

Rube Goldberg Machine Project Report

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Abstract: This project is about analysis of physical phenomenon in blender physics simulation. The Rube Goldberg Machine model was constructed and simulated by blender, data were collected by tracker, and physical evaluation were analyzed by python (JupyterLab). 9 phases of physical phenomenon will be analyzed in this report, which includes circular motion, torque, the law of energy conservation, elastic collision, totally inelastic collision, 2D projectile and free-fall.

Keywords: 3D modeling, physical simulation, engineering, python scientific analysis, freefall

1. Introduction

Blender is an open-source freeware 3D modeling software. It is widely used by cartoon publishers and 3D artists to create animations and 3D models. All simulations in this project were completed by blender. Figure 1 shows a scratch design of the Rube Goldberg Machine. There is a brief video about how the machine works. You download at <https://www.dropbox.com/s/zx6r69xw91kb5cd/blender%20physics%20simulation.mp4?dl=0>.

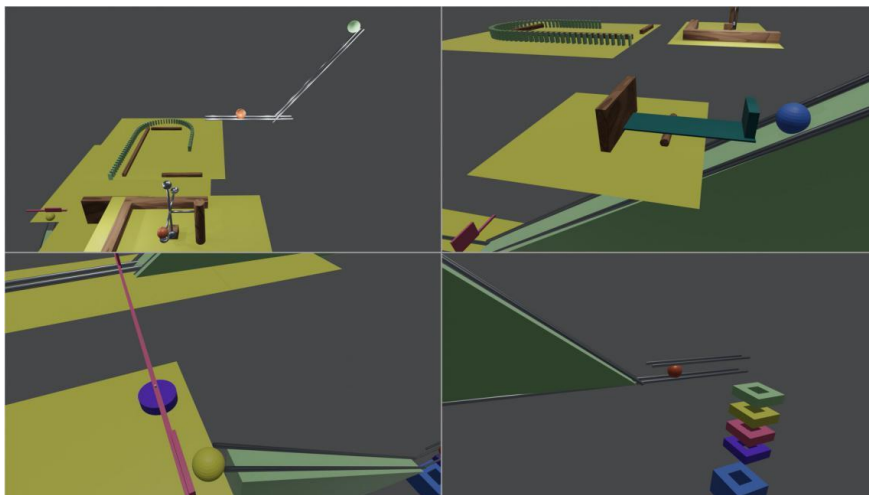


Figure 1 Picture of the modeling in different phase

2. Physical concept

There are several fundamental physical concepts been involved in this project. Explanation of those concepts will be provided at the following subsection. The purpose of this section is to provide a detailed explanation to concepts for readers to comprehend.

2.1 Displacement, velocity, acceleration

These are three of most basic physical concepts. Numerous further physical ideas were developed from and based on the fundamental unit of distance. Distance is a scalar while displacement would be a vector which represents both direction and magnitude. The SI unit for distance and displacement is m.

Deriving from the concept of distance and displacement, there are a unit used to represent the rate of change of distance and displacement - the speed and velocity. The unit for speed and velocity is ms^{-1} . This concept is related to our description of a motion of object. The greater speed an object has, the faster it is moving. Oppositely, the smaller speed an object has, the slower it is moving. Since that velocity is

the rate of change of displacement, in order to find the changing velocity from the function of speed, we can simply calculate its derivative with respect to time $\frac{ds}{dt}$.

The acceleration represents the rate of change of velocity. This unit can measure whether the velocity of an object is increasing or decreasing, which is also a vector. The unit is ms^{-2} . If the acceleration is varying, we can find the function of acceleration from velocity by calculating $\frac{dv}{dt}$.

2.2 Conservation of Energy

Energy is an extremely important concept in physics. The unit for Energy is J which is one of the seven fundamental SI units. The definition of energy is the work that a certain force (gravitational, electromagnetic, etc) can do (sciencedaily.com). To define Energy in formula:

$$\Delta K = \sum_{\text{by all}} \vec{F}$$

Energy will be the ability, or the contained power of an object. In Newton's system, there are mainly two types of energy: Potential energy and Kinetic energy. For gravitational potential energy, it represents that at the height of the object the amount of work that gravity will do to the object from its initial position to ground surface. Here is the formula:

$$PE_g = mgh$$

Kinetic energy represents the amount of energy that an object has for its motion. This is related to the object's mass and velocity. An moving object will carry such amount of kinetic energy, and as it been accelerated or decelerate the energy will change. In real life situation, most motions will be affected by other factors like friction or temperature. Then, some of the kinetic energy will be transferred into other form of energy. The formula to define the kinetic energy:

$$K_{\text{CM}} = \frac{1}{2}mv^2$$

For an isolated system, energy can be transformed from one form into another, but the total amount must remain the same, which means that energy can never be generated but only be transferred between objects or into different forms. This is an essential concept that created a lot of equal relationship for our calculation.

2.3 Collision

Before introducing collision, the concept of momentum needed to be clarified. Momentum describes a property that an object has when it is moving in a specific speed. The formula for momentum is $mv = p$.

During collision, the momentum on an object will be transferred to another object that were collided with it. In a totally inelastic collision, the momentum will be fully transferred from one object to another without any leakage of energy. The formula for totally inelastic collision is shown below:

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

However, in real life situation, according to energy conservation law, the energy is almost impossible to be transferred from one object to another object without any loss of energy. In most situation, the energy will be transferred into heat energy, or a kinetic energy that causes a transform of the object. In order to consider how much energy was transferred and how much wasn't, we use the concept 'Elasticity' to represent the efficiency of the collision. Formula of elasticity is shown below:

$$e = -\frac{V_{1f} - V_{2f}}{V_{1i} - V_{2i}}$$

2.4 Conservation of momentum

This concept is similar to the concept of energy conservation. One of the laws about a totally isolated system is that: the total momentum of any isolated system—no matter how complex—is constant. This

can be represented by a differential equation:

$$\text{For an isolated system, } \frac{d}{dt}(\vec{p}_{tot}) = 0$$

But in real life situation a system is very hard to reach an absolutely isolated statue. Newton then proposed a theory to describe a system that were affected by other forces, stated that: the time rate of change of the momentum of a system is equal to the sum of all the external forces on that system. Here is the formula Newton derived that related the sum of outer force-net force-to the momentum:

$$\text{For a non-isolated system, } \frac{d}{dt}(\vec{p}_{tot}) = \sum \vec{F}_{ext} = \vec{F}_{net}$$

Without any external forces, the total momentum of a system will never be changed. Only when external forces were applied to the system, the total amount of momentum will be changed. This is the conservation of momentum.

2.5 Rotation

Rotational motion is a branch from normal motion. In rotational motions, objects are moving or orbiting a circle. Instead of moving in a straight line, the object will always change its direction of motion in order to keep its track on the circle. As a object moving forwardly, it will have a inertia that makes it moving continuedly forward, and according to newtons first law, the object will not change its direction of motion unless a force was exerted to it. Therefore, in a centripetal motion the object will be keep experiencing a centripetal forces to 'adjust its direction' all the time. Many quantities and units in linear motion can be transferred into rotational motion. Here are some examples:

$$\begin{aligned} \text{Position } x(\text{m}) &\Leftrightarrow \text{Angle } \theta(\text{ radians }) \\ \text{Displacement } \Delta x(\text{m}) &\Leftrightarrow \text{Angular displacement } \Delta\theta(\text{rad}) \\ \text{Velocity } v_x(\text{m/s}) &\Leftrightarrow \text{Angular velocity } \omega(\text{rad/s}) \\ \text{Acceleration } a_x(\text{m/s}^2) &\Leftrightarrow \text{Angular acceleration } \alpha(\text{rad/s}^2) \\ \text{Force } F_x(\text{kg} \cdot \text{m/s}^2) &\Leftrightarrow \text{Torque } \tau(\text{kg} \cdot \text{m}^2/\text{s}^2) \\ \text{Massm (kg)} &\Leftrightarrow \text{Moment of inertia } I(\text{kg} \cdot \text{m}^2) \end{aligned}$$

In linear form, kinetic energy can be represented in form of $K = \frac{1}{2}mv^2$. If the motion was in circular form, then the formula of kinetic energy to $K_{rot} = \frac{1}{2}I\omega^2$. This simply replaced mass (m) in linear form to Moment of inertia (I), and velocity (v) into angular velocity (ω).

3. Data collection

Before evaluating physics procedures in the Rube Goldberg Machine, the data of objects motion are required to be collected for analysis. Tracker is the main tool used for extracting data in this project. The fundamental theory of this software is that it tracks the object that user set for it, record its coordinate with respect to time, and as it progresses it will learn the change of the image of the object, and find their location in each frame of the video.

The main objects to be tracked are balls, which are colored in a distinct color in order to separate it from the background and make Tracker easier to track the object. The whole processes were separated into 9 stages, and an example of data collection of phase 6 and 9 will be shown in the following.

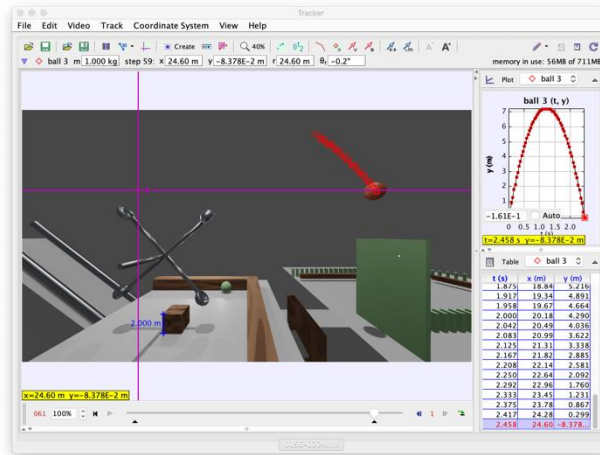


Figure 2 Tracker data of phase 6

In phase 6, ball 3 will be accelerated by the mill, and thrown out, to perform a parabolic motion. The figure 2 shows the process in Tracker.

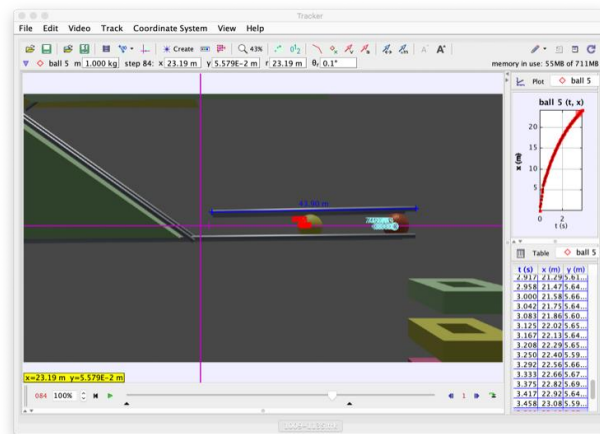


Figure 3 Tracker data of phase 9

In phase 9, ball 5 has roll down to the bottom and then have a elastic collision with ball 6. Ball 6 will be accelerated and eventually drop into the colorful box. The figure 3 shows the process in Tracker

4. Analysis

The following chapter will introduce the physical phenomenon that have been observed during the experiment. The tool used to analyze the data is JupyterLab integrated with python 3.7. The advantage of JupyterLab is that it can calculate my data in the format of coding, and also provide a variety method of graphing, such as Matplotlib.

There are two type of process for analysis. One is that first analyze the data extracted from the experiment and use tools to calculate for a specific data. Then, with the essential data acquired, the following calculation can proceed and calculate for the final value needed.

Another method is first deriving the value that would be measured with physical concepts and formulas, then use technology to calculate for the exact value, and compare them to find out the range and types of error.

Phase 1

From this situation, there are existence of three forces acting on ball 1: The normal force produced by the surface – the pipe, the gravitational force, and a rolling frictional force. Therefore, the netforce and the acceleration will be:

$$F_{net} = F_N - F_f = \sin(\theta) \cdot m_1 g - \mu_k \cdot \sin(\theta) \cdot m_1 g$$

$$a = \frac{F_{net}}{m} = \frac{\sin(\theta) \cdot m_1 g - \mu_k \cdot \sin(\theta) \cdot m_1 g}{m_1} = \sin(\theta) \cdot g - \mu_k \cdot \sin(\theta) \cdot g$$

The unknown variable in this formula is a and μ_k . The next step is use python to analyze the motion data and calculate for acceleration. With the data recorded imported into JupyterLab, the matplotlib generated the graph with itself and its first and second derivative curves where the graph have been shown on left.

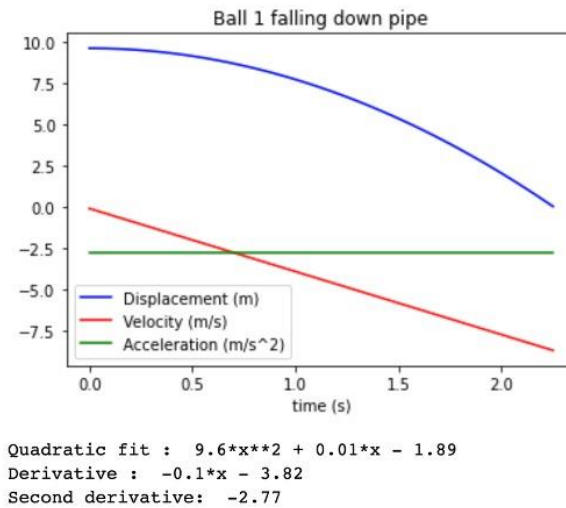


Figure 4 Graph of kinetic motion of ball 1 falling down pipe

The second derivative of displacement, which is the acceleration, is equal to -2.77 . Therefore:

$$a = 2.77 \text{ m/s}^2$$

Phase 3

Phase 3 describes the totally inelastic collision between ball 1 and ball 2 while ball 1 has a initial velocity of 4.191 m/s , which was the final velocity of the surface motion; and ball 2 was at rest initially. According to conservation of momentum, if two totally inelastic objects collides, then $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$. With labels of v_{1i} - initial velocity of ball 1, v_{2i} - initial velocity of ball 2, v_{1f} - final velocity of ball 1 and v_{2f} - final velocity of ball 2:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\because v_{2i} = 0$$

$$\text{and } v_{1f} = 0$$

$$\therefore m_1 v_{1i} = m_2 v_{2f}$$

$$\because m_1 = m_2$$

$$\therefore v_{1i} = v_{2f}$$

Therefore we know that the velocity of ball 2 after collision is 4.191 m/s , same as the velocity of ball 1 before collision.

Phase 4

In this phase, ball 2 is performing a free fall with a initial velocity of 4.191 m/s . The height of the hill is 16.258 m as measured. In order to find the final vertical velocity, we use the free fall formula:

$$\begin{aligned}
 v_{2y} &= \sqrt{2gh} \\
 &= \sqrt{2 \cdot 9.8 \cdot 16.258} \\
 &= \frac{17.851\text{m}}{\text{s}}
 \end{aligned}$$

We can also find the time for the free fall process. According to the formula the time will be:

$$\begin{aligned}
 t &= \sqrt{\frac{2h}{g}} \\
 &= \sqrt{\frac{2 \cdot 16.258}{9.8}} \\
 &= 1.822\text{s}
 \end{aligned}$$

And eventually we can use the time we calculate to find horizontal displacement, since that the horizontal velocity is constant:

$$d_x = v_1 t = 4.191 \cdot 1.822 = 7.634\text{m}$$

Phase 5

In this phase, ball 2 were falling down and hitting the mill. This phase starts at the moment ball 2 hits the mill, and ends at the moment right before ball 3 leaves the mill. To calculate for the torque, the force exerted on it needed to be clarified. By measuring, the initial velocity of ball 2 as it contacts with the torque is 13.568 m/s, and its velocity at the bottom is 14.5 m/s. The duration of time of this process is 0.292 s. With the two velocity and time, acceleration can be calculated, and thus the force can be calculated.

$$\begin{aligned}
 a &= \frac{v - u}{t} \\
 &= \frac{14.5 - 13.568}{0.292} \\
 &= 3.191\text{m/s}^2
 \end{aligned}$$

With newton's second law, we can calculate for the force (labeled as f1) that ball 2 exerted to the mill. The distance from ball 2 to the pivot point (labeled as d1), and distance from ball 2 to the pivot point (labeled as d2) are measured, which will be used to calculate for torque with formula $\tau = fr$, and thus find the force acting on another side f2:

$$\begin{array}{ll}
 f = ma & d_1 = 6.321\text{m} \\
 f_1 = 12 \cdot 3.191 & d_2 = 6.177\text{m} \\
 = 38.289\text{N} & \tau = f_1 d_1 \\
 & = 6.321 \cdot 38.289 \\
 & = 242.014\text{N} \cdot \text{m}
 \end{array}$$

For force at right hand side, and according to the law of conservation of torque, torque at both side is equal:

$$\begin{aligned}
 \tau &= f_2 d_2 \\
 f_2 &= \frac{\tau}{d_2} \\
 &= \frac{242.014}{6.177} \\
 &= 39.18\text{N}
 \end{aligned}$$

Since we found the force that acting on ball 3, we can then analyze its motion. Ball 3 have experienced a circular acceleration before being thrown out. During the acceleration, there are two force acting on it. One is the force exerted by the mill F_2 , and another is its own weight F_w . Then we can calculate for its tangential acceleration among the circle, and we can find its velocity as it left the mill:

$$\begin{aligned}
 m_3 &= 1kg \\
 F_{net} &= ma & d &= 5.606m \\
 f_2 - F_w &= m_3 a & v &= \sqrt{2ad} \\
 f_2 - m_3 g &= m_3 a & v &= \sqrt{2 \cdot 29.38 \cdot 5.606} \\
 a &= \frac{f_2 - m_3 g}{m_3} & v &= 18.15m/s \\
 a &= 29.38m/s^2
 \end{aligned}$$

We can also calculate for the direction of ball 3 (labeled as θ_3) as it leaves the mill:

$$\begin{aligned}
 \theta_2 &= 0.908 \\
 \therefore \theta_3 &= \frac{\pi}{2} - \theta_2 \\
 &= 0.663
 \end{aligned}$$

Ball 3 leaves the mill with a velocity of 18.15 m/s and an angle of 0.663 rad.

Phase 6

In this phase, ball 3 is accelerated by the mill, and thrown out, and performed a parabolic motion. In order to study a parabola, we need to divide the velocity into x and y components:

$$\begin{aligned}
 v_x &= \cos \theta_3 \cdot v \\
 &= \cos(0.663) \cdot 18.15 \\
 &= 14.302m/s \\
 v_y &= \sin \theta_3 \cdot v \\
 &= \sin(0.663) \cdot 18.15 \\
 &= 11.174m/s
 \end{aligned}$$

In a parabola motion, velocity of x-component is always constant, therefore for maximum height of parabola, we use the equation:

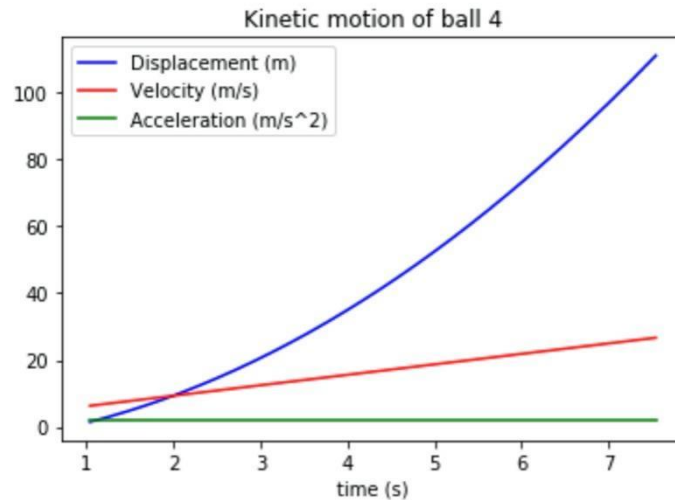
$$h = \frac{v_y^2}{2g} = \frac{11.174^2}{2 \cdot 9.8} = 6.371m$$

For displacement on x-axis, we use y-component velocity to calculate the time of the wholeparabola, and then use it to solve for distance linearly:

$$\begin{aligned}
 t &= \frac{v_y}{g} \cdot 2 & d_x &= v_x t \\
 &= \frac{11.174}{9.8} \cdot 2 & &= 14.302 \cdot 2.28 \\
 &= 2.28s & &= 32.616m
 \end{aligned}$$

Phase 7

In this phase, ball 4 is sliding down the slope. This phase starts at the moment ball 4 starts moving, and ends at the moment before ball 4 hits the surface. This is a similar case as phase 1, the same diagram can illustrate it.



Quadratic fit : $-3.79*x**2 + 3.43*x + 1.57$
 Derivative : $3.03*x + 3.13$
 Second derivative: 2.09

Figure 5 Graph of kinetic motion of ball 4 in phase 7

We have $a = 2.09$ m/s². Using the same formula derived in phase one, we can find its frictional coefficient:

$$a = \sin(\theta) \cdot g - \mu_k \cdot \sin(\theta) \cdot g$$

$$2.09 = \sin(\theta) \cdot g - \mu_k \cdot \sin(\theta) \cdot g$$

$$\mu_k = 0.313$$

Phase 8

In this phase, ball 4 has roll down to the bottom and was going to push the lever. The other side of the lever will push ball 5 down another slope. In order to calculate for its torque, we need first calculate the force exerted by ball 4 to the lever. The method is to find the acceleration of the lever. Since that the level was at rest initially, and then after been collided by ball 4 it started moving, we can calculate its acceleration by finding its final velocity, which is the final velocity of ball 4 after collision.

Here are some prerequisites. m_4 is mass of ball 4; m_5 is mass of ball 5; d_1 is distance moved by left side of lever, the distance that ball 4 pushed it; d_2 is distance moved by right side of lever, pushes the ball 5; r_1 is the distance from ball 4 to the pivot point; r_5 is the distance from ball 5 to the pivot point.

$$m_4 = 5kg$$

$$m_5 = 2kg$$

$$d_1 = 20.744m$$

$$d_2 = 0.475m$$

$$r_1 = 73.2m$$

$$r_2 = 17.69m$$

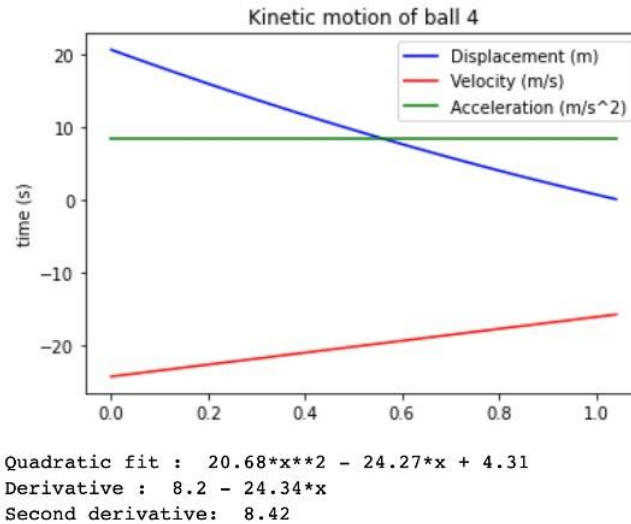


Figure 6 Graph of kinetic motion of ball 4 in phase 8

By importing the data (9) into JupyterLab, matplotlib generated the following graph:

From the graph, we can extract the data that the final velocity will be 16.46 m/s. Thus, we can use acceleration formula to calculate for the acceleration:

$$a = \frac{v^2}{2 \cdot d_1} = \frac{(16.46)^2}{2 \cdot 20.744} = 6.53 \text{ m/s}^2$$

With the acceleration calculated, we can find the force exerted by ball 4 to the lever f_1 ; Then we can calculate for the amount of torque in this system:

$$f_1 = m_4 \cdot a \quad \tau = f_1 r_1 \\ = 5 \cdot 6.53 \quad = 32.652 \cdot 73.2 \\ = 32.652 \text{ N} \quad = 2390.112 \text{ N} \cdot \text{m}$$

According to the conservation of momentum law, the force exerted by the lever to ball 5 will be:

$$f_2 = \frac{\tau}{r_2} = \frac{2390.112}{17.69} = 135.111 \text{ N}$$

With newton second law, we can then calculate for the acceleration of ball 5, and use acceleration formula to calculate its velocity as it left the lever:

$$a = \frac{f_2}{m_5} \quad v = \sqrt{2ad_2} \\ = \frac{135.111}{2} \quad = \sqrt{2 \cdot 67.555 \cdot 0.475} \\ = 67.555 \text{ m/s}^2 \quad = 8.011 \text{ m/s}$$

Phase 9

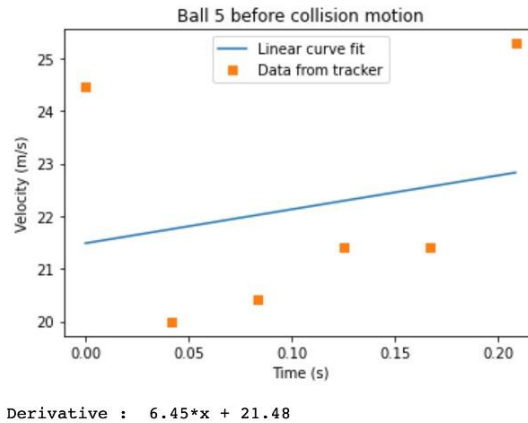


Figure 7 Graph of kinetic motion of ball 5 before collision

In this phase, ball 5 has roll down to the bottom and then have a elastic collision with ball 6. Ball 6 will be accelerated and eventually drop into the colorful box. There will be an inelastic collision happening. Importing data into JupyterLab, and analyze with matlabplot:

From analysis, we can see that velocity of the ball before it collides is 22.828 m/s. Then, analyze the data of ball 5 after the collision:

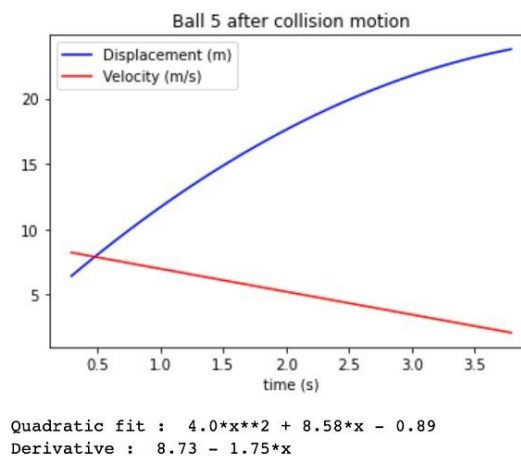


Figure 8 Graph of kinetic motion of ball 5 after collision

From the graph, we can see that the velocity of ball 5 decreased to 8.22 m/s. Then, we can analyze the motion of ball 6 after the collision by importing data of it into JupyterLab:

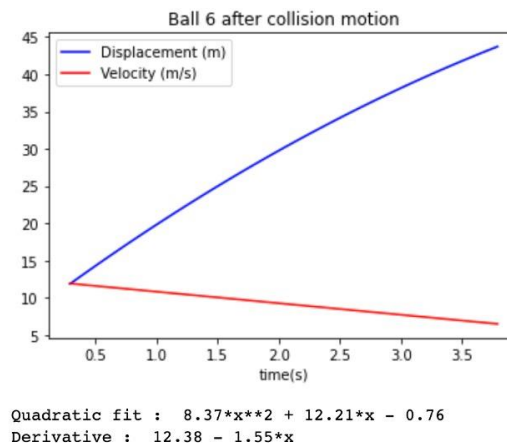


Figure 9 Graph of kinetic motion of ball 6 after collision

From those data, we can then analyze the velocity at different stages. v_{5i} represents the initial velocity of ball 5; v_{5f} represents the final velocity of ball 5 after collision; v_{6i} represents the initial velocity of ball 6; v_{6f} represents the final velocity of ball 6 after collision:

$$\begin{aligned} v_{5i} &= 22.828\text{m/s} \\ v_{6i} &= 0\text{m/s} \\ v_{5f} &= 8.22\text{m/s} \\ v_{6f} &= 11.928\text{m/s} \end{aligned}$$

We can also use the same data to graph for the momentum. Here we compare the momentum of ball 5 before and after the collision:

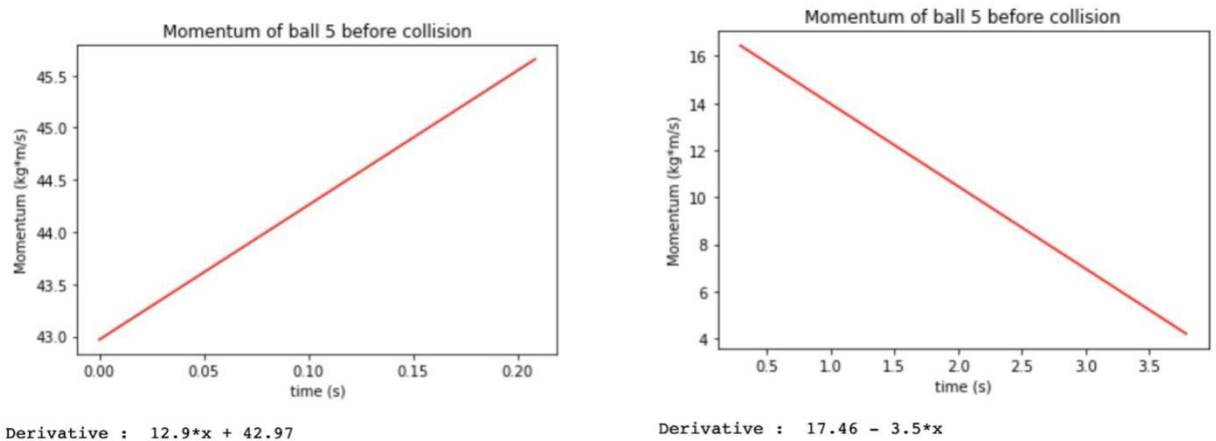


Figure 10 Graph of momentum of ball 5 of its collision

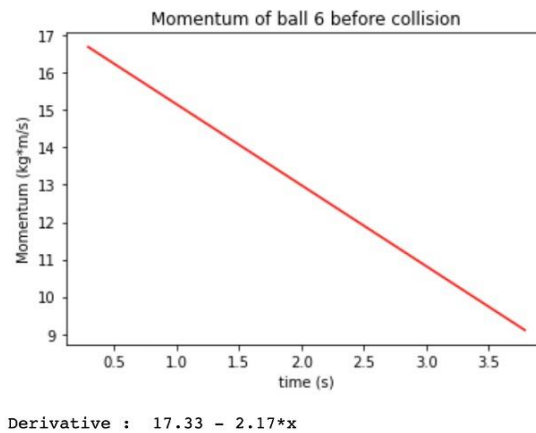


Figure 11 Graph of momentum of ball 6 before collision

Also for momentum of ball 6:

From those, we can get momentum for ball 5 before collision $P_{5 \text{ before}}$, momentum for ball 5 after collision $P_{5 \text{ after}}$, momentum for ball 6 before collision $P_{6 \text{ before}}$, momentum for ball 6 after collision $P_{6 \text{ after}}$; Then, we can compare the change of momentum in ball 5 and ball 6:

$$\begin{aligned} P_{5 \text{ before}} &= 45.667 \text{ kg} \cdot \text{m/s} & \Delta P_5 &= 45.667 - 16.439 \\ P_{5 \text{ after}} &= 16.439 \text{ kg} \cdot \text{m/s} & &= 29.218\text{kg} \cdot \text{m/s} \\ P_{6 \text{ before}} &= 0 & \Delta P_6 &= 17.963 - 0 \\ P_{6 \text{ after}} &= 17.963 \text{ kg} \cdot \text{m/s} & &= 17.963\text{kg} \cdot \text{m/s} \end{aligned}$$

We can find that the change of momentum in two objects are not identical. According to energy conservation law, energy can only be transferred but never be created or destroyed. In this case, the

kinetic energy was lost, it must be transferred into other form of energy. In elastic collision, usually the energy will be transferred into heat or other sources of energy. We can calculate its elasticity with their velocity:

$$\begin{aligned} e &= \frac{v_{1f} - v_{2f}}{v_{1i} - v_{2i}} \\ &= \frac{-8.22 - 11.928}{-22.828 - 0} \\ &= 0.883 \end{aligned}$$

In conclusion, for this inelastic collision, 88.3% of kinetic energy was conserved. The rest will be turned into internal heat or other energy.

5. Ending

The technique used in this project includes Blender 3D models and mechanical analysis, and also Python data processing. The analysis done here shows the ability of this type of calculation. Those technologies would be useful in engineering field that can simulate the physical process of some machines.

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