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Monotone Traveling Waves of a Diffusive Lotka-Volterra Weak Competitive System with Delays

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ABSTRACT. *This paper is concerned with the monotone traveling wave solutions for a diffusive Lotka-Volterra weak competitive system with delays. Using the method of upper-lower solutions, necessary and sufficient conditions are established for the existence and nonexistence of monotone traveling wave solutions connecting the trivial steady state with the coexistence steady state of weak competitive system.*

KEYWORDS: *Traveling waves; Monotone; Delays; Weak competition system*

1. Introduction

In this paper, we consider monotone traveling waves of the following diffusive Lotka-Volterra competitive system with delays

$$\begin{cases} u_t = d_1 u_{xx} + u(t,x)[a_1 - b_1 u(t-\tau_1, x) - c_1 v(t-\tau_2, x)], \\ v_t = d_2 v_{xx} + v(t,x)[a_2 - b_2 u(t-\tau_3, x) - c_2 v(t-\tau_4, x)], \end{cases} \quad x \in R, t > 0 \tag{1.1}$$

where τ_i $(i = 1, 2, 3, 4)$ are delays. In this paper, we assume

$$\text{[A]} \quad \frac{b_1}{b_2} > \frac{a_1}{a_2} > \frac{c_1}{c_2}$$

With condition [A], the system (1.1) has a trivial equilibrium O, two semitrivial equilibria B and C, and positive equilibrium D as follows:

$$O(0,0), \quad B(0, \frac{a_2}{c_2}), \quad C(\frac{a_1}{b_1}, 0), \quad D(\frac{a_2 c_1 - a_1 c_2}{c_1 b_2 - c_2 b_1}, \frac{a_1 b_2 - a_2 b_1}{c_1 b_2 - c_2 b_1}) = (D_x, D_y).$$

Lv and Wang [3] studied traveling waves of the delayed competitive reaction-diffusion system (1.1). But it is remarked that they only considered monotone traveling waves which connected the semi-trivial steady state B with C (strong competition). Monontone traveling waves of competitive system (1.1) will

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become into a cooperative system by making a change of variable in this case. There are many papers which are concerned with the existence of traveling wavefronts of cooperative system. But coexistence of species are extensively observed in natural world, we will look for monotone traveling waves for system (1.1) which connected O and D (weak competition). In [2], Li, Lin and Ruan proposed some weak conditions (WQM) and (WQM*). By using cross-iteration, they reduced existence of traveling waves to existence of an admissible pair of upper and lower solutions. But they didn't get the monotonicity of traveling waves. As we all know, system (1.1) satisfies the condition (WQM*) under the assumption that τ_1, τ_4 are sufficiently small. Fang and Wu [1] obtained a critical value for the existence of monotone traveling wave when $\tau_2 = \tau_3$. However, for non-monotonic models it is not easy to construct a pair of appropriate upper-lower solutions for the application of Schauder's fixed-point theorem. So we should mention the work [6,8] for non-monotonic system. Hence, we consider the necessary and sufficient conditions of monotone traveling waves for system (1.1) which connected O with D and the critical value of delay.

Substituting $U(\xi) = U(x-ct) = u(t,x)$, $V(\xi) = V(x-ct) = v(t,x)$ into (1.1) and setting $\xi = x - ct$, we find that (1.1) has a pair of traveling wave solutions which connect O with D if and only if the following system

$$\begin{cases} L_1[U,V](\xi) = d_1U''(\xi) + cU'(\xi) + a_1U(\xi) - U(\xi)[b_1U(\xi+c\tau_1) + c_1V(\xi+c\tau_2)] = 0, \\ L_2[U,V](\xi) = d_2V''(\xi) + cV'(\xi) + a_2V(\xi) - V(\xi)[b_2U(\xi+c\tau_3) + c_2V(\xi+c\tau_4)] = 0, \end{cases} \tag{1.2}$$

has a pair of solutions $(U(\xi), V(\xi))$ on R, where $L[U,V](\xi) = (L_1[U,V](\xi), L_2[U,V](\xi))$.

2. Preliminaries

For simplification, we assume $\tau_1 = \tau_4,\ \tau_2 + \tau_3 = 2\tau$.

Let $\varphi(\xi) = U(\xi) - D_x$, $\psi(\xi) = V(\xi) - D_y$, Then the system (1.2) becomes

$$\begin{cases} d_1\varphi''(\xi) + c\varphi'(\xi) = [D_x + \varphi(\xi)][b_1\varphi(\xi+c\tau) + c_1\psi(\xi+c\tau_2)], \\ d_2\psi''(\xi) + c\psi'(\xi) = [D_y + \psi(\xi)][b_2\varphi(\xi+c\tau_3) + c_2\psi(\xi+c\tau)], \end{cases} \tag{2.1}$$

The linearization of (2.1) about (0,0) yields

$$\begin{cases} d_1\varphi''(\xi) + c\varphi'(\xi) = b_1D_x\varphi(\xi+c\tau) + c_1D_x\psi(\xi+c\tau_2), \\ d_2\psi''(\xi) + c\psi'(\xi) = b_2D_y\varphi(\xi+c\tau_3) + c_2D_y\psi(\xi+c\tau). \end{cases} \tag{2.2}$$

Then the characteristic equation of (2.2) about the point (0,0) is

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$$\Delta(\lambda,c,\tau)=(d_1\lambda^2+c\lambda-b_1D_xe^{\lambda c\tau})(d_2\lambda^2+c\lambda-c_2D_ye^{\lambda c\tau})-b_2c_1D_xD_ye^{2\lambda c\tau}=0 \quad (2.3)$$

A direct calculation will give the following result:

Lemma 1. When assumption [A] is satisfied, the following statements hold true:

(1) if $\tau=0$, then $\lim_{\lambda\to+\infty}\Delta(\lambda,c,0)=+\infty,\quad \Delta(0,c,0)=(b_1c_2-b_2c_1)D_xD_y>0$;

(2) for fixed $\lambda>0$, then $\lim_{\tau\to+\infty}\Delta(\lambda,c,\tau)=+\infty$.

Indeed, if $\tau=0$, we have

$$\Delta(\lambda,c,0)=(d_1\lambda^2+c\lambda-b_1D_x)(d_2\lambda^2+c\lambda-c_2D_y)-b_2c_1D_xD_y,$$

$$\lim_{\lambda\to+\infty}\Delta(\lambda,c,0)=+\infty,\quad \Delta(0,c,0)=(b_1c_2-b_2c_1)D_xD_y>0.$$

Note that equation d $d_1\lambda^2+c\lambda-b_1D_x=0$(*or* $d_2\lambda^2+c\lambda-c_2D_y$=0), has two real roots: λ_1 and λ_2. And it is easy to see $\lambda_1>0$ and $\lambda_2<0$. Then we have $\Delta(\lambda_1,c,0)=-b_2c_1D_xD_y<0.$ Consequently, there exists $\tilde{\lambda}_1\in(0,\lambda_1)$, such that $\Delta(\tilde{\lambda}_1,c,0)=0.$ Now we can define $\lambda_1^*=\sup\{\lambda\in(0,\lambda_1)|\Delta(\lambda,c,0)=0\}$, then it follows that there exists $\tilde{\lambda}_2\in(\lambda_1^*,\lambda_1)$ such that $\Delta(\tilde{\lambda}_2,c,0)=0$. (2.4)

From Lemma 1(2), for any fixed $\tilde{\lambda}_2>0$, there exists $\tilde{\tau}$ large sufficiently such that $\Delta(\tilde{\lambda}_2,c,\tilde{\tau})>0$. Combining (2.4), there exist $\tau^*\in(0,\ \tilde{\tau})$ such that $\Delta(\tilde{\lambda}_2,c,\tau^*)=0$. Therefore,

$$\tau^*(c)=\sup\{\tau>0|\Delta(\lambda,c,\tau)=0 \text{ has a positive root } \lambda>0\} \quad (2.5)$$

is well defined.

Lemma 2. Assume that [A] holds. If there exist $\xi_1>0$ and decreasing functions (z_1,z_2) defined on $[0,\xi_1+c\tau_0](\tau_0=\max\{\tau_i\})$, which are C^2 on $[0,\xi_1]$, $(z_1,z_2)(0)=(D_{x,}D_y)$, $(z_1',z_2')(0)=0$ and satisfy

$$L[z_1,z_2](\xi)=(L_1[z_1,z_2](\xi),L_2[z_1,z_2](\xi))\ge0,\quad \xi\in[0,\xi_1],$$

and $\qquad T[z_1,z_2](\xi_1)\le(z_1(\xi_1+c\tau),z_2(\xi_1+c\tau)).$

Then $(\overline{\varphi},\overline{\psi})$ is an upper T-solution, where

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$$\overline{\varphi}(\xi)=\begin{cases} z_1(\xi), & \xi\in[0,\xi_1],\\ z_1(\xi_1), & \xi\in[\xi_1,+\infty),\end{cases} \text{ and } \quad \overline{\psi}(\xi)=\begin{cases} z_2(\xi), & \xi\in[0,\xi_1],\\ z_2(\xi_1), & \xi\in[\xi_1,+\infty).\end{cases}$$

Proof. In the case of $\xi\in[0,\xi_1]$, define $H_1=T_1(\overline{\varphi},\overline{\psi})-\overline{\varphi},\ H_2=T_2(\overline{\varphi},\overline{\psi})-\overline{\psi}.$

We can obtain that

$$\begin{aligned} & d_1H_1''(\xi)+cH_1'(\xi)+a_1H_1(\xi)=d_1T_1''+cT_1'+a_1T_1-(d_1z_1''+cz_1'+a_1z_1)\\ & =z_1(\xi)[b_1z_1(\xi+c\tau)+c_1z_2(\xi+c\tau_2)]-(d_1z_1''+cz_1'+a_1z_1)(\xi)\\ & \le 0 \end{aligned}$$

$$d_2H_2''(\xi)+cH_2'(\xi)+a_2H_2(\xi)\le 0, \quad H_1(0)=H_2(0)=0, H_1'(0)=H_2'(0)=0.$$

then we can obtain that

$$H_1(\xi)\le 0, \quad H_2(\xi)\le 0, \quad \text{for all } \xi\in(0,\xi_1).$$

If $\xi\in[\xi_1,+\infty)$, we have

$$T_1(\overline{\varphi},\overline{\psi})(\xi)\le T_1(\overline{\varphi},\overline{\psi})(\xi_1)\le z_1(\xi_1+c\tau)\le z_1(\xi_1)=\overline{\varphi}(\xi),$$

$$T_2(\overline{\varphi},\overline{\psi})(\xi)\le T_2(\overline{\varphi},\overline{\psi})(\xi_1)\le z_2(\xi_1+c\tau)\le z_2(\xi_1)=\overline{\psi}(\xi).$$

It follows that $(\overline{\varphi},\overline{\psi})$ is an upper T-solution.

Lemma 3[9] When n is large enough and $\tau>\tau^*$, there exists a $m>0$ such that $b_1D_x\omega_0(\xi+c\tau)+c_1D_x\omega_0(\xi+c\tau_2)-d_1\omega_0''(\xi)-c\omega_0'(\xi)\ge m\xi^n$,

and $b_2D_y\omega_0(\xi+c\tau_3)+c_2D_y\omega_0(\xi+c\tau)-d_2\omega_0''(\xi)-c\omega_0'(\xi)\ge m\xi^n$,

where $\omega_0(\xi)=\xi^n$.

Lemma 4[9] Assume that [A] holds, the following statements are equivalent:

(1) The traveling wavefront of system (1.1) exists;

(2) (D_x,D_y) is the unique nonincreasing upper T-solution when $\alpha=0$;

(3) There exists a pair of decreasing lower L-solutions defined on $(-\infty,+\infty)$.

3. Main Result

Theorem1 Assume the system (1.1) satisfy [A]. Then for any $c\ge\max\left\{2\sqrt{a_1d_1},2\sqrt{a_2d_2}\right\}$, there exists a critical $\tau^*=\tau^*(c)\in(0,+\infty)$ such that if $\tau\le\tau^*$, system (1.1) has a monotone traveling wave solution; if $\tau>\tau^*$, the system

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(1.1) has no monotone traveling wave solution, where τ^* is defined as (2.5).

Proof. (1) $\tau < \tau^*$.

Let λ be a positive root of (2.4), then $W(\xi) = (C_1, C_2)e^{\lambda\xi}$ is the solution of (2.2). For any ε, denote

$$X(\xi) = (D_x, D_y) - \varepsilon(C_1, C_2)e^{\lambda\xi}. \tag{3.1}$$

Then we have

$$\begin{cases} d_1 X_1''(\xi) + cX_1'(\xi) = b_1 D_x X_1(\xi + c\tau) + c_1 D_x X_2(\xi + c\tau_2) - (b_1 D_x{}^2 + c_1 D_x D_y) \\ d_2 X_2''(\xi) + cX_2'(\xi) = b_2 D_y X_1(\xi + c\tau_3) + c_2 D_y X_2(\xi + c\tau) - (c_2 D_y{}^2 + b_2 D_x D_y) \end{cases}, \tag{3.2}$$

Let us prove Theorem 1 by arguing contradiction. If system (1.1) has no monotone traveling wave solution, we can assume that there exists a nonincreasing upper T-solution $\overline{X}(\xi) \neq (D_x, D_y)$. Choose $\sigma > 0$, such that for small enough $\varepsilon > 0$, we have

$$X(\sigma) = (D_x, D_y) - \varepsilon W(\sigma) > \overline{X}(\sigma). \tag{3.3}$$

Taking the limit on the both side of (3.1), it follows that $\lim_{\xi \to +\infty} X(\xi) = -\infty.$ So $X(\xi)$ becomes negative for large ξ. Let $\sigma_1 > \{c\tau, c\tau_2, c\tau_3\}$ be the point where $X(\sigma_1) = 0$.

Define $\underline{X}(\xi) = \max\{X(\xi), 0\}$. In what follows, we will prove $T(\underline{X}(\xi)) \geq \underline{X}(\xi).$ If $\xi \in [\sigma_1, +\infty)$, then we have that $\underline{X}(\xi) = 0$, and hence T$T(\underline{X}(\xi)) \geq \underline{X}(\xi).$. If $\xi \in [0, \sigma_1),$ we have that $\underline{X}(\xi) = X(\xi)$ is C^2, therefore it is sufficient to prove that $X(\xi)$ is a lower L-solution on the interval of $\xi \in [0, \sigma_1),$, namely,

$L_1[\underline{X}] \leq 0, \quad L_2[\underline{X}] \leq 0.$

If $\xi \in [0, \sigma_1 - c\tau),$ we have

$L_1[\underline{X}] = [D_x - X_1(\xi + c\tau)][-b_1 D_x + b_1 X_1(\xi)] + [D_y - X_2(\xi + c\tau_2)][-c_1 D_x + c_1 X_1(\xi)] \leq 0.$

If $\xi \in [\sigma_1 - c\tau, \sigma_1 - c\tau_2],$ we have $\underline{X}_1(\xi + c\tau) = 0,\ \underline{X}_2(\xi + c\tau_2) = X_2(\xi + c\tau_2),$ Since $\xi \leq \xi + c\tau_2 \leq \sigma_1$, we can get $X_1(\xi) \leq X_1(\sigma_1) = 0,\ X_2(\xi + c\tau_2) \geq X_2(\sigma) = 0$. Then it follows that

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$$L_1[\underline{X}] = b_1 D_x (X_1(\xi + c\tau) - D_x) + c_1 D_x (X_2(\xi + c\tau_2) - D_y) - c_1 X_1(\xi) X_2(\xi + c\tau_2) \le 0.$$

If $\xi \in [\sigma_1 - c\tau_2, \sigma_1]$, we can obtain that $\underline{X}_1(\xi + c\tau) = 0, \underline{X}_2(\xi + c\tau_2) = 0$, Since $X(\xi) = (D_x, D_y) - \varepsilon(C_1, C_2)e^{\lambda\xi}.$, then $X(\xi)$ is decreasing function of ξ and $X(\xi + c\tau) = (X_1(\xi + c\tau), X_2(\xi + c\tau)) \le X(\sigma_1) = 0,$.

Hence we have $L_1[\underline{X}] \le -(b_1 D_x^{\ 2} + c_1 D_x D_y) + a_1 X_1(\xi) \le D_x(a_1 - b_1 D_x - c_1 D_y) = 0.$

It follows that $\underline{X}$ is the lower T-solution. From the definition of $\underline{X}$, we have $\underline{X}(\sigma) > \overline{X}(\sigma)$. This is a contradiction with Lemma 3. Therefore, the system (1.1) has a traveling wavefront with wave speed c for $\tau < \tau^*$.

(2) $\tau = \tau^*$.

In the case where $\tau = \tau^*$, we use a limiting argument. Choose a sequence $\tau_j \in (0, \tau^*)$ such that $\lim_{j\to\infty} \tau_j = \tau^*$, $\tau_{2j} + \tau_{3j} = 2\tau_j$. According to the above arguments, there exists a traveling wavefront (U_j, V_j, τ_j) of (1.1) and for each j, we see from Helly's theorem that there exists a subsequence of (U_j, V_j) converging to monotonic functions (U, V) pointwise. Note that

$$\begin{cases} U_j(\xi) = D_x + \dfrac{1}{d_1}\displaystyle\int_{-\infty}^{\xi} \dfrac{e^{\lambda_2(\xi - s)} - e^{\lambda_1(\xi - s)}}{\lambda_2 - \lambda_1} U_j(s)[b_1 U_j(s + c\tau_j) + c_1 V_j(s + c\tau_{2j})]\mathrm{d}s, \\ V_j(\xi) = D_y + \dfrac{1}{d_2}\displaystyle\int_{-\infty}^{\xi} \dfrac{e^{\mu_2(\xi - s)} - e^{\mu_1(\xi - s)}}{\mu_2 - \mu_1} V_j(s)[b_2 U_j(s + c\tau_{3j}) + c_2 V_j(s + c\tau_j)]\mathrm{d}s. \end{cases}$$

Letting $j \to \infty$ and using Lebesgue's dominated convergence theorem, it follows that $L(U, V) = 0, (U, V)(-\infty) = (D_x, D_y)$. Furthermore, since (U, V) are decreasing, we have $(U, V)(+\infty) = (0, 0)$. Hence when $\tau = \tau^*$, the system (1.1) has a traveling wavefront (U, V).

(3) $\tau > \tau^*$.

In this case, we shall prove the nonexistence of traveling wavefronts for system (1.1) by using Lemma 4. We will construct a nonincreasing upper T-solution which is different from (D_x, D_y) when $\alpha = 0$.

Let $Z(\xi) = (z_1(\xi), z_2(\xi)) = (D_x - \varepsilon\xi^n, D_y - \varepsilon\xi^n)$, where ε is small sufficiently and n is a large integer. In the following argument, we denote $\omega_0(\xi) = \xi^n$ and plan to estimate the $L[Z](\xi) = (L_1[z_1, z_2](\xi), L_2[z_1, z_2](\xi))$.

For any fixed $\xi_1 > 0$, we can choose $\varepsilon > 0$ small enough such that

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$$L[Z](\xi) \geq (\frac{\varepsilon m\xi^n}{2}, \frac{\varepsilon m\xi^n}{2}), \quad \textit{for all } \xi \in [0, \xi_1] \text{ }),$$

Thus $L[Z](\xi)$ can satisfy Lemma 2. Our next step is to choose $\xi_1 > 0$ such that $T[Z](\xi_1) \leq Z(\xi_1 + c\tau)$ .Let $T(D_x - \varepsilon\xi^n, D_y - \varepsilon\xi^n) = T[Z](\xi) = (D_x - \varepsilon\omega_1, D_y - \varepsilon\omega_2)$. Then (ω_1, ω_2) satisfies

$$\begin{aligned} &d_1\omega_1''(\xi) + c\omega_1'(\xi) + a_1\omega_1(\xi) \\ &= a_1\xi^n + b_1 D_x(\xi + c\tau)^n + c_1 D_x(\xi + c\tau_2)^n - b_1\varepsilon\xi^n(\xi + c\tau)^n - c_1\varepsilon\xi^n(\xi + c\tau_2)^n \\ &\geq (a_1 + b_1 D_x + c_1 D_x)\xi^n - b_1\varepsilon(\xi + c\tau)^{2n} - c_1\varepsilon(\xi + c\tau_2)^{2n}, \end{aligned}$$

and

$$d_2\omega_2''(\xi) + c\omega_2'(\xi) + a_2\omega_2(\xi) \geq (a_2 + b_2 D_y + c_2 D_y)\xi^n - b_2\varepsilon(\xi + c\tau)^{2n} - c_2\varepsilon(\xi + c\tau_2)^{2n}.$$

By a direct computation, it is easy to get $(\omega_1, \omega_2)(0) = (0,0), (\omega_1', \omega_2')(0) = (0,0)$. Suppose that $(\underline{\omega}_1, \underline{\omega}_2)$ is a solution of the following problem □ □ □

$$\begin{cases} d_1\underline{\omega}_1''(\xi) + c\underline{\omega}_1'(\xi) + a_1\underline{\omega}_1(\xi) = (a_1 + b_1 D_x + c_1 D_x)\xi^n - b_1\varepsilon(\xi + c\tau)^{2n} - c_1\varepsilon(\xi + c\tau_2)^{2n}, \\ d_2\underline{\omega}_2''(\xi) + c\underline{\omega}_2'(\xi) + a_2\underline{\omega}_2(\xi) = (a_2 + b_2 D_y + c_2 D_y)\xi^n - b_2\varepsilon(\xi + c\tau_3)^{2n} - c_2\varepsilon(\xi + c\tau)^{2n}, \\ (\underline{\omega}_1, \underline{\omega}_2)(0) = (0,0), (\underline{\omega}_1', \underline{\omega}_2')(0) = (0,0). \end{cases} \tag{3.4}$$

It follows that $(\omega_1, \omega_2) \geq (\underline{\omega}_1, \underline{\omega}_2)$. In the following, we will consider the form of $(\underline{\omega}_1, \underline{\omega}_2)$. System (3.4) can be regarded as the perturbation of the following system

$$\begin{cases} d_1\tilde{\omega}_1''(\xi) + c\tilde{\omega}_1'(\xi) + a_1\tilde{\omega}_1(\xi) = (a_1 + b_1 D_x + c_1 D_x)\xi^n, \\ d_2\tilde{\omega}_2''(\xi) + c\tilde{\omega}_2'(\xi) + a_2\tilde{\omega}_2(\xi) = (a_2 + b_2 D_y + c_2 D_y)\xi^n, \\ (\underline{\tilde{\omega}}_1, \underline{\tilde{\omega}}_2)(0) = (0,0), (\underline{\tilde{\omega}}_1', \underline{\tilde{\omega}}_2')(0) = (0,0). \end{cases} \tag{3.5}$$

where ε is a small parameter in the perturbation. By applying the theory of linear ordinary differential equations, we can obtain that the solution of (3.5) can be represented explicitly as a sum of a term coming from the general solution of the homogeneous equation and a particular solution. We can find that the form of the general solution is $(C_1 e^{\lambda_1\xi} + C_2 e^{\lambda_2\xi}, C_1 e^{\mu_1\xi} + C_2 e^{\mu_2\xi})$, where $\lambda_2 \leq \lambda_1 < 0, \mu_2 \leq \mu_1 < 0$. It follows that this exponential function will decay to 0 as $\xi \to +\infty$. On the other hand the particular solution has in the form of

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$$\left(\frac{(a_1+b_1D_x+c_1D_x)\xi^n}{a_1}+(lower\ order\ terms),\frac{(a_2+b_2D_y+c_2D_y)\xi^n}{a_2}+(lower\ order\ terms)\right).$$

(3.6)

Therefore, we can choose $\xi_1 > 0$ large enough that the solution of (3.6) satisfy that

$(\tilde{\omega}_1, \tilde{\omega}_2)(\xi_1) \geq m_1((\xi_1 + c\tau)^n, (\xi_1 + c\tau)^n)$,

where 1 < m1 < $1 < m_1 < \min\left\{\frac{a_1+b_1D_x+c_1D_x}{a_1}, \frac{a_2+b_2D_y+c_2D_y}{a_2}\right\}$.

On the other hand, we can find that when $\varepsilon \to 0$, the solution $(\underline{\omega}_1, \underline{\omega}_2)$ is convergent uniformly to the solution $(\tilde{\omega}_1, \tilde{\omega}_2)$ of (3.5) in $[0, \xi_1]$. Hence we can choose suitable ε such that $(\underline{\omega}_1, \underline{\omega}_2)(\xi_1) > ((\xi_1 + c\tau)^n, (\xi_1 + c\tau)^n)$. Therefore, we have

$T[z_1, z_2](\xi_1) \leq (D_x - \varepsilon(\xi_1 + c\tau)^n, D_y - \varepsilon(\xi_1 + c\tau)^n) = [z_1, z_2](\xi_1 + c\tau).$

It follows that Lemma 2 is valid for $(z_1, z_2)(\xi) > (D_x - \varepsilon\xi^n, D_y - \varepsilon\xi^n)$. We get an upper T-solution $(\overline{\varphi}, \overline{\psi})$ which is different from (D_x, D_y) and nonincreasing.

From Lemma 4, for any $c \geq \max\left\{2\sqrt{a_1d_1}, 2\sqrt{a_2d_2}\right\}$, there exists a critical $\tau^* = \tau^*(c) \in (0, +\infty)$ such that if $\tau \leq \tau^*$, system (1.1) has a monotone traveling wave solution; if $\tau > \tau^*$, the system (1.1) has no monotone traveling wave solution. The proof is completed.

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