inquiry

The core of collaborative inquiry lies in stimulating students' initiative and sense of participation. When encountering difficulties in learning, teachers should not provide direct answers but guide students

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to find solutions together through group cooperation. This process not only teaches students how to collaborate with others but also deepens their understanding of knowledge through practice. Teachers should maintain an open attitude towards students' diverse ideas and provide positive feedback and guidance, further inspiring their creativity and exploratory spirit. In group cooperation, students link scattered knowledge points through discussion and exchange, forming a complete knowledge system. This process of knowledge construction not only improves students' learning efficiency but also teaches them how to integrate and apply what they have learned. Below is an illustration using an excerpt of teacher-student activities for the topic "Ellipses and Their Standard Equations":

Teaching Segment One: Recalling the coordinate method introduced in the previous chapter on general plane analytic geometry, which involves five steps: "establishing a coordinate system, setting up unknowns, finding constraints, substituting known conditions, and simplifying the results." Teachers and students collaboratively explore the process of deriving the standard equation of an ellipse. Through a series of questions, students are guided to think progressively and engage in group cooperative inquiry to discuss the derivation of each step together. When simplifying the equation, different groups may come up with various methods, such as: Method 1: Moving terms and squaring; Method 2: Applying the difference of squares; Method 3: Rationalizing the expression. Teachers respect students' diverse ideas and invite representatives from each group to share their thoughts, allowing everyone to express their opinions and analyze the pros and cons. Additionally, teachers provide students with alternative derivation methods employed by scientists in history, such as "L'Hôpital's rule for sums and differences," "Wright's method using the difference of squares," and "Steele's trigonometric method." Students are encouraged to try out different methods after class, experiencing the problem-solving approaches of mathematicians and seeking resonance with their thoughts [7]. Teachers and students jointly summarize ellipse characteristics with foci on x- and y-axes, as shown in figure 3.

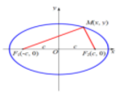
Focus Position	Focus on the x-axis	Focus on the y-axis
Graph		
Standard Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b>0)$.	$\frac{y^2}{a^2}+\frac{x^2}{b^2}=1(a>b>0)$.
Coordinates of Foci	$F_1(-c,\ 0)$, $F_2(c,\ 0)$	$F_1(0,\ -c)$, $F_2(0,\ c)$
Relationship between a、b and c	$a^2=b^2+c^2$	$a^2=b^2+c^2$

Figure 3: Teachers and students collaboratively review and consolidate the findings

Teaching Segment Two: The teacher explains to students the names of the two fixed points on an ellipse and the line segment formed by these points, and then plays a video titled "The Mystery of the Foci." Following this, the teacher sets up an activity called "Creating a Guaranteed-Entry Pool Table [8]." Students watch the video, engage in group discussions, draw the surface of the pool table, the striking point, and the holes on their scrap paper, and then present their designs to the class. The teacher listens attentively to the students' presentations, providing feedback and evaluations based on their designs. Next, the teacher guides students to abstract the real-life pool table into a rigorous geometric figure in mathematics, introducing the next topic from a "mathematized" perspective: the standard equation of an ellipse. The teacher asks students to recall and summarize the "general method for finding the equation of a curve," shifting their focus from studying the ellipse geometrically to studying its standard equation algebraically. Students recall the "general method for finding the equation of a curve," discuss with their group members how to establish a coordinate system, set points, find equivalent relationships, express these relationships in coordinates, and further simplify them.

4.4. Self-summarization

At the end of the class, teachers should guide students to conduct self-summarization, encompassing not only a review of key points but also a distillation of mathematical methods, ideas, and literacy. This summary process helps students systematically organize learning content, reinforce memory, and hone their summarizing and communication skills. Figure 4 presents the specific content of the classroom wrap-up segment for the topic of "Ellipses and Their Standard Equations" in graphical form.

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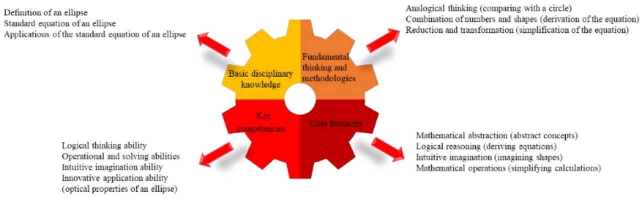

Figure 4: A classroom wrap-up segment for the topic of "Ellipses and Their Standard Equations"

4.5. Design Differentiated Homework

Incorporating the STEAM education concept, differentiated homework is a vital teaching strategy to cater to individual student differences. Teachers should design tasks of varying difficulty and type based on students' abilities, interests, and needs for personalized instruction. Under the STEAM framework, differentiated homework emphasizes interdisciplinary integration and practical application, encouraging students to solve real-world problems using multidisciplinary knowledge. It focuses on hands-on experiences, experimental verification, and project design, allowing students to intuitively understand and apply knowledge while exploring and innovating. This format not only promotes student progress at their respective levels but also stimulates their innovative thinking and comprehensive abilities. Teachers guide exploration, innovation via differentiated homework, fostering holistic development. Figure 5 shows layered assignments for "Ellipses and Their Standard Equations".

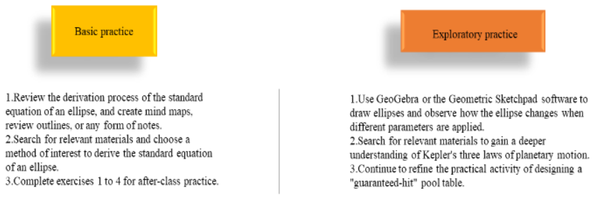

Figure 5: A differentiated assignment segment for the topic "Ellipses and Their Standard Equations"

5. Conclusion

In summary, teaching high school mathematics based on the STEAM education concept is a complex and systematic process. By selecting appropriate content, setting clear objectives, analyzing resources, designing specific content, implementing teaching with timely evaluation, and continuously refining practices, teachers enhance students' core mathematical and STEAM competencies, laying a solid foundation for their future. In this journey, teachers must constantly learn and explore new instructional ideas and methods to meet the demands of time and students' growth.

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