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The Fusion of GeoGebra and High School Mathematics Culture

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Abstract: *Mathematical culture is the embodiment of the value of mathematics and the condensation of the spirit of mathematics. Mathematics culture plays an important role in improving the current situation of mathematics classroom teaching in high school, promoting cultural quality education, responding to the new curriculum standard, and improving students' mathematical literacy requirements. Geogebra is simple to learn and contains all the features of other math software. Geogebra integrates with mathematical culture to visually demonstrate the beauty of culture and the interest in mathematics to students. The author sorted out some examples of mathematics culture in high school mathematics and showed the integration process of Geogebra and high school mathematics culture from four mathematics topics. The results show that the integration of GeoGebra and high school mathematics culture is conducive to enhancing students' attention to and interest in traditional culture, improving classroom efficiency and students' learning effect.*

Keywords: *GeoGebra; High school mathematics; Mathematical culture*

1. Introduction to GeoGebra

Geogebra (GGB) is the co-writing of Geometry and Algebra. It is a completely free, powerful, and dynamic mathematical software with the ability to process geometric drawing and algebra operations at the same time. It was designed by Professor Markus Hohenwarter of Atlanta State University in Florida, US. Compared with the geometry sketchpad familiar to the public, its pages are more concise, more convenient to operate, and more complete in functions. Besides, it has more complex calculation functions and smooth drashixianing of three-dimensional graphics that cannot be realized by a geometry sketchpad[2]. Dynamic classroom demonstration, problem sets, paper production, interaction, exploration, and so on can be realized by the cooperation of GGB tools and instructions. As a kind of dynamic mathematical software, the dynamic is its biggest feature, which can be realized through the function of a "slide bar" when the parameter changes, the target graph changes within the limit range and forms a dynamic image.

GGB has powerful functions, simple operations, and strong interaction. Its functional areas include statistics, differential geometry, graphics, geometry, tabulation, and algebra.[1] These areas can be combined to solve problems and can be used in the production of a variety of courses or projects. Users can draw on the above parabola, hyperbola, ellipse, point, line, plane, directed line segment, and even function image, it also can deal with variables, in the mathematics class using GGB software dynamic display process of change, greatly improving the classroom capacity and teaching effect of a class, promote students inquiry-based learning. The integration of information technology and analytic geometry teaching can deepen learners' perception of geometric figures, promote significant changes in the relationship between education and teaching subjects and break through the difficulties in analytic geometry teaching[3]. Today's learning pays attention to exploratory learning, that is, learning while doing, which requires courseware to be able to provide exploration, interaction, and interaction, and interaction is an important way of exploration. The interaction in GGB can be embodied in many aspects, such as any object can be selected, dragged, and transformed, using variables and sliders, as well as moving points, tracks, etc. Some scholars made drag-type interactive exercises to explore the nature of prime numbers and made some more complex interactive controls with scripts, which reflected the dynamic advantages of GGB[4].

Mathematical culture has a profound influence on social development with its logical way of thinking and its unceasing exploration will. Nowadays, due to the limitation of exam-oriented teaching, in the eyes of most students, mathematics simply exists as an examination question, but ignores the close connection with other subjects, and lacks sufficient understanding of the formation and connotation of mathematical knowledge. We need to integrate the culture of mathematics into learning, especially geometry problems. In this vast country with a splendid mathematical culture, it is particularly important to learn the wisdom of ancient people to view problems dialectically, carry forward China's fine traditional culture, and cultivate cultural confidence in the new era. By integrating mathematics culture into the classroom with the breakthrough point of geometry, we can continuously enrich the theoretical system of mathematics teaching, provide new methods of mathematics teaching, and stimulate teachers' and students' passion for exploring innovative teaching and education. In this paper, the excellent dynamic function of GGB software is combined with mathematical culture to simulate and establish six different types of mathematical models, aiming at helping students to understand the graphic transformation in the topic in an all-round and three-dimensional way and establish concrete thinking in their minds, to seek the answer to the problem.

2. The application of Geogebra to high school mathematics culture

2.1 Equations and Inequalities

The logo of the 24th International Congress of Mathematicians held in Beijing was designed based on the string diagram of the ancient Chinese mathematician Zhao Shuang. As shown in Figure 1, the light and shade of the colors make it look like a windmill, representing the hospitality of the Chinese people. Can you find some equality and inequality relationships in the diagram?

Mathematical culture: Zhou Bi Suanjing, a book foreword by Zhao Shuang, a mathematician in the State of Wu during The Three Kingdoms Period, is where he created a 'circle square diagram' with strings as the side length. The square is composed of 4 identical right triangles plus a small square in the middle. The basic idea of the proof of the "Zhao Shuang String Diagram" is that the area of the graph remains unchanged after cutting and complementing, which is an important principle of area "complementing in and out" in ancient Chinese mathematics. It is one of the characteristics of Chinese ancient mathematics.

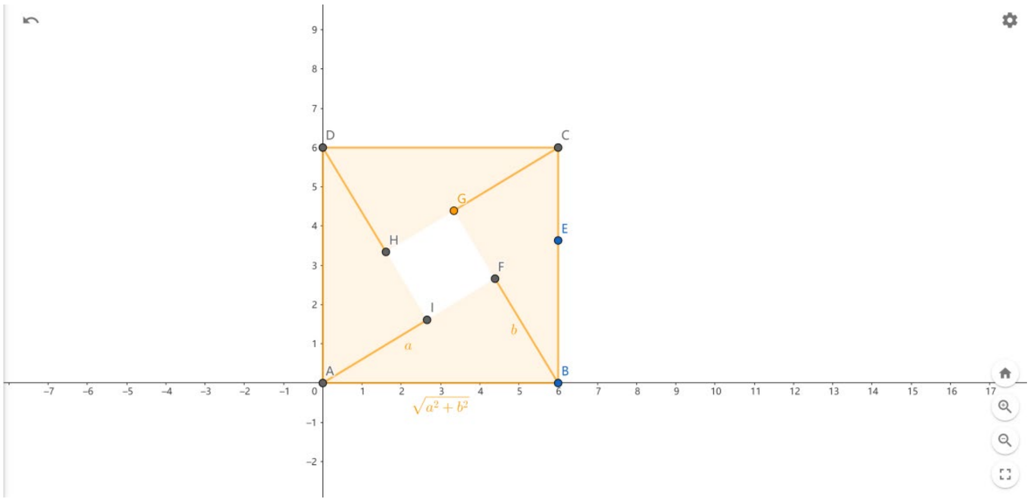

Figure 1 Chords of Zhao Shuang (Drawing module in GeoGebra)

Application process: Using the function of "dynamic transformation", construct a square, take a random point E on the side, connect the line segment AE to form a triangle, pass B as the vertical line of AE, and take the intersection point F, form the first triangle -- ABF, by the same way, you can draw another three triangles. The area of the four triangles can be changed by dragging point E. Let the length of the two sides of a right triangle be.

Teacher activity: Drag point E to achieve the area change of four triangles.

Student activities: Observing the changes in the image, it is found that when point E is at point B, the area of the four triangles is 0, and then square FIGH and square $ABCD$ coincide. During the movement of point E from point B to point C, the area of the four triangles gradually increases until it occupies the whole square $ABCD$, and square $FIGH$ shrinks into a single point. It is concluded that the sum of the areas of the four triangles is less than or equal to the square ABCD, i.e.$a^2+b^2 \geq 2ab$.

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The thought of cut complement, a combination of number and shape, and figure transformation contained in string graphs are excellent carriers for the infiltration of mathematics thought in classroom teaching. Dynamic simulation of its rich geometric features with GGB can make students feel the changes in the figure area more directly, and it has an outstanding and extraordinary effect on students' thinking training. We can see such an introduction in the textbook, which implies the initial idea that the editor wants to combine mathematical culture. However, what the book finally presents is just a static graph, and the dynamic simulation of GGB can be used as a supplement to the changes in the graph that cannot be presented in the book. "Complementing in and out" is a mathematical expression rich in the connotation of traditional Chinese culture, and the understanding of "out" and "in" is the key. With the help of this question type, it extends to the life thinking of "giving up" and "getting", which are all fields that can be tried with mathematical culture.

2.2 Function

Chinese noria is a kind of irrigation tool invented in ancient China. Because of its economy and environmental protection, it is still used in agricultural production. Assuming that each water drum on the Chinese noria moves in a uniform circular motion under a stable water flow, can you use an appropriate functional model to describe the relationship between the relative height of the water drum (regarded as a particle) from the water surface and time?

Mathematical culture: Chinese noria, a tool for irrigating fields with water flow as power. According to historical records[2], the Chinese noria was invented in Sui and flourished in Tang, with a history of more than 1,000 years ago. In the hometown of the lush mountains, streams formed a picture of ancient pastoral spring scenery, for the ancient Chinese people’s outstanding invention.

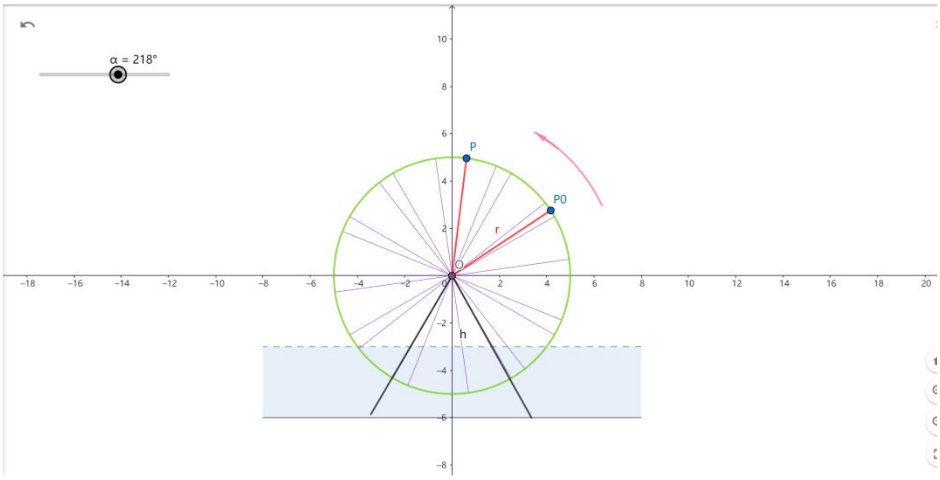

Figure 2 Chinese noria (Geometry module in GeoGebra)

Application process: The “Chinese noria” model built with GeoGebra is shown in Figure 2. Use "dynamic transformation" to establish the coordinate $xoy-$axis, use the function to draw a circle, and use the "sequence" instruction to make 12 line segments as the beam of the Chinese noria. Four points are drawn and connected as the water surface, Then Make two lines over the O point as the support of the Chinese noria. Take two points P and P_0, and connect OP and OP_0. Use the "curve" and "vector" instructions as an arrow indicating the direction of rotation. Let's do another one to slide the Angle of the end edge at OP, use the "rotate" command to rotate the circle with the sliding degree, and retrace point P_0 so that point P_0 does not change with the rotation of the circle. We suppose that after t seconds, the water drum M moves from the point P_0 to the point P, the distance from the center "O" of the Chinese noria runner to the water surface is "h", the radius of the Chinese noria is "r", and the angular speed of the rotation of the Chinese noria is the Angle is "ω", the angle of the end edge of the OP is "α", the initial position of the water drum is "P_0" and the height of the water drum "M" from the water surface is "H".

Teacher activities: Control the rotation Angle of the water drum by manipulating the upper left slider.

Student activities: Observe the changes in the image, and find that as the slider slides to the right, the rotation Angle is α, the Chinese noria and the line segment OP begin to rotate counterclockwise, which is the Angle of beginning edge Ox, and ending edge OP, is $\omega t+\alpha$, and has $y=r\sin(\omega t+\alpha)$. It is

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concluded that the relationship between the height of the water drum H from the water surface and the time t is $H = r\sin(\omega t + \alpha) + h$.

It is of practical significance to introduce trigonometric functions under the background of Chinese noria, which is a very typical function modeling process. The study of the trigonometric function based on the circular motion of the Chinese noria can not only relate to the practice, but also to the analytic expression of the function and the image of the function, and fully reveal the internal logical relationship between them. Using GGB to dynamically simulate the running process of Chinese noria can make students feel the changes of various variables more intuitively, and provide an important platform for improving students' mathematical literacy such as mathematical abstraction, intuitive imagination, and logical reasoning. A trigonometric function is one of the best presentation forms in plane geometry mathematics, and the combination of number and form thinking is also one of the most important learning methods for geometry. As a typical digital-graph combination, GGB dynamically simulates the trajectory of the water drum and calculates its height, which is a combination of life and theory, and a true embodiment of "everywhere mathematics".

2.3 Solid geometry

According to Liu's imagination, when two cylinders with the same base diameter meet vertically, their intersection produces a special geometry, which Liu calls "Mou he square cover".

Mathematical culture: Mou He Fang Gai was first discovered and adopted by Liu Hui, an ancient Chinese mathematician, to calculate the volume of a sphere, similar to the infinitesimal method. With two identical cylinders orthogonal, it is common part "its shape like Mou He square cover", so-called Mou he square cover. Liu Hui proves that the ball volume formula used in Jiuzhang Shuanshu is wrong. Liu Hui could not figure out the volume of the square cover. Later Zu Geng used his progenitor Geng principle to take the ball to a volume is $\frac{\pi}{6}D^3$, as the ball diameter is D.

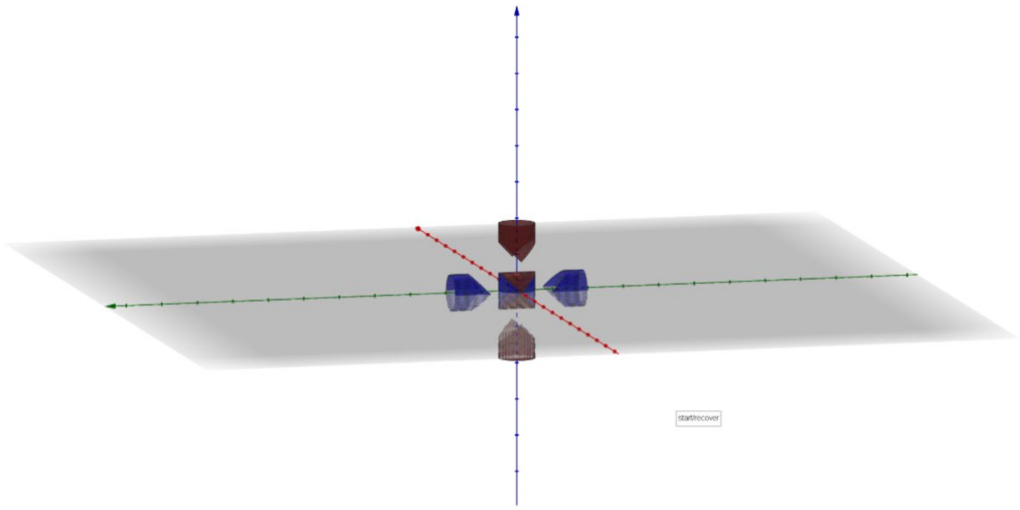

Figure 3 Mu He square cover (3D computer module in GeoGebra)

Application process: The “Mu He square cover” model built with GeoGebra is shown in Figure 3. The "dynamic transformation" is used to divide the drawing of graphics into two processes: graphic making process and animation making process, and the space rectangular coordinate system is established. The "curve" instruction is used to make the middle two intersecting curves of the Mouhe square cover, and the "surface" instruction is used to get the left and right sides of the Mouhe square cover. Similarly, the upper and lower sides of Mouhe square cover can be obtained. Add the absolute value to the coordinates of the above "curve" instruction to get the curve image of the positive half of the Z-axis. Then use the "curve" instruction to make the upper and bottom surface of the cylinder in this direction, and use the "surface" instruction to connect the curve and the corresponding bottom surface of the cylinder. Using the "rotation" command to get the same image in the negative axis half of the Z-axis, positive X-axis, and negative X-axis, the graphic production process is completed. Use the "Pan"

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command to create an animation pan effect, create "text" as a button, add script, name "Start/Resume", click the image starts to extend, and click again to restore.

Teacher activities: Control the process of the Mouhe square cover starting to extend outward and return to its original state by manipulating the "Start/Restore" button in the lower right corner.

Student activities: Observe the changes in the image and find that the intersection line between the extension and the outer surface of the spherical shell is the tangent line between the inner tangential ball and the Mouhe square cover.

GGB visually demonstrates the production principle of Mou He square cover, which enables students to deeply experience the wisdom of ancient people and the beauty of mathematics and cultivates students' mathematical abstract and intuitive imagination ability, which is conducive to the study and understanding of solid geometry. "Mouhe Square cover" is a relatively complex spatial solid geometry polyhedron, it is not easy to come up with graphics by imagination alone. With the static multi-frame presentation of GGB, you can see its changes in different stages, and then realize its "beginning" and "end" connection by dynamic transformation. By combining GGB with examples in such classic works, students can think more, guide their thinking divergence, and realize cultural exchange and understanding across time and space through a graph expressed by software. Why the knowledge already existed a thousand years ago is now shining in the light, in the development of our country's culture, it is easy to ignore the development of the natural sciences, a large part of it is because the feudal society can do no help to people's thought, and being content with the status quo is the fundamental reason why we are beaten. If history can change "beginning" and "end" with the help of buttons like Mou He Fang Gai, how can we break through the shackles of this layer of thought? It's worth thinking about.

2.4 Statistics and probability

Bean experiment: There is an experiment in the Monte Carlo method, which randomly scatters a handful of beans. By examining the number of beans in its tangent circle, the approximate value of PI P can be calculated by the random simulation method. The “Bean-spread experiment” model built with GeoGebra is shown in Figure 4.

Mathematical culture: Monte Carlo a city in the Principality of Monaco, is a world-famous "gambling city", in the 1940s, the members of the United States "Manhattan Project" S.M. Ulam and J. von Neumann first named the method of calculating the deterministic values of interest by probabilistic events as "Monte Carlo Method". The core idea is to calculate a quantity we are interested in through the probability of probability experiment. One of the experiments in Monte Carlo's method is the bean-spreading experiment.

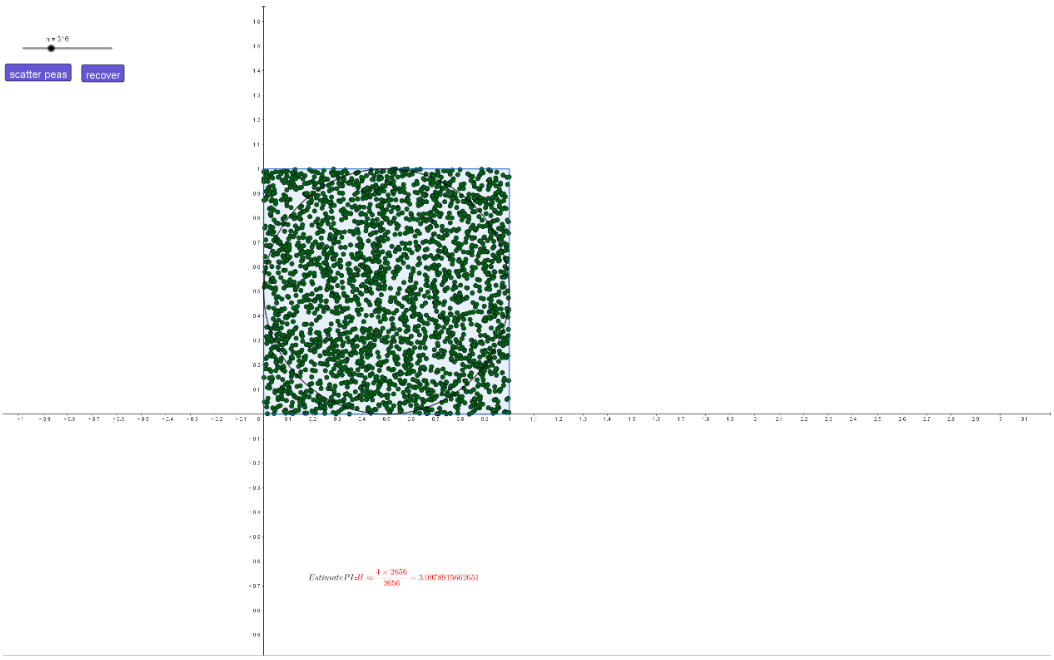

Figure 4 Bean-spreading experiment (Drawing module in GeoGebra)

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Application process: Using the two functions of "dynamic transformation" and "calculation and measurement", firstly draw a square with a side length of 1 and its tangent circle with the tool, and then create an empty list to store the position of each sprinkling bean, and create a slider to control the number of each sprinkling bean. The "whether in the region" instruction and "mapping" instruction are used to judge whether the beans are in the tangent circle, and then the "sum" instruction is used to calculate the number of beans in the tangent circle. Create button "scatter beans", add script "sequence" command to control scatter beans, create button "restore", and add script empty list. Add another text to estimate PI, and write the estimating formula, according to the different number of beans, the estimated value of PI will change accordingly.

Teacher's activities: We select the number of beans by manipulating the slider in the upper left corner, click the "Sprinkle beans" button to start to sprinkle beans, and continue to click the "sprinkle beans" button will scatter more beans on the original basis, and last click the "Restore" button to recover all beans.

Student activities: Observe the image changes, find that click "scatter beans", beans randomly scattered in the square and tangent circle, and click "restore". The scattered beans are emptied. By spreading a total of a bean into the square, the number of beans in the circle is b. From the law of large numbers, using frequency to approximate $\frac{b}{a} \approx \frac{\Pi}{4}$, that is, $\Pi \approx \frac{4b}{a}$.

The experiment of "scattering beans" is a very interesting mathematical exploration, reflecting the relationship between frequency and PI with infinite thought. GGB is used to visually present the changes in the number of beans and the change process of PI so that students can feel the limited idea and expand their thinking. In the process of dynamic demonstration, students can understand the essential meaning of probability, and experience the application of the classical scheme and geometric scheme in practical problems. Exploring the reason for the formation of a theorem inevitably involves the story of the mathematical culture behind it. The formation of a lot of truth comes from the experiment of "scattering beans", which was not considered at first. In the process of tracing back to the origin of theorems like this, we can gradually realize the innovation contained in "truth", which is also of great help to the diffusion of our thinking.

2.5 Conic curve

What is the trajectory of an aerial ladder used in ancient battles to scale the wall, placed in a corner, which slowly slides down until it falls to the ground?

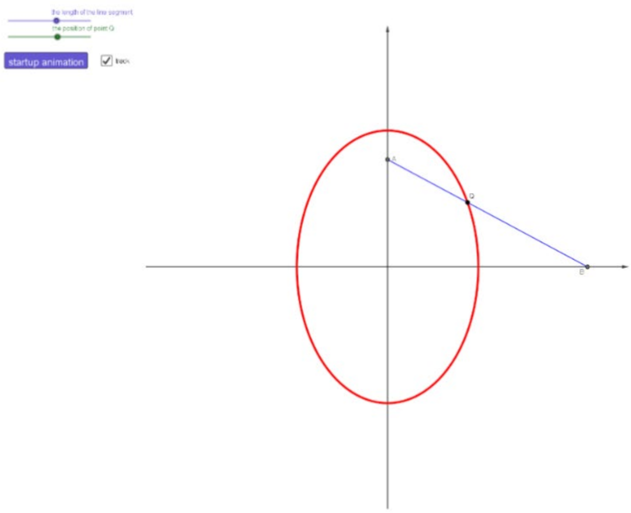

Figure 5 Ladder sliding (Geometry module in GeoGebra)

Mathematical culture: The aerial ladder in ancient China, with wheels under it, can be driven, so it is also called a "ladder car", equipped with a shield, winch, grappling hook, and other equipment, some with pulley lifting equipment. The ladder has existed since the Shang and Zhou Dynasties. During the Spring and Autumn Periods, a skilled craftsman named Lu Ban improved the State of Lu. At that time, King Hui of Chu ordered the first ladder in history to be made by the Gong Shupan to become the king. In Huainan Zi · Soldier Training, Xu Shen notes that an"aerial ladder can stand on Evian, so you can look down on the enemy city", indicating that another use of an aerial ladder can climb high and look far to spy on the enemy.

Application process: The trajectory model of the “Ladder sliding” established by GeoGebra is shown in Figure 5. The aerial ladder is transformed into a line segment by using the two functions of "dynamic transformation" and "trajectory tracking", and a position on the aerial ladder is regarded as a point. We create two vertical vectors, use the tool to draw A circle, trace a point P on the circle, and take two points A and B on the two vectors, so that the two points have the same ordinate and horizontal coordinates as point P respectively. We connect the two points to form line segment AB, take a point Q on line segment AB, and create two Slide bars, one to control the length of the line segment, and one to control the position of the point on the line segment. Input "trajectory" instruction to track the trajectory of point Q. We create a checkbox, associate trajectories, and create a button to control the start of the animation.

Teacher activities: We control the length of the line segment by manipulating the purple slider in the upper left corner, control the position point Q by manipulating the green slider in the upper left corner, and click the "Start animation" button, and the animation starts to run, last, check the "track" box to display the motion track of Q point.

Student activities: We observe the changes in the image and find that the line segment starts to move from the positive semi-axis of the y-axis and rotates clockwise until the line segment reconnects with the positive semi-axis of the x-axis. Assume that the included Angle between the ladder and the x-axis is α, and the length of the line segment is l. By analyzing the relationship between these variables, we can calculate the trajectory equation of the ladder sliding.

GGB visually presents the slide of the ladder and the trajectory of points, turns abstraction into intuition, turns difficult into easy, and cultivates students' mathematical abstract and intuitive imagination ability. "Ladder truck" is a common offensive weapon in ancient battlefields. The mathematical theory behind it reflects the ancients' continuous exploration of weapon improvement, and it is the wisdom crystallization of many military strategists. Because of the chaotic times, they want to improve the capability of weapons for self-defense or expansion. There are countless examples of applying mathematical knowledge to tools. We can see the essence through the phenomenon.

2.6 Series

In "Zhuangzi Tianxia", it is mentioned that "Take one half of a foot-long stick every day, never finish". It vividly shows that things are infinitely divisible.

Mathematical culture: mentioned in the "Zhuangzi Tianxia": "Take one half of a foot-long stick every day, never finish" is the earliest passage reflecting the thought of calculus in the ancient books of our country. Half-taken today, half taken tomorrow, half taken the day after tomorrow. If "day and a half" are taken, half is always left, so "inexhaustible".

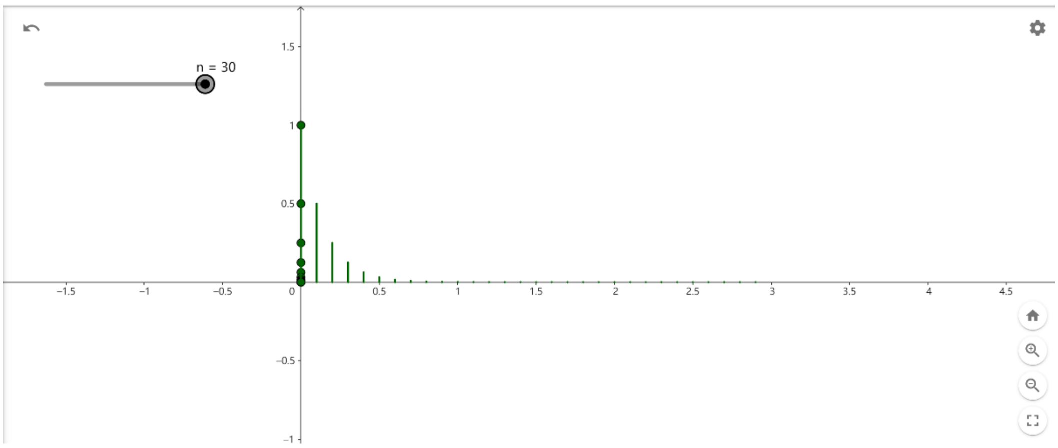

Figure 6 Maul of a ruler (Geometry module in GeoGebra)

Application process: “Maul of a ruler” model is built by GeoGebra and is shown in Figure 6. The two functions of "dynamic transformation" and "operation and measurement" were used to create a coordinate system and create sliders to control days. The "iteration" instruction was used to ensure that each iteration

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was at the midpoint between the origin O and the original point. The "sequence" instruction was input to display the line segment moving to the right and getting shorter and shorter, and the cycle continued.

Teacher activities: We select the number of days passed by manipulating the slider in the upper left corner is n.

Student activities: We observe the changes in the image, and find that the teacher manipulates the slider in the upper left corner to select the days passed as n, and the image shows the number of line segments is $n+1$. Take "Maul" as a line of length 1, when time is $n=1$, the line length is 0.5, when time is $n=2$, the line length is 0.25... And so on. You get a set of numbers:1, $\frac{1}{2}$, $\frac{1}{4}$, $\cdots\cdots$, and you get the conclusion that each day the length of the line segment is $1/2$ the length of the previous day.

Transform the abstract problem into mathematic and mathematic problem into intuitionistic, to help students understand the concept of geometric series. After the dynamic transformation of GGB, we can still see the enlarged maul, which reflects the ancient people's infinite divisible thinking about things. From another point of view, a thing is constantly weakened. Though it still exists, its quantity is equal to nothingness. In the idea of limit, the value tending to infinitesimal is equal to zero. Therefore, all things have related attributes, but we pay more attention to the factors that have the greatest influence on "it". If the quantity is large, the influence will be large, while if the quantity is small, the influence will be small. This is similar to the thoughts of Taoist Mohist Legalists.

3. Conclusion and enlightenment

Using GGB for tutoring, to improve students' spatial understanding of three-dimensional shapes and geometric question types, learning enthusiasm, and promote teaching to improve teaching quality. Based on this, the interactive integration research of GGB and mathematical culture is of positive significance. When GGB is applied to more complex high school mathematics, we should also guide students to combine the cultural connotation of mathematics with the topic while solving the problem. The above questions listed the wisdom crystallization of our ancestors: equation inequality, function, solid geometry, statistics and probability, conic curve, series of six senior high school mathematics question types, respectively explained the origin of mathematics culture behind and contemporary teachers and students of problem-solving thinking. In the mathematical universe of stars, we have extremely fruitful results. The exploration thinking of predecessors is vividly presented through the dynamic simulation of GGB. What is transformed here is not only different forms of graphics but also another collision of ideas through thousands of years. With the help of GGB, we can face the ubiquitous mathematical culture in life more intuitively and with more visual impact. How to combine cultural ideas with actual life is the key and basic requirement of using software to explore the topic.

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