

Research on Lane Change Decision Model of Intelligent Connected Vehicles in Multi-Vehicle Game

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Abstract: This paper builds a decision model for intelligent networked vehicles (ICV) lane changing behavior based on the idea of game theory. Focusing on the complex traffic conditions of three-lane highway with multi-vehicle interaction and competition, we solve the ICV lane change decision problem by pairing ICVs two by two, formulating a multi-group game and comparing the game gain results to find the best. Based on Time-to-Collision (TTC), we design the payoff function of this decision model game so that the game results in a pure strategy Nash equilibrium (PSNE), and obtain the unique optimal lane change decision for a given state by designing the objective function of the lane change game. The joint simulation by PreScan/Simulink/CarSim software verifies the rationality of the payoff function and the validity and applicability of the lane-changing model.

Keywords: Intelligent Connected Vehicle, Game theory, Nash equilibrium, Lane change decision

1. Introduction

The continuous development of Telematics and Vehicle-Circuit Collaboration (VCC) technology makes real-time information exchange and cooperative control between vehicles possible^[1]. Compared with self-driving cars, intelligent networked vehicles can exchange their own state with the surrounding vehicles and release their driving intentions in real time, which greatly reduces their perception and reaction time. The information exchange and collaboration between intelligent networked vehicles can improve the efficiency of lane changing and avoid the occurrence of collisions.

Lane-changing behavior is the basic behavior of the vehicle driving process, the vehicle needs to obtain a better driving environment in the driving process to produce the intention of lane-changing and thus lane-changing, according to the vehicle's motivation to lane-changing behavior is divided into free lane-changing and forced lane-changing^[2].

In this paper, a new free lane-changing decision-making scheme is proposed based on game-theoretic ideas by focusing on the complex traffic conditions of highway three-lane multi-vehicle interaction competition with intelligent networked vehicles as the carrier. Different from the computationally complex multi-vehicle game lane-changing decision-making scheme in the current literature, without affecting the reliability of the solution, this paper disassembles the multi-vehicle game into several sets of two-two pairwise games, and aims at the pure-strategy Nash equilibrium solution, designs a computationally simple and fast decision-making payoff function algorithm that effectively reduces the computational burden of the on-vehicle computer, and puts forward a method for quickly determining the optimal decision from the solutions to multiple games, avoiding the computation of the optimal decision, and avoiding the computation of the optimal decision. A method to determine the optimal decision from multiple game solutions is proposed, which avoids the computationally complex n-dimensional game solutions. Considering the unexpected conditions that may occur in the actual driving environment, the decision model is allowed to reconsider its decision by repeating the game with the newly collected real-time vehicle data exchanged between intelligent networked vehicles at a certain time period, in order to cope with the unexpected conditions and to improve the applicability of the lane-changing decision model.

2. Game Theory

Game theory is a mathematical mathematical theory and methodology for studying optimal decision

making in the interaction of rational intelligences. The goal of a participant in a game is to obtain the highest possible payoff by choosing an appropriate strategy, taking into account that all other participants will also try to maximize their own payoffs.

The game payoffs are represented by a payoff matrix, as shown in Figure 1. The game has two participants A and B, each with two different strategies. Participant A can execute either U or D. Participant B can execute either L or R. Participants A and B will receive gains a_{ij} and $b_{ij}(i=1,2; j=1,2)$ for a particular combination of strategies.

		B	
		L	R
A	U	a_{11}, b_{11}	a_{12}, b_{12}
	D	a_{21}, b_{21}	a_{22}, b_{22}

Figure 1: Game payoff matrix.

The set of strategies available to participant i is denoted as $A_i = \{s_i^1, \dots, s_i^n\}$, where s_i^j denotes the j th strategy available to player i [3]. The symbol s_i is used to denote the strategy of participant i . The symbol s_{-i} is used to denote the strategies adopted by all other players except participant i , i.e., $s_{-i} = \{s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_N\}$.

The gain that participant i obtains in the game is denoted as $J_i(s) = J_i(s_i, s_{-i})$. Define strategy s_i^* as the best strategy of participant i for a given combination of strategies s_{-i} , and the gain of participant i under the best strategy $J_i(s_i^*, s_{-i}) \geq J_i(s_i, s_{-i})$. A pure strategy Nash equilibrium (PSNE) is said to be achieved in the game if all participants make the best strategy for each other, i.e., all participants choose strategies s_i^*, s_{-i}^* such that inequality (1)(2) holds.

$$J_i(s_i^*, s_{-i}^*) \geq J_i(s_i, s_{-i}^*) \tag{1}$$

$$J_i(s_i^*, s_{-i}^*) \geq J_i(s_i^*, s_{-i}) \tag{2}$$

When participants are in NE, no participant can unilaterally change his or her strategy to increase his or her payoff, and NE results in maximizing the payoffs of the game to each other.

3. Game Formulation For Highway Lane-Changing

3.1. Lane-Changing State Variables

Vehicle (E) is traveling on a multi-lane highway shared by several other vehicles as shown in Figure 2. The vehicles around E are denoted as left-back (LB), right-back (RB), and center-front (MF) vehicles. The state variables are defined as the longitudinal distance d and speed v of each vehicle with respect to E. The symbol d_i is used to denote the longitudinal distance between E and vehicle i , where $i \in \{MF, LB, RB\}$. v_i denotes the speed of vehicle i , where $i \in \{E, MF, LB, RB\}$.

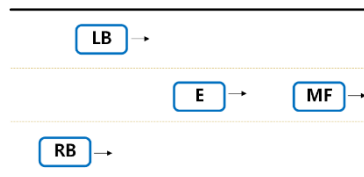


Figure 2: Lane-changing scenario in a multi-lane highway

For this particular scenario, there are three participants making decisions, namely E, LB, and RB. The car in front of E is unlikely to respond to E's demand to change lanes. Thus, the set of participants in the game is $N = \{E, LB, RB\}$.

E has three strategies, keeping the current lane (K), changing lanes to the left (L) and changing lanes to the right (R). Thus, $A_E = \{K, L, R\}$. LB has two strategies, allowing lane change (Y) and disallowing lane change (N). RB has the same strategy as LB, i.e., $A_{LB} = A_{RB} = \{Y, N\}$.

3.2. Pairwise Games Formulation

In existing studies, the three-vehicle game uses three-dimensional arrays as the payoff matrix, which is computationally complex. In this paper, the three-vehicle game is decomposed into two sets of two-vehicle games to solve this problem, i.e., let E play with LB and RB respectively. E plays the first set of games with LB to decide whether it can change lanes to the left or not, and the corresponding payoff matrices are shown in Figure 3. At the same time, E plays the second set of games with RB to decide whether it can change lanes to the right, and the corresponding payoff matrix is shown in Figure 4. If the solution of both sets of games is that E keeps driving in the current lane, then this is the optimal strategy for E. If the solution of one and only one of the sets of games is to change lanes, then E changes lanes to the corresponding side; if the solution of both sets of games is that E changes lanes, then E changes lanes to the side with the higher payoff. The optimal strategy for E can be quickly derived by comparing the payoff values of the two sets of games.

		LB				RB	
		Y	N			Y	N
E	L	a_{11}^1, b_{11}^1	a_{12}^1, b_{12}^1	E	R	a_{11}^2, b_{11}^2	a_{12}^2, b_{12}^2
	K	a_{21}^1, b_{21}^1	a_{22}^1, b_{22}^1		K	a_{21}^2, b_{21}^2	a_{22}^2, b_{22}^2

Figure 3: Game 1. E plays against LB only.

Figure 4: Game 2. E plays against RB only.

3.3. Repeated Play with Changing Payoffs

By appropriately designing the payoff function, the solution of the game generates optimal decisions in each given state. However, the state of vehicles in the highway lane changing game may change suddenly, so the optimal strategy at a given moment may not be optimal after some time. In order to cope with unexpected situations and avoid performing unsafe maneuvers, this paper designs a decision algorithm that allows E to repeat the game every 0.1 seconds, so that E can quickly reconsider the decisions made based on the most recently obtained data.

4. Payoff Design

4.1. Paper Times-to-Collision as Decision Variables

The TTC between E and any other vehicle i is represented by $T_i^{[4-5]}$.

$$T_{MF} = \frac{d_{MF}}{v_E - v_{MF}} \tag{3}$$

$$T_{LB} = \frac{d_{LB}}{v_{LB} - v_E} \tag{4}$$

Figure 5 shows the definition of collision times for E and other vehicles in the center and left lanes. Similarly, the same definition is given for E and right lane vehicles, which will not be repeated in this paper.

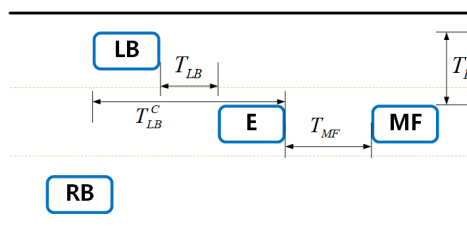


Figure 5: Definition of variables for the agents payoff.

In addition to TTC, define T_H as the lane change time of a human-driven car on a highway, and according to the literature^[6], $T_H = 5.6s$ is selected.

Define T_{LB}^C as the time when LB overtakes E, l_{LB} as the length of LB and l_E as the length of E.

$$T_{LB}^C = \frac{d_{MF} + l_{LB} + l_E}{v_{LB} - v_E} \quad (5)$$

Define T_{MF}^I as the impending collision time (ITTC) between E and the vehicle MF, i.e., the value that the TTC will have when E is traveling at its desired speed $v_{E,d}$.

$$T_{MF}^I = \frac{d_{MF}}{v_{E,d} - v_{MF}} \quad (6)$$

4.2. Payoffs for Lane-Changing Games

Due to the symmetry of the two sets of games in Figures 4 and 5, the following focuses on the design of the game payoff function between E and LB.

If $T_{MF}^I < T_H$, E switches to the left, and if $T_H > T_{LB}$, E stays in the original lane; if $T_{LB}^C < T_H$, E is not allowed to switch lanes, and if $T_{LB} > T_H$, E is allowed to switch lanes. Based on the above rule, define:

$$a_{22} = (T_{MF} - T_{MF}^I) + (T_H - T_{LB}) \quad (7)$$

$$a_{11} = a_{12} = T_H - T_{MF}^I \quad (8)$$

$$b_{11} = T_{LB}^C - T_H \quad (9)$$

$$b_{12} = b_{22} = T_{LB} - T_H \quad (10)$$

In any case, K/Y is an undesired outcome of the game, i.e., E's choice to keep the original lane strategy and LB's choice to allow a lane change is never an optimal strategy combination. Therefore, define:

$$a_{21} = a_{11} - \varepsilon \quad (11)$$

$$b_{21} = b_{12} - \varepsilon \quad (12)$$

ε is an arbitrary positive constant, and $\varepsilon = 1$ is chosen in this paper. a_{21} and b_{21} are designed to prevent K/Y from being a Nash equilibrium of the game. Moreover, the design ensures that there must be a pure strategy Nash equilibrium (PSNE) in the game.

4.3. Objective Function for Lane-Changing Games

With the above payoff function, the PSNE in the game can be quickly found according to the following rules.

- 1) When $a_{22} \geq a_{12}, b_{11} \geq b_{12}$, PSNE is L/Y and K/N;
- 2) When $a_{22} < a_{12}, b_{11} < b_{12}$, PSNE is L/N;
- 3) when $a_{22} \geq a_{12}, b_{11} < b_{12}$, PSNE is K/N;
- 4) When $a_{22} < a_{12}, b_{11} \geq b_{12}$, PSNE is L/Y.

There are two PSNEs in Rule 1), and the lane-switching decision model requires that the final decision is uniquely determined. This problem is solved by designing the objective function.

According to the definition of Nash equilibrium, when the participants are in NE, no participant can unilaterally change his/her strategy to increase his/her payoff. The outcome of NE is to maximize the payoffs of the game to each other. Based on this, define the objective function:

$$J_{ij}^1 = \max(a_{ij}^1 + b_{ij}^1) \quad (13)$$

J_{ij}^1 denotes the maximum value of the sum of the gains of the E and LB games under the PSNE

strategy, and the lane-changing decision model selects the optimal strategy based on the solution of the objective function. Similarly, the game between E and RB also results in a solution of the objective function J_{ij}^2 . By comparing the solutions of the objective functions of the two sets of games in search of an optimal solution, the final lane-changing decision of E is arrived at, and then the vehicle E executes to carry out the corresponding lane-changing operation.

5. Simulated Implementation

Based on PreScan/Simulink/CarSim software, a joint simulation platform is constructed, and three working conditions are set up to simulate and verify the validity and applicability of the game lane-changing decision-making model in different lane-changing scenarios.

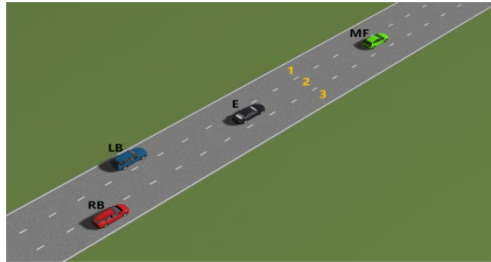


Figure 6: Three-lane highway scene.

In a three-lane highway scenario, as shown in Figure 6, E is traveling in the middle lane (lane 2), and the vehicle ahead, MF, is traveling too slowly to meet E's speed expectation (in this simulation, the desired speed of the controlled vehicle E is set to be $v_{E,d}=110\text{km/h}$), and E generates the motivation to change lanes. The parameter configuration of each vehicle is shown in Table 1.

Table 1: Vehicle parameter configuration.

Vehicle	Lane	Vehicle model	Vehicle length (m)
E	2	Audi A8 Sedan	5.21
MF	2	Fiat Bravo Hatchback	4.34
LB	1	BMW X5 SUV	4.79
RB	3	Ford Focus Stationwagon	4.56

Case 1: The initial information of the vehicle is shown in Table 2.

Table 2: Initial vehicle information for Case 1.

Case 1	E	MF	LB	RB
v_i (km/h)	90	80	100	100
d_i (m)	0	25	30	40

Based on the initial state information of each vehicle in Table 2, the vehicle simulation test is launched, and the payoffs of vehicles is shown in tables 3 and 4.

Table 3: The payoffs of Game 1 in Case 1.

	LB	Y	N
E			
L		2.602,8.788	2.602,5.191
K		1.602,4.191	0.804,5.191

Table 4: The payoffs of Game 2 in Case 1.

	RB	Y	N
E			
R		2.602,12.303	2.602,8.788
K		1.602,7.788	-2.793,8.788

From Table 3, the PSNE of Game 1 is L/Y and $J_{11}^1=11.39$, so the decision of vehicle E in Game 1 is to change lane to the left. From Table 4, the PSNE for Game 2 is R/Y and $J_{22}^2=14.905$, so the decision of vehicle E in Game 2 is to change lane to the right. Because of $J_{11}^1 < J_{22}^2$, the final decision of the

game lane change decision model is for vehicle E to change lane to the right, and the system outputs the target lane 3 as shown in *Figure 7*.

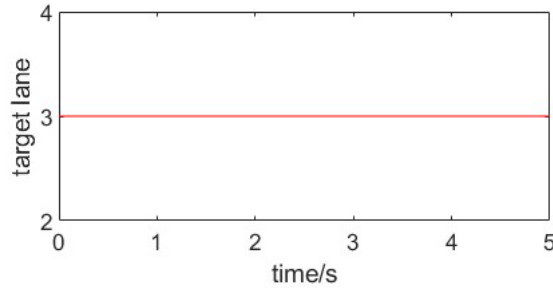
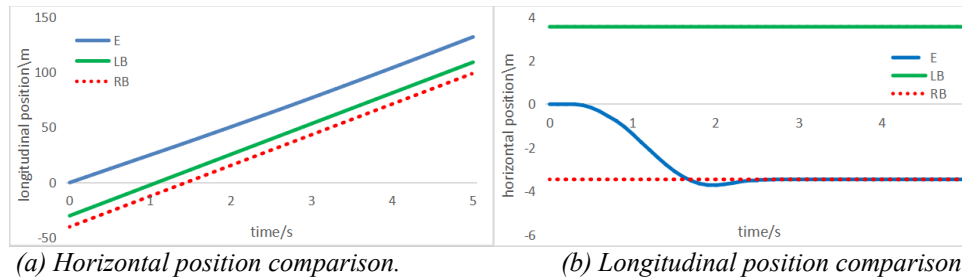


Figure 7: Target Lane for Vehicle E in Case 1.

A comparison of the horizontal and longitudinal positions of vehicle E, LB and RB traveling is shown in *Figure 8*. As can be seen in *Figure 8(a)*, Vehicle E received a decision command to change lanes to the right (lane 3) and then started the lane changing behavior to the right lane. *Figure 8(b)*, it can be seen that E and LB always maintain a safe distance during the lane change and no collision occurs.



(a) Horizontal position comparison.

(b) Longitudinal position comparison.

Figure 8: Comparison of horizontal and longitudinal positions of vehicles in Case 1.

Case 2: Increase the speed of vehicle RB from 100km/h to 120km/h, and keep the rest of the variables the same as case 1 to start the simulation. The payoffs of vehicles is shown in tables 5 and 6.

Table 5: The payoffs of Game 1 in Case 2.

	LB	Y	N
E			
L		2.602,8.788	2.602,5.191
K		1.602,4.191	0.804,5.191

Table 6: The payoffs of Game 2 in Case 2.

	RB	Y	N
E			
R		2.602,0.375	2.602,-0.798
K		1.602,-1.798	6.793,-0.798

From *Table 5*, the PSNE of Game 1 is L/Y and $J_{11}^1 = 11.39$, so the decision of vehicle E in Game 1 is to change lanes to the left. From *Table 6*, the PSNE for Game 2 is R/Y and $J_{22}^2 = 5.995$, so the decision of vehicle E in Game 2 is to maintain the current lane. Because of $J_{11}^1 > J_{22}^2$, the final decision of the game lane change decision model is for vehicle E to change lane to the left, and the system outputs the target lane 1 as shown in *Figure 9*.

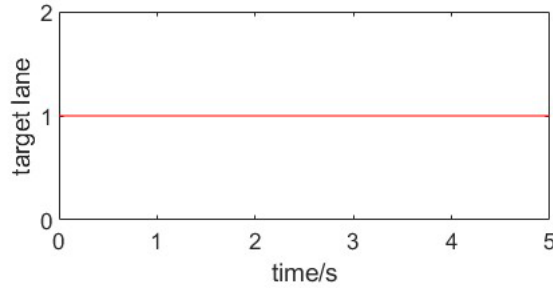
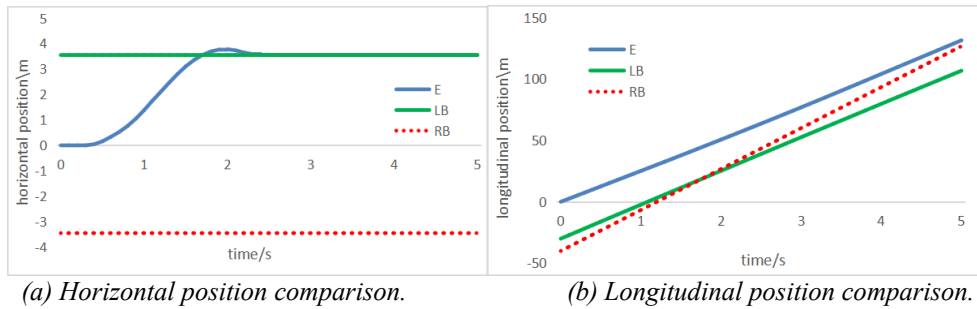


Figure 9: Target Lane for Vehicle E in Case 2.

A comparison of the horizontal and longitudinal positions of vehicle E, LB and RB traveling is shown in Figure 10. As can be seen in Figure 10(a), Vehicle E received a decision command to change lanes to the left (lane 1) and then started the lane changing behavior to the left lane. From Figure 10(b), it can be seen that E and LB always maintain a safe distance during the lane change and no collision occurs.



(a) Horizontal position comparison.

(b) Longitudinal position comparison.

Figure 10: Comparison of horizontal and longitudinal positions of vehicles in Case 2.

Case 3: The initial state information of each vehicle is kept the same as that of Case 1. In order to test the applicability of the repeated game function of this game lane-changing model, vehicle RB is allowed to travel at an initial speed of 100km/h for 0.2s and then suddenly accelerate at an acceleration of 4m/s².

As shown in the simulation results in Figs. 11 and 12, vehicle E firstly changes lanes to the right, then starts to change lanes to the left after 1.5s, and finally completes the behavior of changing lanes to the left.

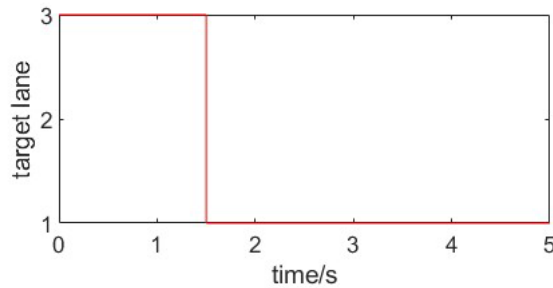


Figure 11: Target Lane for Vehicle E in Case 3.

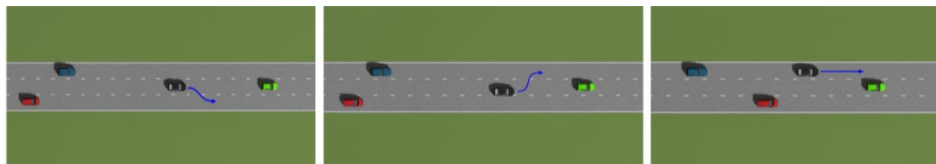


Figure 12: Lane change scenario in Case 3.

At the beginning of the simulation, the model outputs the decision instruction to change lane to the right (lane 3), and vehicle E starts to change lanes to the right. 0.2s later vehicle RB suddenly accelerates. The game lane changing model repeats the game every 0.1s. The payoffs for each vehicle at 1.5s are shown in Tables 7 and 8. $J_{11}^1 = 170.876$ in Game 1, $J_{11}^2 = 6.704$. $J_{11}^1 > J_{11}^2$, so the final decision of the

game lane changing model for vehicle E to the left (lane 1) lane changing. At this time, vehicle E is located in the position of the right lane demarcation line, and receives the decision instruction to change lane to the left, and then vehicle E successfully completes the behavior of changing lanes to the left.

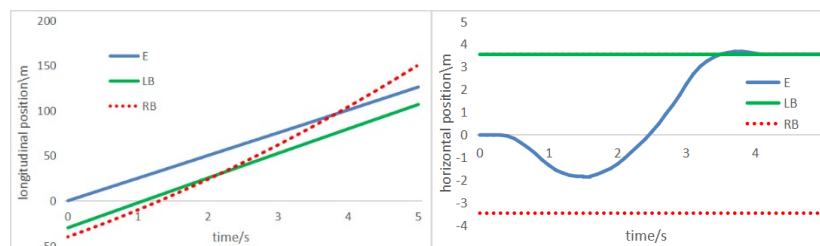
Table 7: The payoffs of Game 1 in Case 3.

E \ LB	Y	N
L	3.371,167.505	3.371,122.051
K	2.371,121.051	-121.064,122.051

Table 8: The payoffs of Game 2 in Case 3.

E \ RB	Y	N
R	3.371,3.333	2.602,1.386
K	2.371,0.386	-0.382,1.386

As can be seen in *Figure 13*, Vehicle E and RB and LB always maintain a safe distance during the lane changing process and no collision occurs.



(a) Horizontal position comparison.

(b) Longitudinal position comparison.

Figure 13: Comparison of horizontal and longitudinal positions of vehicles in Case 23.

6. Conclusions

This paper utilizes the advantages possessed by intelligent networked vehicles and uses them as a carrier to conduct research on free lane-changing decision-making model of intelligent networked vehicles for highway scenarios, and puts forward a game-theoretic scheme to realize optimal decision-making in multi-lane and multi-vehicle lane-changing scenarios, which greatly reduces the computational complexity required for solving the multi-player N-dimensional game. A multi-vehicle game lane-changing decision-making model is designed, and a joint simulation platform is built based on three simulation software, PreScan/Simulink/CarSim, and the simulation verifies the reasonableness of the designed game payoff function, the validity of the multi-vehicle game lane-changing decision-making model, and the applicability of the game to deal with the unexpected situation.

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