

Research on UAV Formation Positioning Method Based on Relative Angle Information

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Abstract: In this paper, the localization problem of UAVs flying at the same altitude is investigated. Firstly, a rectangular coordinate system based on two signal transmitting UAVs is established, and four different choices of signal transmitting UAVs are identified. Then, several circular orbit equations are constructed by analyzing the relative angles between the signal transmitting UAV and the receiving UAV and the fixed positions of the signal transmitting UAV. By solving these equations together, the current position of the UAV with deviation can be accurately determined and the UAV positioning can be realized. Further, this study analyzes the relative positions of the UAV formation in the ideal state, and obtains the directional information α_1 and α_2 , which should be captured when the UAV receives the signal at the target position. Because the position deviation of the receiving signal UAV is small, the relative Angle information captured by the receiving signal UAV changes only in a small range, which makes it possible to determine the number of the signaling UAV by comparing the expected relative Angle information in the four cases. Finally, the UAV positioning model is used to determine the location of all UAVs, and the validity of the model is verified by geometric theorems such as sine and cosine. The results show that only one additional UAV can transmit signals to achieve effective positioning of the entire UAV formation. This discovery provides a new method for UAV formation positioning and lays a foundation for further research and application.

Keywords: UAV formation, Positioning, Relative Angle information, Signal emission, Geometry theorem

1. Introduction

1.1 Research Background

UAV group has developed rapidly in recent years. As an excellent air combat platform, UAV group can carry all kinds of equipment and perform various tasks. UAV formation collaborative control methods can be broadly divided into centralized control and distributed control [1]. Distributed control has the characteristics of high error tolerance and strong plasticity [2]. In this case, the UAV adopts distributed control, and the direction information received by the UAV receiving signals is stipulated as follows: the Angle between the UAV and any two UAVs transmitting signals (as shown in Figure 1). For example, drones numbered FY01, FY02, and FY03 transmit signals, and drones numbered FY04 receive directional information of α_1 , α_2 , and α_3 .

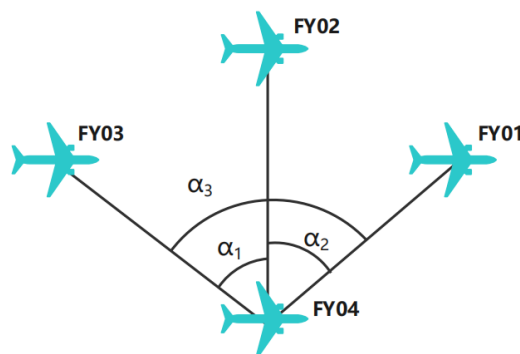


Figure 1: Diagram of the directional information received by the drone

1.2 Research Problems

Assume that 10 UAVs form a circular formation, the UAVs numbered FY01 ~ FY09 are evenly distributed on a certain circle, and the UAVs numbered FY00 are located in the center of the circle (Table 1). This study aims to solve the following problems:

(1) The UAV FY00 and any two UAVs with unbiased positions in the formation are used as the signal transmitting source, and the UAVs with slightly biased positions in the rest of the formation are established according to the directional Angle information in the received signals.

(2) In addition to receiving signals launched by FY00 and FY01, UAVs with position deviation in the formation can also receive direction information launched by other UAVs with unknown numbers but accurate positions. It needs to be solved that several such UAVs are also needed as signal sources to achieve effective positioning of the overall formation.

Table 1: Initial position of the drone

Drone number	Polar coordinates (m, °)	Drone number	Polar coordinates (m, °)
0	(0,0)	5	(98,159.86)
1	(100,0)	6	(112,199.96)
2	(98,40.10)	7	(105,240.07)
3	(112,80.21)	8	(98,280.17)
4	(105,119.75)	9	(112,320.28)

2. Research methods

The goal of this research is to select a certain number of signal transmitting UAVs, so that the signal receiving UAVs can adjust their positions according to the differences in direction information, and finally realize that 9 UAVs are evenly distributed on the circumference centered on FY00.

(1) It is stipulated that FY00 and the other two UAVs in the formation fly in the correct position, and they not only know the number, but also can send direction signals to the UAVs in the wrong position. In order to better describe the position of the UAV in the solution process, the plane coordinate system can be introduced. Considering the different choices of the other two signal transmitting UAVs, the classification discussion is considered. The location of the UAV with deviation can be realized by using the accuracy of the position of the UAV transmitted by the signal and the Angle relationship between the UAV.

(2) Only the numbers of the two signal transmitting drones FY00 and FY01 are known, but the numbers of the unknown signal transmitting drones can be deduced backwards according to the direction information captured by the receiving signal drones; The deviation position of UAV can be obtained by using the idea of solving triangle, and the positioning of UAV can be realized effectively.

3. Model establishment and solution

3.1 Research on UAV formation positioning and distribution optimization methods based on directional information

In the case of 10 UAVs flying in their respective positions without deviation, observing the position of the other 9 UAVs relative to it from the perspective of FY00 UAV, we can find that the interval between them is 40° evenly distributed on the circumference, regardless of the radius of the circumference. What we do know is that each drone must be on a ray starting with FY00. In order to better describe the position information of each UAV, this paper stipulates that the direction of FY00→FY01 is the positive X-axis, and the vertical line is the Y axis. Therefore, it can be seen that the orbit of UAV FY02 should be on the ray where the positive X-half axis is rotated 40° counterclockwise. The orbit FY03 is supposed to be on a ray that rotates 80° counterclockwise on the positive half axis of X. The FY0x drone (1 < x ≤ 9) was supposed to fly in an orbit rotated counterclockwise by 40(x-1) on the positive x-half axis. On the ray it's in. Just by obtaining the position information and number of FY00 UAV and any other UAV, the coordinate system can be naturally established in space.

Since there are two drones capable of transmitting signals in addition to the FY00 drone, the model can be divided into four scenarios: ①FY00, FY0x, FY0(x+1) drones can transmit signals, ②FY00,

FY0x, FY0 (x+2) drones can transmit signals, ③FY00, FY0x, FY0 (x+3) drones can transmit signals, (4) FY00, FY0x, FY0 (x+4) unmanned aerial vehicles can transmit signals. In order to achieve visualization, the following case is taken as an example and described in Figure 2.

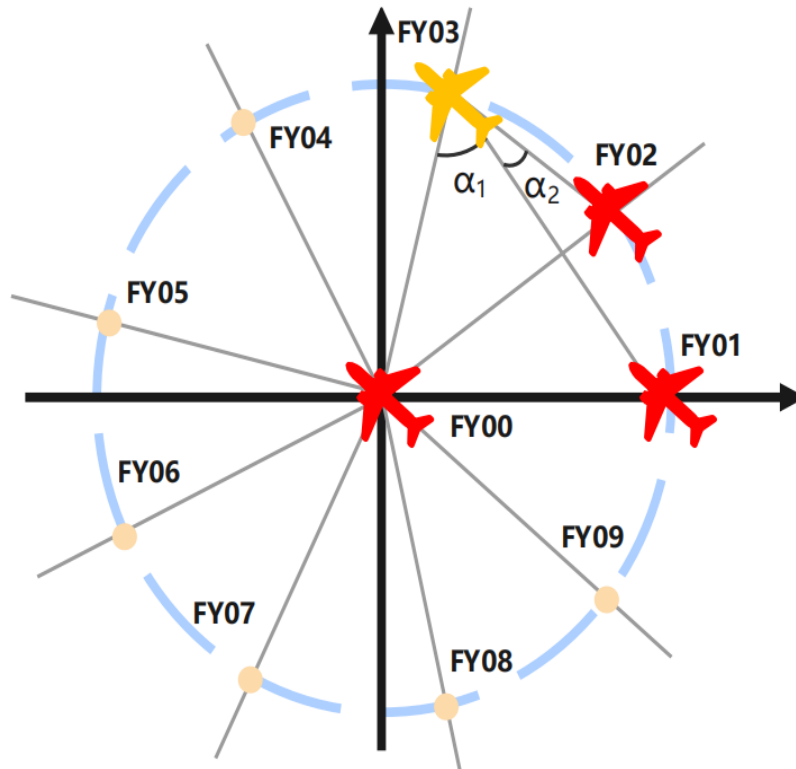


Figure 2: The establishment of the coordinate system and the explanation of the Angle information

For the three UAVs that transmit signals, their flight position is correct. By capturing the time t from the signal sent by UAVs FY00 to the signal captured by any other UAVs and the speed v of electromagnetic wave propagation in the air, the circular radius $r=v \times t$ of the UAVs formation at this time can be calculated. Therefore, the position of UAV FY0x should be on its corresponding track and the distance distance of UAV FY00 is r , so as to obtain the absolute position of each UAV in the coordinate system, as shown in Table 2 below.

Table 2: Cartesian coordinates when the UAV is at the target position

Drone number	Rectangular coordinate	Drone number	Rectangular coordinate
FY00	(0,0)	FY01	(r,0)
FY02	(rcos40°,rsin40°)	FY03	(rcos80°,rsin80°)
FY04	(rcos120°,rsin120°)	FY05	(rcos160°,rsin160°)
FY06	(rcos200°,rsin200°)	FY07	(rcos240°,rsin240°)
FY08	(rcos280°,rsin280°)	FY09	(rcos320°,rsin320°)

In the four cases of ①, ②, ③, and ④, $x=1$ is taken as an example to list the positions of the UAs to be sought relative to FY0x and FY0 (x+m) UAs (in which, m equals 1, 2, 3, and 4 respectively in these four cases), and the relative position relationship between the passive receiving signal UAs and the two transmitting signal UAs except FY00 can be obtained. As shown in Table 3 below.

$$\left\{ \begin{array}{l} \alpha_1: \text{The Angle between the desired UAV and FY00 connection} \\ \quad \text{and the desired UAV and FY01 connection} \\ \alpha_2: \text{The Angle between the desired UAV and FY01 connection} \\ \quad \text{and the desired UAV and FY0(1+m) connection} \end{array} \right.$$

Table 3: Relative Angle information of UAV at target position

Situation ①			Situation ②		
Drone should be in position (Relative to FY01 and FY02)			Drone should be in position (Relative to FY01 and FY03)		
Drone number	$\alpha 1(^{\circ})$	$\alpha 2(^{\circ})$	Drone number	$\alpha 1(^{\circ})$	$\alpha 2(^{\circ})$
FY03	50	20	FY02	70	140
FY04	30	20	FY04	30	40
FY05	10	20	FY05	10	40
FY06	10	10	FY06	10	40
FY07	30	10	FY07	30	40
FY08	50	20	FY08	50	40
FY09	70	20	FY09	70	40
Situation ③			Situation ④		
Drone should be in position (Relative to FY01 and FY04)			Drone should be in position (Relative to FY01 and FY05)		
Drone number	$\alpha 1(^{\circ})$	$\alpha 2(^{\circ})$	Drone number	$\alpha 1(^{\circ})$	$\alpha 2(^{\circ})$
FY02	70	120	FY02	70	100
FY03	50	120	FY03	50	100
FY05	10	60	FY04	30	100
FY06	10	60	FY06	10	80
FY07	30	60	FY07	30	80
FY08	50	60	FY08	50	80
FY09	70	60	FY09	70	80

After determining the number and position of the three transmitting signal UAVs, the remaining UAVs FY0x can receive signals to determine their unique position in the coordinate system, the specific process is as follows:

Given the coordinates of two points A (x_1, y_1), B (x_2, y_2) and the Angle α_1 between FY0x and the line between the two points, the distance between AB can be obtained from the distance formula between two points $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, as shown in Figure 3. Two fixed circles can be drawn from two fixed points and an Angle. When α_1 is an acute Angle, the radius of the circle $r = \frac{d}{2 \sin \alpha_1}$ can be obtained by constructing a right triangle. Assuming that the equation of the circle is $(x - a)^2 + (y - b)^2 = r^2$, by substituting the coordinates of AB, two sets of solutions can be obtained as a1, b1 and a2, b2. At this time, the position trajectory of FY0x is on the superior arc of the two ends of the circle divided by AB.

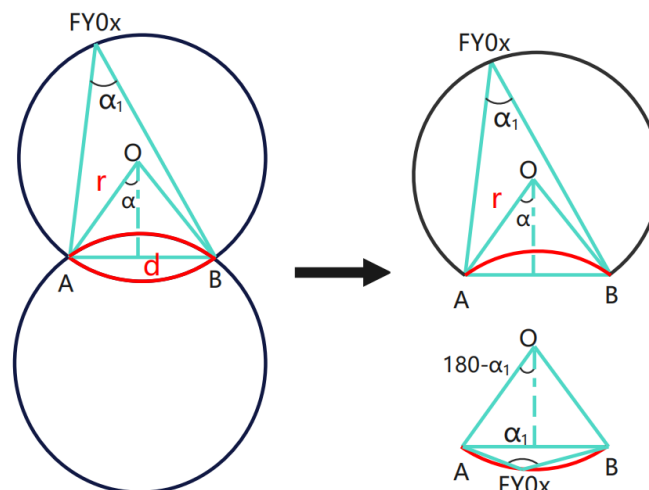


Figure 3: Relative position of UAV

Similarly, when the detected α_1 is obtuse Angle, in addition to $r = \frac{d}{2 \sin \alpha_1}$, the motion trajectory of FY0x also changes to the inferior arc of the two ends of the circle divided by AB. The trajectory formula of the superior arc and the inferior arc are also explained accordingly:

As shown in Figure 4, the dividing line between superior arc and inferior arc is line AB:

$$(x - x_1)(x_1 - x_2) - (y - y_1)(y_1 - y_2) = 0 \tag{1}$$

When the coordinates of the center of the circle (x_1, y_1) are substituted into the equation of the line greater than 0, the points on the superior arc are the points on the circle into the equation of the line greater than 0, and vice versa, the corresponding formulas of the superior arc and the inferior arc can be obtained, that is, the trajectory formula of FY0x obtained from α_1 .

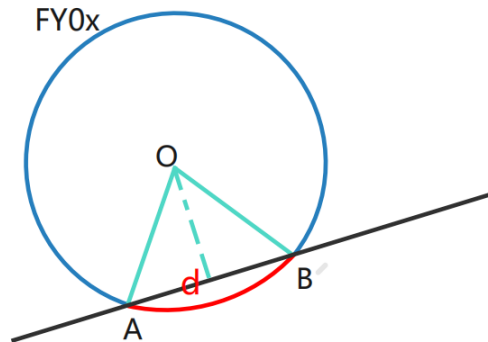


Figure 4: Division of superior arc and inferior arc

Three angles can be obtained from three UAVs transmitting signals, and three sets of trajectory equations can be obtained. It is known that there is no deviation in the position of UAVs transmitting signals in the problem, that is, there is no collinear situation of three UAVs. As shown in Figure 5, a unique position point, that is, the position of UAVs receiving signals, can be obtained through three sets of trajectory equations.

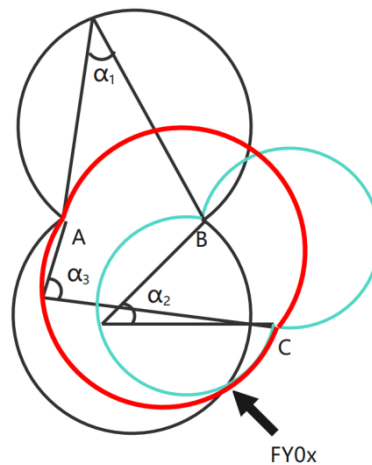


Figure 5: Determine the location of the drone receiving the signal

In the case of establishing the coordinate system, after the position of the UAV with deviation is calculated, the moving plan for correcting the error [3-5] can be obtained according to the correct position of the UAV in the queue, so as to realize the effective positioning and position correction of the UAV.

3.2 Research on the application of signal launching UAV selection and positioning strategy in UAV formation

It is assumed that the UAV can distinguish the different signal angles generated by FY00 and FY01 when receiving the signal (Figure 6). Assume that the signal Angle generated by FY00 and the unknown signal source is α_3 , and the signal Angle generated by FY01 and the unknown signal source is α_2 . When the offset point and the unknown signal source are on the same side, $\alpha_3 > \alpha_2$; On the opposite side, $\alpha_3 < \alpha_2$.

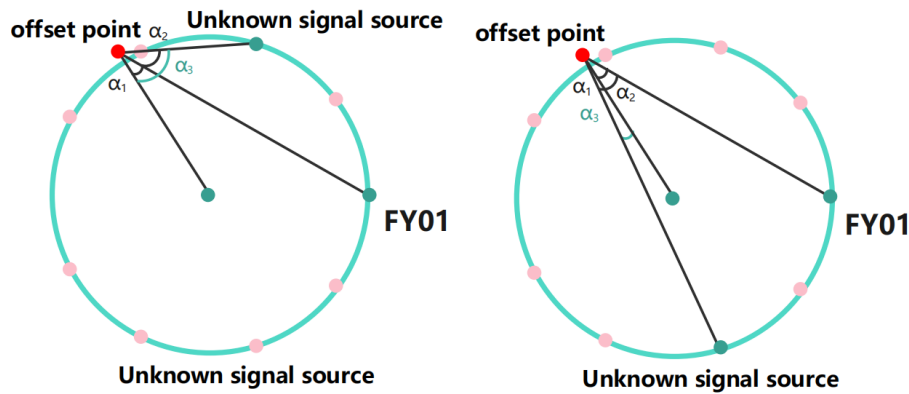


Figure 6: The difference of signal Angle between the same side and the other side

It is necessary to solve that in addition to FY00 and FY01, several UAVs need to transmit signals to achieve effective positioning of UAVs. First, no new signal source is introduced. What is known in the question is the Angle between the offset UAV and FY00 and FY01, that is, the signal Angle; According to Table 3, the Angle of each UAV relative to FY01 when it is in the correct position can be obtained, that is, the positioning Angle. In particular, when the size of the signal Angle is exactly the size of the positioning Angle, it is known that the position of the UAV to be measured has no deviation.

Then assume that a new signal transmitting source is introduced, that is, a UAV with unknown number and unbiased position. Since the number is unknown, coordinate positioning cannot be adopted. After introducing the new transmitting source, the UAV to be tested will acquire two more signal angles, as shown in FIG. 7. That is, the situation of the new launch source and the UAV to be tested on the same side and the other side of the FY00 and FY01 connection segment.

At the same time, the number of the newly introduced signal source can be obtained by using the α_2 Angle of different combination with FY01 in Table 3, combined with the number of the point to be measured. As shown in Figure 7, two triangles can be obtained according to known angles. In order to obtain the exact position of the point to be measured, the polar coordinate system is established with FY00 as the origin, and the position of FY01 is $(R_m, 0^\circ)$.

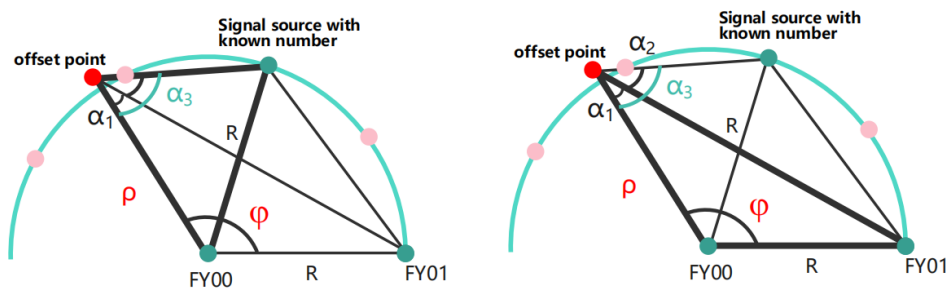


Figure 7: The offset point coordinates are solved

Since it is assumed that the position of the UAV of the signal source has no deviation, the Angle formed between the UAV and FY01 at FY00 can be obtained from the number, set the Angle as θ , and the number of the signal source as k . The specific formula is as follows:

According to the triangle sine theorem, the following formula can be listed:

$$\begin{cases} \frac{R}{\sin \alpha_3} = \frac{\rho}{\sin(\pi - \alpha_3 - (\varphi - \theta))} \\ \frac{R}{\sin \alpha_1} = \frac{\rho}{\sin(\pi - \alpha_1 - \varphi)} \end{cases} \quad (2)$$

Work out

$$\arcsin\left(\frac{\rho \sin \alpha_1}{R}\right) - \arcsin\left(\frac{\rho \sin \alpha_3}{R}\right) = \alpha_1 + \theta \quad (3)$$

In order to verify the feasibility of the equation derived from the model, assume that the signal source is FY00, FY01 and FY03, the offset point to be measured is FY04, and the known $\alpha_1=27^\circ$, $\alpha_2=38^\circ$, $\alpha_3=65^\circ$ Formation radius $R=100m$, the equation is programmed by MATLAB, and the value near R value is randomly calculated. Set $y = \arcsin\left(\frac{\rho \sin \alpha_1}{R}\right) - \arcsin\left(\frac{\rho \sin \alpha_3}{R}\right) - (\alpha_1 + \theta)$ for deviation factor, as shown in figure 8, solving the rho optimal value is 109, when the deviation coefficient of 0.028 or so, substitution and $\varphi = 123.3$. Thus, the positioning of the UAV in the polar coordinate system is realized.

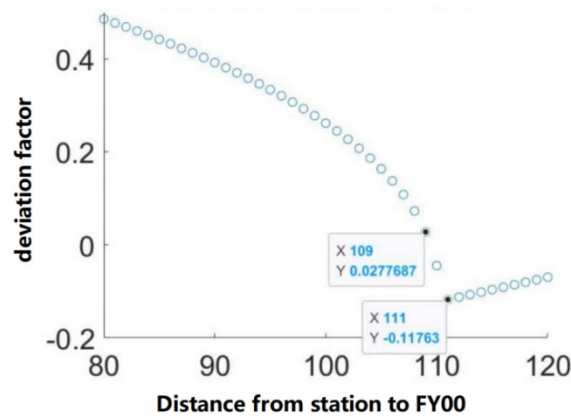


Figure 8: The change of the deviation coefficient when the ρ value is taken

Therefore, it is also necessary to send a signal from a drone to achieve effective positioning of the drone. In fact, when the size of the signal Angle is exactly the size of the positioning Angle, there will be a special situation, that is, when the offset point is offset to its correct position and the circle formed by FY00 and FY01, the signal Angle is equal to the positioning Angle, so the deviation can not be detected. When a new signal transmitting source is introduced and its serial number is obtained. The effective positioning of the offset point can be realized to avoid this special situation.

After the launch signal of a UAV is introduced, the Angle formed between the UAV and FY00 at the correct position of the offset point to be measured can be obtained according to Table 3, and a fixed circle system can be formed according to this Angle. Moreover, this circle system has only two points of intersection with the circle system formed by FY00 and FY01 in the correct position. As shown in Figure 9, one point of intersection is FY00 and the other is the correct position.

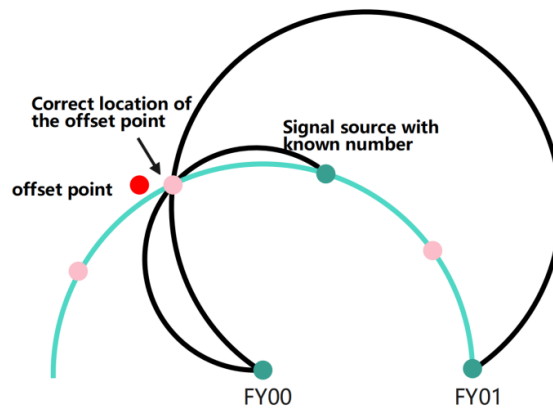


Figure 9: Determine the correct location of the offset point

After obtaining the correct position, the correction scheme can be solved by shifting the corresponding position of the UAV in the polar coordinate system and the change of signal Angle, which can further verify that only one UAV can transmit signals to achieve effective positioning of the UAV.

4. Conclusion

Aiming at the positioning problem of UAVs flying at the same altitude, a positioning method based on relative Angle information is proposed in this paper. By establishing a plane rectangular coordinate

system based on the signal transmitting UAV, and analyzing the relative Angle between the signal transmitting UAV and the receiving UAV with unbiased position and the fixed position of the signal transmitting UAV, several circular orbit equations are constructed, so as to accurately determine the current position of the UAV with biased position. In addition, by analyzing the relative position of the UAV formation in the ideal state, the direction information that the UAV should capture when receiving signals is at the target position is obtained. The results show that only one additional UAV can transmit signals to achieve effective positioning of the entire UAV formation. This method provides a new idea for UAV formation positioning and has a wide application prospect.

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