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A study of emergency medical service system based on a two-stage risk-averse model

Yebin Tang*

Shanghai Maritime University, Logistics Engineering College, Shanghai, 201306, China
**Corresponding author: 1208916185@qq.com*

Abstract: *An efficient EMS system provides out-of-hospital emergency medical services for sick and injured patients and maximizes the ability to respond quickly to calls, transfer, provide timely treatment and save lives in the event of an emergency. In this paper, an EMS system consisting of multiple EMS stations and demand sites is studied. Rescue supplies such as ambulances are stored at each EMS site to meet the uncertain demand from the demand location. The planning and allocation of emergency medical service sites and site storage ambulance resources under uncertainty of emergency needs is studied.*

Keywords: *Emergency Medical Services System, Value at Risk, Two-Stage Stochastic Programming Model*

1. Introduction

An efficient EMS system provides out-of-hospital emergency medical care for sick and injured patients and maximizes rapid response to calls, transfers, timely treatment, and life-saving treatment in the event of an emergency. In emergency medicine, the ten minutes after an emergency event is called the emergency platinum ten minutes. Ten minutes of effective medical treatment for a patient will greatly improve the survival rate of the patient, and it is estimated that for every minute of delay in treatment, the chance of survival is reduced by as much as 10% [1]. This requires that the ambulance should arrive at the patient as soon as possible to initiate timely treatment. To achieve this, it is more critical to effectively plan the layout of emergency stations [1].

In recent years, there have been many studies on emergency medical service systems; Rania Boujemaa et al, considering the inherent uncertainty of EMS demand, proposed a two-stage stochastic planning location assignment model based on cost minimization [1], which calculated the expected travel cost between the first stage of setting up an ambulance station and ambulance deployment for fixed and second stage demand points and ambulance stations and the expected travel cost due to unsatisfied demand and the penalty cost incurred due to unsatisfied demand is composed; Kanglin Liu, Qiaofeng Li, Zhi-Hai Zhang proposed a distributed robust model for ambulance location, number and demand allocation in EMS system based on minimization of expected total cost [2]. However, the optimal solution obtained from the model solution is highly likely to be broken by small probability high risk events because risk aversion aspects are not taken into account. In considering risk aversion, Özgün Elçi and Nilay Noyan proposed a risk-neutral two-stage stochastic programming model by considering the importance of the decision maker's risk preference on the impact of stochastic outcomes in the presence of uncertainty with few catastrophic events and characterized by the average risk objective with the conditional value at risk (CVaR) as the risk measure [3]; in the EMS siting and sizing problems with a two-stage risk-averse model with joint opportunity constraints.Nilay Noyan focused on stochastic preference relations based on widely used risk measures, conditional value at risk (CVaR) and second-order stochastic dominance (SSD), and proposed single- and two-stage stochastic optimization problems with such risk-averse preference relations [4]. Most of the existing related studies are based on stochastic optimization or robust optimization, but stochastic optimization is often too ideal because it requires a known probability distribution, and robust optimization is less feasible because of overconsideration of the most extreme cases[5-6].In this paper, we propose a two-stage risk-averse model that solves the following three problems by combining the above studies:

(1) The siting of the ambulance stations.

(2) Which areas of need will be served by each ambulance station.

(3) The problem of ambulance allocation for ambulance stations.

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And to summarize the contributions of this paper as follows.

(1) The application of a two-stage sequential decision framework is closer to reality.

(2) Considering one more unmet demand penalty cost on top of the conventionally considered two-stage cost, and caring more about the performance of EMS systems in terms of demand service rate.

(3) Consider both large-scale uncertain demand and uncertain scenario probability distributions for a more realistic portrayal of realistic data.

(4) Considering decision maker risk preferences, incorporating CVaR risk metric effectively hedges the serious risks caused by unmet demand.

2. Problem description and formulation

2.1 Problem description

The design of an emergency medical service network can be described as a two-stage stochastic planning problem: The first stage determines the network structure, where the location of emergency stations and the resources of ambulances stored at the stations are determined prior to the operation of ambulance vehicles. The second stage is the ambulance vehicle logistics problem after setting up the stations and allocating the resources, and after the random emergency demand appears with random parameters realized[7]. Due to the uncertainty of the first aid demand, we have to need to dispatch ambulance vehicles reasonably according to the time, location and scale of the occurrence of the first aid demand[8-9]. The decision to dispatch ambulance vehicles is made after the randomness of the demand is revealed[10].

Assume that symbol I is the set of first-aid demand nodes and J is the set of potential first-aid station location nodes; symbol S is the set of scenarios where demand occurs; decision variable Y_j indicates whether to set emergency station at node j. If a station station is set at node j, Y_j= 1, otherwise Y_j= 0. Under the assumption of infinite capacity of first-aid stations, each station opens the unit fixed cost of each station opening is f_j and the unit cost of deploying ambulances at each emergency station is V_j; assuming that the total number of stations is capped at Γ and the total cap of ambulances is K.

2.2 Model formulation

To facilitate model building, we first make some assumptions.

(1) The number of candidate first-aid stations and demand points in the plane is determined and the location coordinates are known.

Table 1: Indices and parameters

Notation	Description
I	The set of all demand points i
J	The set of all candidate first aid stations j
S	The set of all scenarios s
Parameter	
f_j	Unit fixed cost of site opening
V_j	Unit deployment cost of ambulance at first aid site j
Γ	Upper limit of total number of stations
K	Maximum total number of ambulances
M_j	Upper limit of the number of ambulances that can be accommodated at site j
C_{ij}	Distance from site j to demand point i
W_{ij}	Unit cost of ambulance transport from site j to demand point i
M	An infinite number
N_{ij}	Average number of EMS demands that can be served by each ambulance
E_i	Penalty cost per unit of unmet demand at demand point i
$d_i{}^s$	The number of patients at demand point i for scenario s
p^s	The probability of occurrence of scenario s
α	Confidence level

(2) The number of first aid stations and station ambulances are definitively known.

(3) The distance between the candidate first-aid stations and the demand points is measured using the Euclidean distance.

(4) Each emergency station can serve multiple demand points, each demand point can be served by multiple emergency stations, and each ambulance can only serve one demand point.

Indices and parameters are shown in table 1. Decision variables are shown in table 2.

Table 2: Decision variables

Notation	Description
Y_j	Scenario s under the first aid station j, take this station as 1, otherwise 0
Z_j	The number of ambulances parked at site j
$X_{ij}{}^{s}$	The number of ambulances dispatched from site j to demand point i under scenario s
$U_i{}^{s}$	The number of patients who do not meet demand point i

The proposed model assumes that each demand point can be served by multiple ambulance stations (which are independent of each other), and that ambulances located at a given ambulance station can serve all demand points. Assuming that the variable $X_{ij}{}^{s}$ denotes the number of patients served by ambulances located at point j to demand point i under scenario s, the total number of ambulances assigned to station j is:

$$\max \sum_{i\in I} \sum_{j\in J} X_{ij}{}^{s}, \forall s \in S$$

We first give the traditional risk-neutral two-stage stochastic programming model that does not take into account the decision maker's risk preferences in the general case.

$$min_{\boldsymbol{Y},\boldsymbol{Z}} \sum_{j\in J} f_j Y_j + \sum_{j\in J} V_j Z_j + \mathbb{E}[Q(\boldsymbol{Y},\boldsymbol{Z},s)] \tag{1}$$

s.t.

$$Z_j \le M_j Y_j, \forall j \in J \tag{2}$$

$$\sum_{j\in J} Z_j \le K \tag{3}$$

$$Y_j \in \{0,1\}, \quad \forall j \in J \tag{4}$$

The meaning of the objective function (1) is to minimize the sum of the cost of the first stage and the expected cost of the second stage. The cost of the first stage consists of the fixed cost of setting up an ambulance station and the fixed cost of storing unit ambulances at the station. The expected cost of the second stage consists of the expected ambulance travel cost between the demand point and the ambulance station and the expected and risky value of the penalty cost due to unmet demand.

Constraint (2) indicates that ambulances can only be deployed in open facilities and receive capacity constraints. Constraint (3) indicates that the total number of ambulances deployed at all sites does not exceed the maximum number of ambulances. Constraint (4) indicates the value range of the decision variable y, with 1 for adopted sites and 0 for non-adopted sites.

Where the second stage model is expressed as:

$$Q(\boldsymbol{Y},\boldsymbol{Z},s) = min \sum_{i\in I} \sum_{j\in J} C_{ij} W_{ij} X_{ij}{}^{s} + \sum_{i\in I} E_i U_i{}^{s} \tag{5}$$

$$d_i{}^{s} \le U_i{}^{s} + \sum_{j\in J} N_{ij} X_{ij}{}^{s}, \forall i \in I, \quad \forall s \in S \tag{6}$$

$$\sum_{i\in I} X_{ij}{}^{s} \le Z_j, \quad \forall j \in J \tag{7}$$

$$U_i{}^{s}, \quad X_{ij}{}^{s} \in \mathbb{Z}^{+}, \forall j \in J, \forall i \in I, \quad \forall s \in S \tag{8}$$

The objective function (5) means the expected total cost of travel and penalty costs in the rescue process after the emergence of EMS demand. Constraint (6) indicates that the total EMS demand consists of the amount of unmet demand and the amount of completed rescues. Constraint (7) indicates that the total number of ambulances allocated out of each station must not exceed the number of ambulances placed at that station. Constraint (8) indicates the domain of the second stage decision variables.

The traditional risk-neutral stochastic programming model is widely used because it takes risk factors into account, but the model fails in extreme cases, so we further consider the following risk-averse model

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- conditional risk-averse model (CVaR).

2.3 Risk Aversion Model

CVaR is the average of the portfolio's losses conditional on the portfolio's losses being greater than some given value of VaR. If the stochastic loss of portfolio Z is f(X), then the CVaR of this portfolio can be expressed as:

$$\mathrm{CVaR}_{\alpha} = E[f(X)|f(X) > \mathrm{CVaR}_{\alpha}]$$

For a random variable X, $\mathrm{F_X}$ (-) denotes its cumulative distribution function. The value-at-risk (Val) at the confidence level α is defined as:

$$\inf\{\eta \in \mathbb{R} : \mathrm{F_X}(\eta) \ \geq \ \alpha\} \tag{9}$$

The conditional value at risk (CVaR) of the random variable X at confidence level α is:

$$\mathrm{CVaR}_{\alpha}(\mathrm{X}) = \inf_{\eta\in\mathbb{R}} \ \{\eta + \frac{1}{1-\alpha}\mathbb{E}([\mathrm{X}-\eta])\} \tag{10}$$

We consider the risk of averting too much unmet demand by measuring and minimizing the CVaR risk value of the unmet demand in the second stage. The following expanded form of the two-stage risk-averse stochastic optimization model can be obtained:

$$min\sum_{j\in J} f_j Y_j + \sum_{j\in J} V_j Z_j + (1-\lambda)\mathbb{E}[Q(\boldsymbol{Y},\boldsymbol{Z},s)] + \lambda min_{\eta}\mathrm{CVaR}[Q(\boldsymbol{Y},\boldsymbol{Z},s)]$$

Also according to the linear defining equation of CVaR, by introducing auxiliary variables v^{s} and given a confidence level $\alpha \in (0,1)$, the following two-stage stochastic equivalence model for the uncertain large-scale demand scenario can be obtained

$$min\sum_{j\in J} f_j Y_j + \sum_{j\in J} V_j Z_j + (1-\lambda)\sum_{s\in S} p^s\left(\sum_{i\in I}\sum_{j\in J} C_{ij}\, W_{ij}\, {X_{ij}}^s + \sum_{i\in I} E_i {U_i}^s\right) + \lambda\left(\eta + \frac{1}{1-\alpha}\sum_{s\in S} p^s v^s\right)$$

$$\mathrm{v}^{s} \geq \sum_{i\in I}\sum_{j\in J} C_{ij}\, W_{ij}\, \mathrm{X_{ij}}^{s} + \sum_{i\in I} E_i U_i - \eta \ \ ,\forall s \in S \tag{11}$$

$$\eta \in \mathbb{R}, \mathrm{v}^{s} > 0, \forall s \in S \tag{12}$$

Constraint (11) calculates the tail loss v^{s} for scene s. Constraint (12) states that the considered loss must exceed η. From this we construct two models, which are next verified numerically using examples.

3. Numerical experiments

3.1 Case Study

Jinan city now handles an average of more than 600,000 120 emergency calls per year, and the yearly increase in emergency demand has posed a greater challenge to the pre-hospital emergency system in Jinan city. Since it is difficult to directly obtain the specific emergency data from the Jinan 120 emergency center, the representative data of the key high-risk morbidity population in Jinan is used instead of the specific emergency data, and the representative data is selected to analyze the location and quantity distribution of the emergency demand in Jinan. It is known from the survey that one of the main reasons for emergency trips is cardio-cerebral vascular diseases, among which the incidence of cardio-cerebral vascular diseases such as hypertension, stroke, diabetes and chest pain is high and the risk of resuscitation is high, so patients with these diseases can be used as a representative of emergency demand data to a certain extent.

Using Python software, we crawled the location and number of key high-risk morbidity population. However, since the number of

The data volume is too large to be solved, so we simplified the data processing. Acute Care Needs Matrix is shown in table 3.

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Table 3: Acute Care Needs Matrix

						106			
		618	185	224		118	108		
	113	36	78	524	696	405	16		
	168	398	276	1074	749	887	219	207	
451	12	288	186	130	583	213	293	239	86
838	1103	1930	3193	744	133	342	473	218	498
	3719	14821	77004	32653	4804	1885	797	558	275
	2163	30335	170128	152353	32679	6781	2978	1955	
		6144	34315	29546	14207	18057	5199	979	
			1767	2379	3132	2305	2785		
				1701	596	735			
				618	554	575			

The EMS demand matrix shows that the five central districts of Jinan show the distribution characteristics of great demand in the central area and less demand in the surrounding remote areas.

Parameter f_j,the annual operating cost of each first aid site. Here it is assumed that the operating cost of each station is the same. The construction cost of each first aid station is 1.5 million yuan, and the depreciation is 50,000 yuan per year for a service life of more than 30 years. Adding the daily operation and maintenance cost of each first aid station of 100,000 yuan per year, the total is f_j of 150,000 yuan.

Parameter V_j, the annual operating cost per ambulance. The purchase cost of an ambulance plus medical facilities.

The purchase cost of an ambulance plus medical facilities is 400,000 RMB, and the service life is more than 10 years. Each ambulance is equipped with an average of five first responders, each with an average salary of RMB 0.5 million per month. Therefore, V_j is calculated to be 240,000 yuan.

Parameter W_{ij}, annual ambulance round-trip transportation costs per kilometer per year. The fuel consumption per kilometer for a medium-sized vehicle is 1 yuan.

The average number of trips to the emergency center in Jinan city is about 180,000 per year, and the average number of trips per vehicle is 2,500 per year, counting round trips, W_{ij} is about 0.5 million yuan.

Parameter J, the range of sites: 38 emergency centers (stations) plus 12 level II+ hospitals and 45 level I hospitals, constituting 95 candidate sites.

The parameter M_j, which is the maximum number of ambulances per emergency station, is based on the comprehensive strength of the hospital.

Parameter d_i^s, the number of patients at demand point i under scenario s.

Parameter Z_j, the number of ambulances owned by each emergency site.

Parameter N_{ij}, the demand that one ambulance can serve, i.e. the priority high-risk morbidity population, and each ambulance should meet the demand of 11,200,000 high-risk morbidity population.

Parameter C_{ij}, the distance between the emergency station and the demand point is calculated based on the latitude and longitude coordinates of each point with the help of the distance function of MATALAB software.

3.2 Case Results

The results of the two-stage model were analyzed, and the data were substituted into the model and solved using CPLEX, as shown in Table 4.

Table 4: The results of the two-stage model

						1			
		2	3	4		5	6		
	7	8	9	10	11	12	13		
	14	15	16	17	18	19	20	21	
22	23	24	25	26	27	28	29	30	31
32	33	34	35	36	37	38	39	40	41
	42	43	44	45	46	47	48	49	50
	51	52	53	54	55	56	57	58	
		59	60	61	62	63	64	65	
			66	67	68	69	70		
				71	72	73			
				74	75	76			

						1			
		2	3	4		5	6		
	7	8	9	10	11	12	13		
	14	15	16	17	18	19	20	21	
22	23	24	25	26	27	28	29	30	31
32	33	34	35	36	37	38	39	40	41
	42	43	44	45	46	47	48	49	50
	51	52	53	54	55	56	57	58	
		59	60	61	62	63	64	65	
			66	67	68	69	70		
				71	72	73			
				74	75	76			

4. Conclusions

In this paper, we propose a new two-stage risk avoidance model based on cost minimization for the siting of emergency medical services, taking into account the risk preference factor of decision makers, and introduce a risk measure CVar in the two-stage model, which can better avoid the occurrence of low-probability high-risk events and make the model closer to the real situation. Examples are introduced for model solving and sensitivity analysis is performed to prove that the model is more advantageous in meeting the demand.

The research in this paper has some limitations and does not consider improving the robustness of the model as well as the type of ambulance. Further consideration can be given to the introduction of a distributed Ruben model in conjunction with demand and cost as well as different types of rescue dispatch tasks for different ambulances.

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