

Remaining Useful Life Prediction of Nonlinear Wiener Process-Based Degradation Model Based on Multi Source Information with Considering Random Effects

Fengfei Wang, Liang Li*, Hui Ye

High-Tech Institute of Xi'an, Xi'an, Shaanxi, PR China
xzj_921@163.com

*Corresponding author

Abstract: Accurate parameters estimation is the premise of accurate remaining useful life (RUL) prediction. This paper proposes a RUL prediction method of nonlinear Wiener process based on multi source information with considering random effects. First, a nonlinear Wiener process with considering random effects is used to model the degradation process of equipment. Then, according to the nature of parameters estimation, nonlinear parameter can be obtained based on historical degradation data. After that, the expectation maximization (EM) algorithm is used to calculate fixed parameter and random coefficient in model with fusing prior degradation information and prior failure time data information. Finally, fatigue crack data are used for experimental verification. Compared with the method based on historical degradation data or failure time data, the method based on multi source information with considering random effects can effectively improve the accuracy of parameters estimation and RUL estimation.

Keywords: Multi source information, Random effects, Wiener process; Remaining useful life, Nonlinear

1. Introduction

In recent years, electromechanical equipment such as subway, automobile and aircraft are widely used in daily civil and military fields. However, due to the increase of equipment manufacturing accuracy and the lack of natural resources, the manufacturing cost of equipment increases significantly. Therefore, the whole life cycle of equipment needs to be optimized from design, manufacturing, sales, use, maintenance to scrapping [1]. For some key equipment in significant fields, once the parts fail during operation, it will cause significant economic losses, and even a serious disaster.

In the actual operation process, due to the complex working environment and surrounding random impact interference [2], the parts of equipment would undergo a gradual deterioration process, resulting in degradation of their performance [3]. If the maintenance personnel could predict the remaining useful life (RUL) of equipment during operation according to the performance degradation status or failure time data of parts and formulate corresponding maintenance strategies, it can greatly prevent the sudden accident and reduce the preventive maintenance cost of replacing parts in advance. Prognostics and health management (PHM) is an effective technology to solve such problems [4].

RUL prediction is one of the important components in PHM [5]. Its purpose is to obtain the failure probability of equipment parts after running for a period of time according to the relevant degradation data, failure time data of the congeneric equipment and field degradation data of the evaluated equipment. In recent years, with the improvement of the reliability of complex equipment, RUL prediction method, especially RUL prediction method based on Wiener process, has attracted great attention of scholars [6].

Accurate parameters estimation can improve the accuracy of RUL prediction. Tang et al. [7] and Wang et al. [8] analyzed the natures of parameters estimation based on Wiener process and random coefficient regression model respectively, and pointed out that the accuracy of parameters estimation can be improved by fusing more prior information of congeneric equipment. Generally speaking, the prior information of equipment is mainly divided into historical degradation data and failure time data of congeneric equipment. How to make rational use of these two data is the key to improve the accuracy of parameters estimation and RUL prediction. Recently, the method based on the historical degradation data

and the failure time data, i.e., based on multi source information, has been applied in RUL prediction [9-11]. However, these literatures ignore the existence of random effects, that is, the unit-to-unit variability. Peng et al. [12] had proved that the penalty of mis-fitting a random-effect model by a fixed-effect model was more serious than that of mis-fitting a fixed-effect model by a random-effect model. In addition, the degradation trend of most equipment is nonlinear, such as lithium-ion batteries [13], gyroscope [14], aero-turbofan engines [15] and so on. Therefore, it is necessary to add random effects and nonlinear features to the Wiener process model based on multi source information.

Through the studies of the above literatures regarding fusing multi source information, it is found that there are still some problems to be solved. To address these problems, the nonlinear Wiener process is used to model degradation process, in which the random coefficient is regarded as a random variable to represent the unit-to-unit variability. Then, the nonlinear parameter in model can be calculated based on the historical degradation data. After that, EM algorithm is used to calculate other unknown parameters of the model with fusing historical degradation data and failure time data. Finally, fatigue crack data are used to prove the effectiveness of this method compared with the method based on historical degradation data, the method based on failure time data and the method based on multi source information without considering random effects.

The remainder of this paper is organized as follows. Section 2 develops a nonlinear Wiener process-based model with considering random effects, and obtains the parameters estimation results by the EM algorithm based on multi source information. A case studies is provided in Section 3; and Section 4 draws the main conclusions.

2. RUL Prediction of Nonlinear Wiener Process-Based Model Based on Multi Source Information

Nonlinearity is an important characteristic of degradation process. Therefore, in this section, a RUL prediction method with fusing multi source information based on the nonlinear Wiener process with considering random effects is given.

2.1. Degradation Model

The degradation process based on the nonlinear Wiener process can be expressed as follows

$$Y(t) = y_0 + \lambda\Lambda(t; \theta) + \sigma_B B(\Lambda(t; \theta)) \quad (1)$$

where y_0 denotes the initial degradation state, θ denotes the nonlinear parameter, λ is the drift coefficient, which is assumed to be a random variable to characterize the unit-to-unit variability, i.e., $\lambda \sim N(\mu_\lambda, \sigma_\lambda^2)$, σ_B is the diffusion coefficient and $B(t)$ is the standard Brownian motion. Without loss of generality, y_0 is assumed as zero.

Let w denote the failure threshold of equipment. The PDF of lifetime can be written as [16]

$$f_T(t | \lambda) \approx \frac{w}{\sqrt{2\pi\sigma_B^2\Lambda(t; \theta)^3}} \exp\left(-\frac{(w - \lambda\Lambda(t; \theta))^2}{2\sigma_B^2\Lambda(t; \theta)}\right) \quad (2)$$

When the random effects are added to the Wiener process, the PDF of lifetime can be expressed as [17]

$$f_T(t) = \frac{w}{\sqrt{2\pi\Lambda(t; \theta)^3(\sigma_\lambda^2\Lambda(t; \theta) + \sigma_B^2)}} \exp\left(-\frac{(w - \mu_\lambda\Lambda(t; \theta))^2}{2\Lambda(t; \theta)(\sigma_\lambda^2\Lambda(t; \theta) + \sigma_B^2)}\right) \frac{d\Lambda(t; \theta)}{dt} \quad (3)$$

2.2. RUL Prediction

Based on Bayesian theory, random coefficient can be updated to adapt to the degradation characteristics of individuals. Given the field degradation data $y_{1:k} = [y_1, y_2, \dots, y_k]$ and the prior information of λ , the posterior distribution of random coefficients can be expressed as

$$\lambda | y_{1:k} \sim N\left(\frac{y_k\sigma_\lambda^2 + \mu_\lambda\sigma_B^2}{\Lambda(t_k; \theta)\sigma_\lambda^2 + \sigma_B^2}, \frac{\sigma_\lambda^2\sigma_B^2}{\Lambda(t_k; \theta)\sigma_\lambda^2 + \sigma_B^2}\right) \quad (4)$$

Then, the PDF of the RUL can be obtained as

$$f_{L_k}(l_k) = \frac{w - y_k}{\sqrt{2\pi\Delta v(l_k)^3}} \frac{1}{\sqrt{(\sigma_{\lambda,k}^2 \Delta v(l_k) + \sigma_B^2)}} \frac{d\Lambda(l_k + t_k; \theta)}{dl_k} \exp\left(-\frac{(w - y_k - \mu_{\lambda,k} \Delta v(l_k))^2}{2\Delta v(l_k)(\sigma_{\lambda,k}^2 \Delta v(l_k) + \sigma_B^2)}\right) \quad (5)$$

where $\Delta v(l_k) = \Lambda(l_k + t_k; \theta) - \Lambda(t_k; \theta)$.

Then, $\Theta = (\mu_\lambda, \sigma_\lambda^2, \sigma_B^2, \theta)$ are the prior parameters of nonlinear Wiener process. If the "FMINSEARCH" function of MATLAB is used to search four unknown parameters directly, it is easy to fall into the local optimum. Therefore, based on the natures of parameters estimation of nonlinear Wiener process, this section proposes a two-step parameters estimation method with fusing multi source information, which is shown in Section 2.3.

2.3. Parameters Estimation of the Nonlinear Wiener process

It is assumed that there are n' sets of failure time data $T_{1:n'} = [T_1, T_2, \dots, T_{n'}]$ and the failure time data of the v th unit is T_v . In addition, suppose that there are n units for testing and each unit is monitored at the same times t_1, t_2, \dots, t_m , then, let $y_{i,j}$ denote the degradation data of the i th unit at time $t_{i,j}$, where $1 \leq i \leq n$ and $1 \leq j \leq m$. Let $\Delta y_{i,j} = Y_i(t_j) - Y_i(t_{j-1})$, then, $\Delta y_{i,j}$ can be expressed as

$$\Delta y_{i,j} = \lambda_i \Delta v_j + \sigma_B (\Delta v_j) \quad (6)$$

where $\Delta v_j = \Lambda(t_j; \theta) - \Lambda(t_{j-1}; \theta)$ and λ_i denote the mean degradation rate of i th unit. Let $\Delta \mathbf{v} = (\Delta v_1, \Delta v_2, \dots, \Delta v_m)'$, $\Delta \mathbf{y}_i = (\Delta y_{i,1}, \Delta y_{i,2}, \dots, \Delta y_{i,m})'$ and $\mathbf{Y} = (\Delta \mathbf{y}'_1, \Delta \mathbf{y}'_2, \dots, \Delta \mathbf{y}'_n)'$. Then, $\Delta \mathbf{y}_i$ is multivariate normal with mean $\mu_\lambda \Delta \mathbf{v}$, and variance $\Sigma = \sigma_\lambda^2 \Delta \mathbf{v} \Delta \mathbf{v}' + \sigma_B^2 \mathbf{\Omega}$, where $\mathbf{\Omega} = \text{diag}(\Delta v_1, \Delta v_2, \dots, \Delta v_m)$.

If the MLE method is used to calculate the unknown parameters in model, the log-likelihood function based on multi source information can be written as

$$\begin{aligned} \ln L(\Theta | \mathbf{Y}, T_{1:n'}) &= n' \ln w + \sum_{v=1}^{n'} \ln \Lambda'(T_v; \hat{\theta}) - \frac{n' \ln 2\pi}{2} - \frac{3}{2} \sum_{v=1}^{n'} \ln \Lambda(T_v; \hat{\theta}) - \frac{1}{2} \sum_{v=1}^{n'} \ln (\sigma_\lambda^2 \Lambda(T_v; \hat{\theta}) + \sigma_B^2) \\ &\quad - \sum_{v=1}^{n'} \frac{(w - \mu_\lambda \Lambda(t; \hat{\theta}))^2}{2\Lambda(t; \hat{\theta})(\sigma_\lambda^2 \Lambda(t; \hat{\theta}) + \sigma_B^2)} - \frac{mn \ln 2\pi}{2} - \frac{n(m-1)}{2} \ln \sigma_B^2 - \frac{n}{2} \ln (\sigma_B^2 + \sigma_\lambda^2 \Delta \mathbf{v} \mathbf{\Omega}^{-1} \Delta \mathbf{v}) \\ &\quad - \frac{n}{2} \sum_{j=1}^m \ln \Delta v_j - \frac{1}{2\sigma_B^2} \sum_{i=1}^n (\Delta \mathbf{y}_i - \hat{\lambda}_i \Delta \mathbf{v})' \mathbf{\Omega}^{-1} (\Delta \mathbf{y}_i - \hat{\lambda}_i \Delta \mathbf{v}) - \frac{1}{2} \frac{\Delta \mathbf{v} \mathbf{\Omega}^{-1} \Delta \mathbf{v}}{\sigma_B^2 + \sigma_\lambda^2 \Delta \mathbf{v} \mathbf{\Omega}^{-1} \Delta \mathbf{v}} \sum_{i=1}^n (\mu_\lambda - \hat{\lambda}_i)^2 \end{aligned} \quad (7)$$

From Equation (7), it is found that it is not easy to calculate the unknown parameters in model based on MLE method. Recently, Tang et al. [7] proposed a two-step MLE method with fusing failure time data. Then, Inspired by Tang et al [7], we give a two-step parameters estimation method with fusing multi source information.

The parameters estimation method with fusing multi source information for nonlinear Wiener process is divided into two steps, which are as follows:

Step 1: Estimating the nonlinear parameter θ based on the historical degradation data

The estimation of θ can be obtained by maximizing Equation (8) through the "FMINSEARCH" function of MATLAB, more details can be found in [18].

$$\begin{aligned} \ln L(\theta | \mathbf{Y}) &= -\frac{mn}{2} \ln 2\pi - \frac{n(m-1)}{2} \ln \left(\frac{1}{n(m-1)} \sum_{i=1}^n \left[\Delta \mathbf{y}'_i \mathbf{\Omega}^{-1} \Delta \mathbf{y}_i - \frac{1}{\Delta \mathbf{v} \mathbf{\Omega}^{-1} \Delta \mathbf{v}} (\Delta \mathbf{y}'_i \mathbf{\Omega}^{-1} \Delta \mathbf{v})^2 \right] \right) \\ &\quad - \frac{n}{2} \ln \left[\frac{\Delta \mathbf{v} \mathbf{\Omega}^{-1} \Delta \mathbf{v}}{n} \sum_{i=1}^n \left(\hat{\mu}_\lambda - \frac{\Delta \mathbf{y}'_i \mathbf{\Omega}^{-1} \Delta \mathbf{v}}{\Delta \mathbf{v} \mathbf{\Omega}^{-1} \Delta \mathbf{v}} \right)^2 \right] - \frac{n}{2} \sum_{j=1}^m \ln \Delta v_j - \frac{mn}{2} \end{aligned} \quad (8)$$

Step 2: Calculating the random coefficient $\mu_\lambda, \sigma_\lambda^2$ and fixed parameter σ_B^2 based on multi source

information

The log likelihood functions of $\Theta = (\mu_\lambda, \sigma_\lambda^2, \sigma_B^2)$ based on multi source information can be obtained as

$$\begin{aligned} \ln L(\Theta | \mathbf{Y}, \mathbf{T}_{1:n'}, \boldsymbol{\lambda}) = & -\frac{n' \ln 2\pi}{2} + n' \ln w - \frac{n'}{2} \ln \sigma_B^2 - \frac{3}{2} \sum_{v=1}^{n'} \ln \Lambda(T_v; \hat{\theta}) - \sum_{v=1}^{n'} \frac{(w - \lambda_v \Lambda(T_v; \hat{\theta}))^2}{2\sigma_B^2 \Lambda(T_v; \hat{\theta})} - \frac{n' \ln 2\pi}{2} - \frac{n'}{2} \ln \sigma_\lambda^2 \\ & - \frac{1}{2\sigma_\lambda^2} \sum_{v=1}^{n'} (\lambda_v - \mu_\lambda)^2 - \frac{mn}{2} \ln 2\pi - \frac{nm}{2} \ln \sigma_B^2 - \frac{n}{2} \sum_{j=1}^m \ln \Delta v_j - \frac{1}{2\sigma_B^2} \sum_{i=1}^n \sum_{j=1}^m \frac{1}{\Delta v_j} (\Delta y_{i,j} - \lambda_i \Delta v_j)^2 \quad (9) \\ & - \frac{n}{2} \ln 2\pi - \frac{n}{2} \ln \sigma_\lambda^2 - \frac{1}{2\sigma_\lambda^2} \sum_{i=1}^n (\lambda_i - \mu_\lambda)^2 \end{aligned}$$

Given $\hat{\Theta}^{(k)} = \{\hat{\mu}_\lambda^{(k)}, \hat{\sigma}_\lambda^{2(k)}, \hat{\sigma}_B^{2(k)}\}$ as the estimation of Θ in the k th step based on \mathbf{Y} and $\mathbf{T}_{1:n'}$, EM algorithm can be implemented as follows:

E-step: calculating the expectation of the complete log-likelihood function

$$\begin{aligned} L(\Theta | \hat{\Theta}^{(k)}) = & E_{\lambda_i | T_{1:n'}, \hat{\Theta}^{(k)}, \lambda_j | X, \hat{\Theta}^{(k)}} \ln L(\mu_\lambda, \sigma_\lambda^2 | \mathbf{T}_{1:n'}, \mathbf{Y}, \boldsymbol{\lambda}) \\ = & -\frac{n' \ln 2\pi}{2} + n' \ln w - \frac{n'}{2} \ln \sigma_B^2 - \frac{3}{2} \sum_{v=1}^{n'} \ln \Lambda(T_v; \hat{\theta}) - \sum_{v=1}^{n'} \frac{(w - \lambda_v \Lambda(T_v; \hat{\theta}))^2}{2\sigma_B^2 \Lambda(T_v; \hat{\theta})} \\ & - \frac{n' \ln 2\pi}{2} - \frac{n'}{2} \ln \sigma_\lambda^2 - \frac{1}{2\sigma_\lambda^2} \sum_{v=1}^{n'} [(\lambda_v - \mu_{\lambda,v})^2 + \sigma_{\lambda,v}^2] \quad (10) \\ & - \frac{mn}{2} \ln 2\pi - \frac{nm}{2} \ln \sigma_B^2 - \frac{n}{2} \sum_{j=1}^m \ln \Delta v_j - \frac{1}{2\sigma_B^2} \sum_{i=1}^n \sum_{j=1}^m \frac{1}{\Delta v_j} [(\Delta y_{i,j} - \mu_{\lambda,i} \Delta v_j)^2 + \sigma_{\lambda,i}^2 \Delta v_j^2] \\ & - \frac{n}{2} \ln 2\pi - \frac{n}{2} \ln \sigma_\lambda^2 - \frac{1}{2\sigma_\lambda^2} \sum_{i=1}^n [(\mu_{\lambda,i} - \mu_\lambda)^2 + \sigma_{\lambda,i}^2] \end{aligned}$$

where

$$\mu_{\lambda,i} = E(\lambda_i | \Delta y_i, \hat{\Theta}^{(k)}) = \frac{y_{i,m} \sigma_\lambda^{2(k)} + \mu_\lambda^{(k)} \sigma_B^{2(k)}}{\Lambda(t_m; \hat{\theta}) \sigma_\lambda^{2(k)} + \sigma_B^{2(k)}}, \sigma_{\lambda,i}^2 = \text{var}(\lambda_i | \Delta y_i, \hat{\Theta}^{(k)}) = \frac{\sigma_B^{2(k)} \sigma_\lambda^{2(k)}}{\Lambda(t_m; \hat{\theta}) \sigma_\lambda^{2(k)} + \sigma_B^{2(k)}} \quad (11)$$

$$\mu_{\lambda,v} = E(\lambda_v | T_v, \hat{\Theta}^{(k)}) = \frac{w \sigma_\lambda^{2(k)} + \mu_\lambda^{(k)} \sigma_B^{2(k)}}{\Lambda(T_v; \hat{\theta}) \sigma_\lambda^{2(k)} + \sigma_B^{2(k)}}, \sigma_{\lambda,v}^2 = \text{var}(\lambda_v | T_v, \hat{\Theta}^{(k)}) = \frac{\sigma_B^{2(k)} \sigma_\lambda^{2(k)}}{\Lambda(T_v; \hat{\theta}) \sigma_\lambda^{2(k)} + \sigma_B^{2(k)}} \quad (12)$$

M-step: maximizing $L(\Theta | \hat{\Theta}^{(k)})$

$$\hat{\Theta}^{(k+1)} = \arg \max_{\Theta} L(\Theta | \hat{\Theta}^{(k)}) \quad (13)$$

Taking the first partial derivatives $L(\Theta | \hat{\Theta}^{(k)})$ with respect to μ_λ , σ_λ^2 and σ_B^2 , and setting these derivatives to zero, the MLE of μ_λ , σ_λ^2 and σ_B^2 can be obtained as follows

$$\hat{\mu}_\lambda^{(k+1)} = \frac{\sum_{v=1}^{n'} \mu_{\lambda,v} + \sum_{i=1}^n \mu_{\lambda,i}}{n' + n} \quad (14)$$

$$\hat{\sigma}_\lambda^{2(k+1)} = \frac{\sum_{v=1}^{n'} [(\mu_{\lambda,v} - \hat{\mu}_\lambda^{(k+1)})^2 + \sigma_{\lambda,v}^2] + \sum_{i=1}^n [(\mu_{\lambda,i} - \hat{\mu}_\lambda^{(k+1)})^2 + \sigma_{\lambda,i}^2]}{n' + n} \quad (15)$$

$$\hat{\sigma}_B^{2(k+1)} = \frac{\sum_{v=1}^{n'} \frac{(w - \mu_{\lambda,v} \Lambda(T_v; \hat{\theta}))^2 + \sigma_{\lambda,v}^2 \Lambda(T_v; \hat{\theta})^2}{\Lambda(T_v; \hat{\theta})} + \sum_{i=1}^n \sum_{j=1}^m \frac{1}{\Delta v_j} [(\Delta y_{i,j} - \mu_{\lambda,i} \Delta v_j)^2 + \sigma_{\lambda,i}^2 \Delta v_j^2]}{n' + mn} \quad (16)$$

Then, the above E-step and M-step are iterated until $\|\Theta^{(k+1)} - \Theta^{(k)}\|$ is sufficiently small.

The flow chart of this method is shown in Figure 1, in which how to fuse historical degradation data and failure time data is a key step.

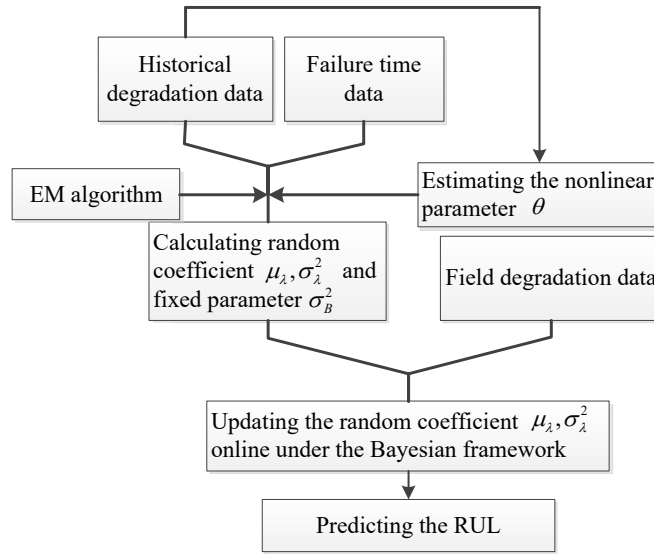


Figure 1: The flow chart of RUL prediction based on multi source information for nonlinear Wiener process

Remark 1: By analyzing the natures of parameters estimation, Tang et al. [7] pointed out that the more prior information related to congeneric equipment is fused, the more accurate the parameters estimation are. Therefore, we synthesize the PDF of lifetime of the nonlinear Wiener process, and propose a two-step parameters estimation method based on multi source information, in which the nonlinear parameter θ are obtained by the MLE method based on the historical degradation data, and the random coefficient and fixed parameter σ_B^2 are obtained based on multi source information to improve the accuracy of parameters estimation.

3. Experimental Studies

In this section, fatigue crack data are used to demonstrate the effectiveness of RUL prediction of nonlinear Wiener process with considering random effects based on multi-source information. There are 21 sets of degradation data, which are shown in Figure 2. For detailed description, please refer to Wang et al. [19].

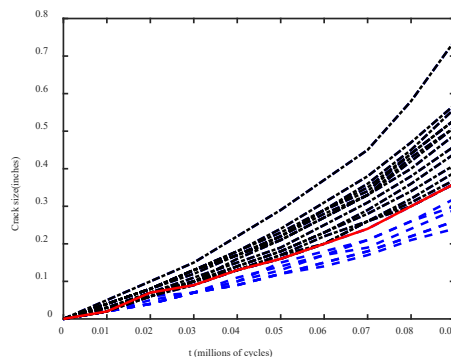


Figure 2: Degradation paths of fatigue crack

The method based on multi source information is referred to as M_0 , the method based on historical degradation data of congeneric equipment is referred to as M_1 , and the method with fusing failure time

data is referred to as M_2 . After that, the thirteenth group of fatigue crack data is selected as the object of lifetime and RUL prediction, which is represented by the red solid curve in Figure 2. Then, the lifetime distribution and the RUL distribution can be obtained based on these three methods, which are shown in Figure 3 and Figure 4. The MSEs at some points are in Figure 5.

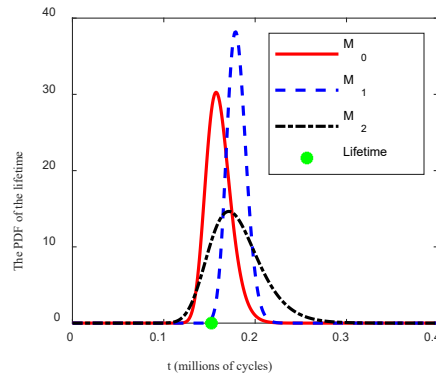


Figure 3: The estimated lifetime by M_0 , M_1 and M_2

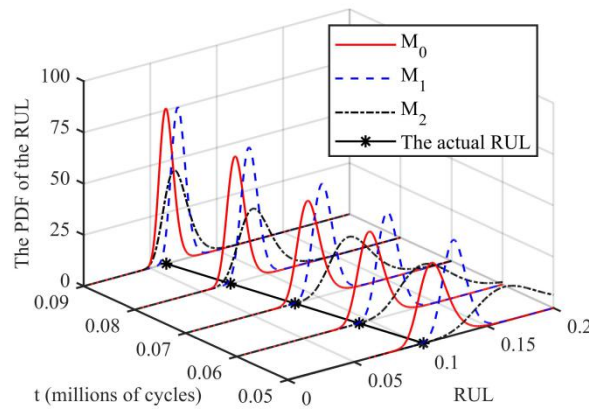


Figure 4: The estimated RULs by M_0 , M_1 and M_2

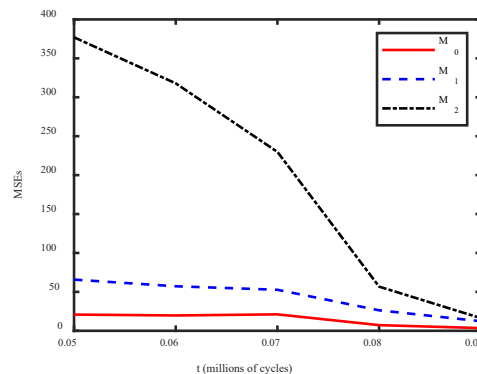


Figure 5: Degradation paths of fatigue crack

The results in Figure 3, Figure 4 and Figure 5 all show that the RUL prediction method based on multi source information is suitable for the degradation process based on nonlinear Wiener process. Then, let M_3 represent the RUL prediction method without considering random effects based on multi source information. The estimated RUL and MSEs at some measurement points are shown in Figure 6 and Figure 7. The results show that the estimated RULs with considering random effects are more concentrated and considering random effects can improve the accuracy of RUL prediction for the nonlinear degradation process.

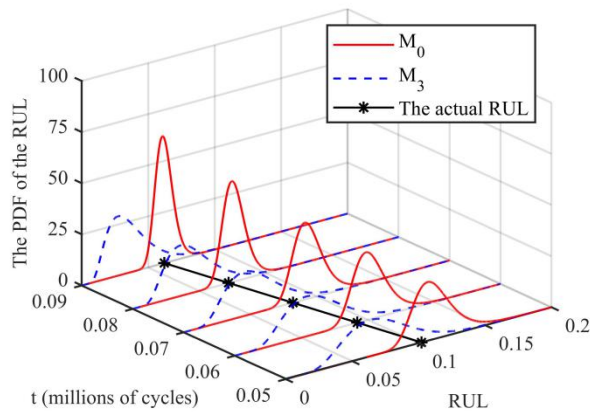


Figure 6. The estimated RULs at some points by M_0 and M_3

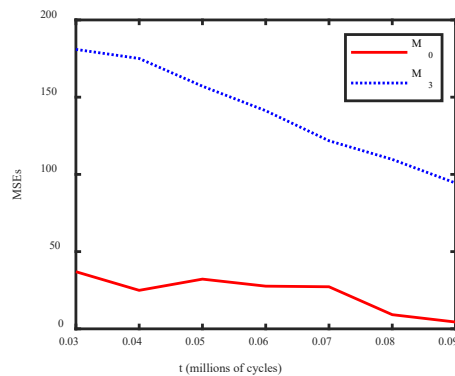


Figure 7. The MSEs at some points by M_0 and M_3

4. Concluding Remarks

Unit-to-unit variation is an important characteristic for degradation process of equipment. Many studies have shown that it is necessary to consider unit-to-unit variation, i.e., random effects. In order to improve the accuracy of RUL prediction, this paper fuses the historical degradation data and failure time data reasonably to estimate the unknown parameters of nonlinear Wiener process with considering random effects. First, the nonlinear parameter of Wiener process is calculated based on historical degradation data. Then, EM algorithm is used to calculate other unknown parameters. Based on these parameters estimation, RUL prediction is carried out. The results show that the method of nonlinear Wiener process model with considering random effects based on multi source information can effectively improve the accuracy of RUL prediction.

References

- [1] MAZHAR MI, KARA S, KAEBERNICK H. Remaining life estimation of used components in consumer products: Life cycle data analysis by Weibull and artificial neural networks[J]. *Journal of Operations Management*, 2007, 25(6):1184-1193.
- [2] CAI Z. Remaining useful lifetime prediction for equipment based on nonlinear implicit degradation modeling[J]. *Journal of Systems Engineering and Electronics*, 2020:194-205.
- [3] JI S, HAN X, HOU Y, et al. Remaining Useful Life Prediction of Airplane Engine Based on PCA-BLSTM[J]. *Sensors (Basel)*, 2020, 20(16).
- [4] WANG Y, PENG Y, ZI Y, et al. A Two-Stage Data-Driven-Based Prognostic Approach for Bearing Degradation Problem[J]. *IEEE Transactions on Industrial Informatics*, 2016, 12(3):924-932.
- [5] ZHANG J-X, HU C-H, HE X, et al. A Novel Lifetime Estimation Method for Two-Phase Degrading Systems[J]. *IEEE Transactions on Reliability*, 2019, 68(2):689-709.

- [6] TANG S, YU C, WANG X, et al. Remaining Useful Life Prediction of Lithium-Ion Batteries Based on the Wiener Process with Measurement Error[J]. *Energies*, 2014, 7(2):520-547.
- [7] TANG S, XU X, YU C, et al. Remaining Useful Life Prediction With Fusing Failure Time Data and Field Degradation Data With Random Effects[J]. *IEEE Access*, 2020, 8:11964-11978.
- [8] WANG F, TANG S, SUN X, et al. Remaining useful life prediction based on nonlinear random coefficient regression model with fusing failure time data[J]. *Journal of Systems Engineering and Electronics*, (to be published).
- [9] SUN M, JING B, JIAO X, et al. Research on Life Prediction of Airborne Fuel Pump Based on Combination of Degradation Data and Life Data[C]. *2018 Prognostics and System Health Management Conference (PHM-Chongqing)*, 2018:664-668.
- [10] ZHAO Q, JIA X, GUO B, et al. Real-Time Bayes Estimation of Residual Life Based on Multisource Information Fusion[C]. *2018 Prognostics and System Health Management Conference (PHM-Chongqing)*, 2018:215-222.
- [11] ZHANG Y, JIA X, GUO B. Bayesian framework for satellite rechargeable lithium battery synthesizing bivariate degradation and lifetime data[J]. *Journal of Central South University*, 2018, 25(2):418-431.
- [12] PENG C Y, TSENG S T. Mis-Specification Analysis of Linear Degradation Models[J]. *IEEE Transactions on Reliability*, 2009, 58(3):444-455.
- [13] CHEN X, LIU Z, WANG J, et al. An Adaptive Prediction Model for the Remaining Life of an Li-Ion Battery Based on the Fusion of the Two-Phase Wiener Process and an Extreme Learning Machine[J]. *Electronics*, 2021, 10(5).
- [14] WANG X, HU C, SI X, et al. An Adaptive Prognostic Approach for Newly Developed System With Three-Source Variability[J]. *IEEE Access*, 2019, 7:53091-53102.
- [15] WANG Z Z, CHEN Y X, CAI Z Y, et al. Methods for predicting the remaining useful life of equipment in consideration of the random failure threshold[J]. *Journal of Systems Engineering and Electronics*, 2020, 31(2):415-431.
- [16] WANG H-W, XU T-X, WANG W-Y. Remaining Life Prediction Based on Wiener Processes with ADT Prior Information[J]. *Quality and Reliability Engineering International*, 2016, 32(3):753-765.
- [17] HUANG Z, XU Z, WANG W, et al. Remaining Useful Life Prediction for a Nonlinear Heterogeneous Wiener Process Model With an Adaptive Drift[J]. *IEEE Transactions on Reliability*, 2015, 64(2):687-700.
- [18] TANG S, WANG F, SUN X, et al. Unbiased parameters estimation and mis-specification analysis of Wiener process-based degradation model with random effects[J]. *IEEE Transactions on Reliability*, (under review).
- [19] WANG X, BALAKRISHNAN N, GUO B. Residual life estimation based on a generalized Wiener degradation process[J]. *Reliability Engineering & System Safety*, 2014, 124:13-23.