ISSN 2522-6398 Vol. 3, Issue 11: 121-128, DOI: 10.25236/FER.2020.031121

On the Application of Mathematical Model in the Pricing of Financial Derivatives

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ABSTRACT. *Mathematical model is widely used in the field of finance. The mathematical model can clearly distinguish the relationship between markets and the internal logic of financial markets. This paper will introduce some mathematical models and their application in financial markets.*

KEYWORDS: *Financial market, Mathematical model, Portfolio model, Capm model*

1. Introduction

With the development of mathematics level and computer technology, mathematical models are more and more applied in the field of economy, especially in the field of finance. In the financial field, trading and investment in financial markets are undoubtedly the core and most complex part. How to match different capital flows to different products according to the needs of investors in the vast investment target, not only need to be familiar with different products in the financial market, but also need to identify the characteristics of their risks and returns on this basis, and combine different products properly. In order to maximize the return of funds at acceptable risk level. Applying mathematical models to financial markets helps to express the nature of the financial market system and make appropriate investment decisions.

2. Overview of Financial Markets

2.1 What Are Financial Markets

Financial market is a broad market for capital circulation. In this market, four kinds of institutions, such as the supply side of capital, the demand side of capital, the financial intermediary, the supervisor and so on, are active. Under the supervision of the government or other institutions, the supply and demand side of the funds, through the services provided by the intermediary, complete the matching of the supply and demand of the funds, and realize the maximum utilization of the capital resources.

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2.2 Classification of Financial Markets

The financial market can be divided into primary market and secondary market according to the different trading methods. For example, the listed company issues shares for the first time, which belongs to the primary market, while in the stock exchange, The circulation of the stock belongs to the secondary market. According to the different financial products, the financial market can also be divided into stock market, bond market, futures market, foreign exchange market, insurance market and so on. Among them, investment and financial management, stocks and funds, lending and insurance, is closely linked with the broad masses of our people.

3. Overview of Mathematical Models

Mathematical model is a scientific or engineering model constructed by mathematical logic method and mathematical language. Mathematical models abstract and simplify complex practical problems, so as to express and solve them in mathematical language. Mathematical models can be simple, such as, in economics, describing product markets with supply curves and demand curves, and in biology, describing population changes over time with J font curves, or complex, such as solving optimization problems with neural network algorithms. In modern financial analysis, quantitative and qualitative analysis through mathematical models to find the potential laws in financial activities and to guide practice has become a more and more common phenomenon and effective technical means.

4. Examples of the Application of Mathematical Models in the Field of Finance

4.1 Portfolio Models

In 1952, American economist Markowitz first put forward portfolio theory and carried out systematic, in-depth and fruitful research, so he won the Nobel Prize in Economics. The theory of portfolio of securities first examines the income and risk of single securities. From the point of view of probability theory, the price of securities is regarded as a random variable, and the mathematical expectation of the random variable is used to describe the return of securities, and the variance is used. For a portfolio composed of a variety of securities with different return risks, the model holds that the return of the portfolio is the weighted average of the returns of these securities, but its risk needs to consider the risk of single securities and the correlation between them. By building such a model, it can be seen that the portfolio can reduce risk.

4.1.1 Expected Return Model of the Portfolio

Take the securities price in the portfolio as a random variable and use its mean value to represent the return.

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$$E(r_p) = \sum_{i-1}^{n} xiE(ri)$$

4.1.2 Variance Model of Portfolio

Use variance to represent the relationship between various benefits

The variance depicts the risk of the portfolio, and the greater the variance, the greater the volatility of the actual return of the portfolio from the expected return, and the greater the risk faced by the investor. The advantage of this model is that the mathematical model can be used to clearly and intuitively see the relationship between the risk and return of various securities, and at the same time, the smaller the correlation between the returns of different assets, the lower the risk of the combination as a whole. This is what we often say “don't put eggs in the same basket “. But this is only a general relationship, specific investment needs specific analysis. By means of the simple and intuitive expression of mean-variance, the model scientifically explains the decentralized investment in modern finance The idea.

4.2 Capital Asset Pricing Model (Capm Model)

The other major achievement of the application of the mathematical model in the financial market is the capital asset pricing model, which is Capital Asset Pricing Model referred to as CAPM, by William·Sharp, John·Lintner created the development together to study how the market price of securities is determined. The model converts all products in the market into a market portfolio according to their market value, and based on the risk of the market portfolio, it depicts the relationship between the price of any asset (or any portfolio) and its risk.

CAPM model can be expressed by the following formula:

$$E(\text{ri}) = \text{rf} + \beta\text{im}(\text{E}(\text{rm}) - \text{rf})$$

CAPM model:

4.2.1 Concept

CAPM is based on the Markowitz model, and the assumptions of the Markowitz model are naturally included in it:

1). Investors hope that the more wealth the better, the utility is a function of wealth, and wealth is a function of the rate of return on investment, so it can be considered that utility is a function of the rate of return.

2). Investors can know in advance that the probability distribution of investment returns is a normal distribution.

3). Investment risk is identified by the variance or standard deviation of the

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investment yield.

4). The main factors that affect investment decisions are expected return and risk.

5). Investors all abide by the Dominance rule, that is, under the same level of risk, they choose securities with higher yields; under the same level of yield, they choose securities with lower risks.

6). Funds can be borrowed or loaned without restriction at the risk-free discount rate R.

7). All investors have the same view on the probability distribution of securities returns, so there is only one efficiency boundary in the market.

8). All investors have the same investment period, and there is only one period.

9). All securities investments can be subdivided unlimitedly, and any investment portfolio can contain non-integer shares.

10). There is no tax burden and transaction cost when buying and selling securities.

11). All investors can obtain sufficient market information in time for free.

12). There is no inflation and the discount rate remains unchanged.

13). Investors have the same expectation, that is, they have the same expectation value for the expected rate of return, standard deviation, and the covariance between securities.

The above assumptions show that: first, investors are rational and diversify investments strictly in accordance with the rules of the Markowitz model, and will choose their portfolio from somewhere on the effective frontier; second, the capital market is perfect/complete in the market, no friction hinders investment.

4.2.2 The Advantage of Capm

The biggest advantage of CAPM is simplicity and clarity. It divides the price of any kind of risky security into three factors: risk-free rate of return, risk price, and risk calculation unit, and organically combines these three factors.

Another advantage of CAPM lies in its practicality. It allows investors to evaluate and select various competitive financial assets based on absolute risk rather than total risk. This method has been widely adopted by investors in the financial market to solve general problems in investment decisions.

4.2.3 The Limitation of Capm

Of course, CAPM is not perfect, and it has certain limitations; appears in:

First of all, the premise of CAPM is difficult to achieve. For example, at the beginning of this section, we summarize the assumptions of CAPM into six aspects.

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One of the assumptions is that the market is in a state of perfect competition. However, it is difficult to achieve a perfectly competitive market in actual operation, and “market making” occurs from time to time. The second assumption is that investors have the same investment period and do not consider the situation after the investment plan period. However, there are so many investors in the market that their asset holding periods cannot be exactly the same, and more and more investors are engaged in long-term investments, so the second assumption has become less realistic. The third assumption is that investors can borrow at a fixed risk-free interest rate without restrictions, which is also difficult to do. The fourth assumption is that the market is frictionless. But in fact, the market has problems such as transaction costs, taxation and information asymmetry. The fifth and the sixth assumption are rational person hypothesis and consensus expectation hypothesis. Obviously, these two assumptions are just an ideal state.

Secondly, the β value in CAPM is difficult to determine. Due to the lack of historical data for certain securities, its β value is not easy to estimate. In addition, due to the continuous development and changes of the economy, the β value of various securities will also change accordingly. Therefore, the β value estimated based on historical data should also be discounted for future guidance. In short, due to the above-mentioned limitations of CAPM, financial market scientists continue to explore more accurate capital market theories than CAPM. At present, there have been some other distinctive capital market theories (such as arbitrage pricing models), but there is no theory that can match CAPM.

4.3 Apt Arbitrage Model:

Arbitrage is the act of using different prices of the same or similar physical assets or financial assets to obtain risk-free benefits. It is achieved by buying securities with a high yield and selling securities with a low yield. The key point is to buy and sell at the same time. For example, if you see an egg in the school cafeteria for seven cents, the same egg is bought outside the school gate for a dollar, then you buy the eggs from the cafeteria and go to the school gate and sell it. This is arbitrage. If you buy a bunch of eggs, don’t sell them, and wait until the price of the New Year’s eggs increase. This way of making money is not arbitrage.

Arbitrage pricing theory (APT) talks about an expression that should be satisfied by each security pricing under ideal market conditions. The ideal market conditions mentioned here mean that there are no transaction costs in the market, and traders in the market avoid risks and have consistent views on the expected return of securities. Then the security pricing satisfies the following expression

$$r_i = \alpha_i + \beta_i^{(1)} F^{(1)} + \cdots + \beta_i^{(K)} F^{(K)} + \epsilon_i$$

The essence of the expression form and the meaning represented by the parameters is the same as the multi-factor model. The [formula] for each stock is obtained through time series regression.

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If an asset portfolio meets the following three requirements, we call it an arbitrage portfolio

The initial investment is zero,

which means that to the portfolio $P = (w_1, \cdots, w_N)$ have $\sum_{i=1}^{N} w_i = 0$

The risk of the investment portfolio is zero, and the risks are divided into systemic risks and non-systematic risks. The systemic risk is zero, that is, for any $k \in [K]$, $\sum_{i=1}^{N} \beta_i^{(k)} w_i = 0$; the non-systematic risk requires that the types of investment are sufficiently large and dispersed, which can be obtained by the theorem of large numbers $\sum_{i=1}^{N} w_i \epsilon_i = 0$

Of course, the most important point is that the profit is positive, that is, $\sum_{i=1}^{N} w_i r_i = 0$ .Combining the first two, there is $\sum_{i=1}^{N} w_i \alpha_i = 0$.

When this arbitrage combination exists in the market, we say that there are arbitrage opportunities in this market.

4.4 Fama-French Three-Factor Model:

In asset pricing and portfolio management, the three-factor model refers to the Fama-French three-factor model, an improved theory of the capital asset pricing model. The proposed model is based on the results of empirical research on the historical rate of return in the US stock market, with the purpose of explaining which risk premium factors affect the average rate of return in the stock market. Model designers, Eugene Fama and Kenneth French both worked at the University of Chicago Booth School of Business.

Under traditional theories such as the Capital Asset Pricing Model (CAPM), the entire risk premium of the investment portfolio is represented by the Beta coefficient. However, this model has encountered many challenges in explaining the reality of stock market returns, such as the January effect. Fama and French (1992) observe that two types of companies with a smaller market value and a higher book value are more likely to achieve an average rate of return better than the market. Therefore, the three-factor model introduces two new explanatory variables [1]: price-to-book ratio, company size, estimate the return level of stocks together with the market index in the CAPM, namely:

$$r = R_f + \beta_3 (R_m - R_f) + b_s \cdot SMB + b_v \cdot HML + \alpha$$

Where r is the expected rate of return of the investment portfolio, Rf is the market risk-free rate of return, Rm is the rate of return of the market portfolio, and the coefficient β of the three variables to be estimated is the market portfolio risk

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premium, scale premium, and price-to-book ratio premium. The impact of changes on the expected rate of return. The beta concept of the market portfolio risk premium is close to the beta coefficient in the CAPM model. The firm size variable SMB refers to the difference between the return of a portfolio consisting of companies with small market capitalization and that of companies with large market capitalization. The price-to-book ratio premium HML is the difference between the return on the portfolio of companies with higher book value and the return on the portfolio of companies with lower ratios. α is the excess return rate. Under ideal circumstances, the excess return of the portfolio will be explained by three factors, so α should be equal to 0 in a statistical sense.

In regression analysis, the three-factor data divides all companies on the stock market into 10 investment portfolios of equal market capitalization based on the price-to-book ratio and the size of the company, and uses their historical data to calculate the respective premium levels of the three factors. These data can still be found on Kenneth French's website.

When the scale premium and price-to-book ratio premium are determined, their coefficients can be calculated by linear regression. The calculation results of Fama and French show that about 70% of the rate of return can be calculated through this grouping method through the CAPM model; and the rate of return of more than 90% can be calculated through the revised model, Fama-French three-factor model is explained. The regression coefficients of scale premium and price-to-book ratio premium are statistically significant, which means that the three-factor model may capture information that the market portfolio risk premium cannot explain. In addition, the coefficient of the scale premium is positive, which means that the portfolio of companies with smaller market capitalization can be expected to bring higher returns and higher risks.

Griffin’s research proves that the three-factor model is a country-specific model, and the impact of global economic variables on the level of return of each country’s stock market is not as significant as that of internal economic variables. Therefore, some empirical studies based on domestic stock markets in various countries have also made relevant progress, such as the United Kingdom, Germany, and Switzerland. In fact, as an application of arbitrage pricing theory, the improved Fama-French three-factor model is also used by scholars to explain GDP growth rates and bond market yields.

Er i is the expected rate of return on the asset i; r f is the risk-free rate;βThe im is called the Beta coefficient, that is, the systemic risk of asset i, which is determined by the correlation between portfolio and market portfolio; Er m is the expected market return on the market; Er m -r f is the market risk premium (market risk premium), that is, the difference between expected market return and risk-free return.

It can be seen from the above formula that the premium of any portfolio relative to the risk-free return is proportional to the premium of the market portfolio relative to the risk-free return, and the proportional coefficient is the correlation between the asset and the asset and the market portfolio. The greater the

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correlation, the closer the risk premium to the market portfolio. This simple linear model clearly and intuitively illustrates the relationship between income and risk, which is of great significance to the pricing of capital assets in the future. It provides a model that can measure the size of risk to help investors determine the relative size of risk and return. This model also implies the classical political economy of Marxism, asset price fluctuates around asset value, and is refined into correlation.

4.5 Summary

At present, the international financial field is constantly developing, many mathematical models and in the financial market have been widely used. Therefore, we should study mathematics seriously and apply it more in finance so that it has a better and wider application prospect in the field of finance.

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