

# QLIKE and VaR evaluation approaches for volatility forecasts of exchange rates using structural models in US and UK

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**Abstract:** This paper evaluates the time-varying volatility forecasting performance of a range of macroeconomic theory-based models, explains the exchange rates in the US and UK, and non-theory-based models using univariate GARCH and bivariate GARCH models. This paper uses a traditional statistical criterion and a more practical Value at Risk (VaR) approach to judge these models' performance, using out-of-sample data and one-step-ahead forecasts. The results are mixed: some models perform well in terms of one criterion but badly in terms of another. The best model considering both criteria is the model based on the efficient market hypothesis (EMH), but it does not outperform the other models significantly, especially in terms of the VaR method, which gives the EMH model the same performance as GARCH (1,1).

**Keywords:** Exchange rate volatility forecasting; Vector Error Correction Model; Autoregressive model; Time-varying volatility; Value at Risk

## 1. Introduction

There are extensive studies investigating time-varying volatility forecasting models of the exchange rate that capture the “volatility-clustering” effect found in different sets of currencies (see Diebold & Nerlove, 1989; Baillie & Bollerslev, 1989, for example)<sup>[1]</sup>. Here, the “volatility-clustering” effect refers to the periods during which the exchange rate is volatile tending to be followed by a volatile period, and periods during which the exchange rate is stable tending to be followed by a stable period as if the volatile periods cluster together rather than being separated across the time horizon. Furthermore, the most traditional ways of modeling the time-varying volatility of the exchange rate are using the GARCH model and its variants such as IGARCH or multivariate GARCH models (see Baillie & Bollerslev, 1991; Bollerslev, 1990; Vilasuso, 2002, for example)<sup>[2]</sup> as empirically the GARCH family models outperform other models in various sets of currencies using either economic criteria or statistical criteria (see West et al., 1993)<sup>[20]</sup>.

Nevertheless, most of the literature focuses on using pure econometric models instead of economically meaningful models to make forecasts. However, some studies are already integrating economically meaningful relationships into the models to forecast the conditional mean of exchange rates (see Garratt & Lee, 2010, for example)<sup>[12]</sup>. Garratt and Lee's research used three macroeconomic relationships (efficient market hypothesis, purchasing power parity, and monetary fundamentals term) to build multivariate vector error correction models to make conditional mean forecasts of exchange rates between the US, UK, and Japan. They let these models compete against autoregressive (AR) and random walk models. The result of their paper reveals that although the more structural economic models underperform the AR and random walk judged on their root mean squared error, a pure statistical criterion, they systematically outperform AR and random walk models judged on economic criterion. This criterion is based on the utility obtained using each model to make investment decisions in an artificial investment scenario. However, they did not apply these theory-based models in volatility forecasting of exchange rates, which is a gap found in the literature on exchange rate volatility forecasting models. Therefore, few papers have formally studied the forecasting ability of economically meaningful structural models and compared them against an atheoretical GARCH model, which is what this paper aim to do in this empirical study.

In this paper, the author extends Garratt and Lee's (2010) work on forecasting the conditional mean

of the exchange rate using theoretical and atheoretical models to forecast the conditional variance of the exchange rate in the US and UK<sup>[12]</sup>. Author compares their forecasting performance against GARCH(1,1) models with an autoregressive process and a plain process that is only regressed on a constant in the mean equations. Then, this paper takes two evaluation approaches to judge the models' forecasting performance, with one being a traditional statistical criterion (QLIKE loss function) and the other being a practical VaR criterion, which is popularly applied in financial risk management practices. The result suggests that each model's relative forecasting performance differs when judged on various criteria. For example, the model-based monetary fundamental term gives the most accurate forecasts when judged by statistical criteria but performs poorly when judged by VaR criteria. This result is analogous to Garratt and Lee's (2010) findings that the statistical criterion gives significantly different results from the alternative economic criterion<sup>[12]</sup>. The evaluation method applied in this research is illustrated in detail and can be used to evaluate other financial volatility forecasting exercises. The innovative part of this paper includes the VaR approach as an alternative evaluation method of exchange rate volatility forecasts and the incorporation of the economic relationship into standard econometric forecasting models.

The paper's plan is as follows: Section 2 fully describes the models for the mean equation, volatility forecasting, and evaluation of volatility forecasting performance. Section 3 discusses some essential information from the data. Section 4 describes the estimation of the models using US and UK data for the in-sample period 1990m1-2014m12 and the evaluation using model forecasts for the out-of-sample period 2015m1-2021m6. Section 5 concludes.

## 2. Models

### 2.1. Models for Mean Equation Estimation

Estimating an exchange rate volatility forecasting model requires us first to estimate the mean equation upon which volatility models could be built. This paper considers five alternative mean equation models: three theoretical models containing economically meaningful relationships and two atheoretical models from pure econometric literature. By assuming variables in  $Z_t$  are  $I(1)$ , the mean equation models can be written in the vector error correction (VECM) form according to Garratt & Lee (2010)<sup>[12]</sup>:

$$\Delta Z_t = a + \sum_{i=1}^p \Gamma_i \Delta z_{t-i} + \alpha \beta' z_{t-1} + \mu_t \quad (1)$$

- Where:
- $Z_t = (z_{1t}, z_{2t})'$  is a  $2 \times 1$  vector of variables of interest including at least  $e_t$ , defined as  $e_t = \log(E_t)$ . And I use  $\Delta e_{t+1} = \log\left(\frac{E_{t+1}}{E_t}\right) \approx \frac{E_{t+1} - E_t}{E_t}$  as an approximation of the percentage exchange rate return. This approximation method is widely applied in the literature on exchange rate data (see Garratt & Lee, 2010, for example)<sup>[12]</sup>.

- $\Gamma_t$  and  $\alpha$  are two vectors of parameters to be estimated by the maximum likelihood method.
- $\beta$  is the cointegration vector and is model-specific depending on the economic theory associated with the model. Here, the paper uses the maximum likelihood estimation method, which is different from Garratt & Lee's (2010) work where they impose restriction  $\beta = (1, -1)$ <sup>[12]</sup>.
- $\mu_t$  is a vector of innovation terms that is assumed to follow white-noise processes.
- For the general VECM model, this paper considers five specifications corresponding to different theories, which are described as follows (Garratt & Lee, 2010)<sup>[12]</sup>:
  - $M_E$ : Efficient Market Hypothesis (EMH);
  - $M_M$ : Monetary Fundamentals model (MF);
  - $M_P$ : Purchasing Power Parity (PPP);
  - $M_A$ : Autoregressive model of  $e_t$  in first difference (AR);
  - $M_n$ : Plain model of  $e_t$  in first difference (N).

In the EMH, this paper defines  $z_{1t} = e_t$  and  $z_{2t} = f_t$ , where  $f_t$  is the logarithm of the one-month forward exchange rate (US/GBP). This model reflects the literature proposing that the foreign exchange market is efficient, and all information is reflected in the current price. Hence, the expected price of a currency in one month is equal to the current price. If the equality does not hold, the arbitrator will take arbitrage transactions, closing the gap between spot and future prices. Moreover, some empirical evidence suggests that at least some information is contained in the term structure of exchange rate (see Clarida & Taylor, 1997)<sup>[7]</sup>.

In the MF, this paper defines  $z_{1t} = e_t$  and  $z_{2t} = x_t$ , where  $x_t = (m_t - m_t^*) - (y_t - y_t^*)$ .  $m_t$ ,  $m_t^*$ ,  $y_t$ , and  $y_t^*$  denote the logarithms of the domestic (US) narrow money supply, domestic real income, foreign (UK) narrow money supply, and foreign real income, respectively. This specification is widely applied in the determination of exchange rates (see Frenkel, 1976)<sup>[11]</sup>.

In the PPP, this paper defines  $z_{1t} = e_t$  and  $z_{2t} = p_t - p_t^*$ , where  $p_t$  and  $p_t^*$  denote the logarithms of domestic and foreign consumer price indices, respectively. This specification relates to the law of one price, which is a crucial assumption in open macroeconomic studies. While the one-to-one relationship often does not hold in reality, there is at least some information contained in this specification (Garratt et al., 2006)<sup>[13]</sup>.

The AR model is simply an atheoretical autoregressive model of exchange rate return to-compete against the above three more structural theory-based models when the conditional variances of exchange rate returns are forecasted. In this model,  $Z_t = (e_t)$ .

The last plain model is a univariate model, the same as the AR model, except that the exchange rate return is only regressed on a constant. This model serves as a baseline model to judge the performance of the above four models. This approach is similar to Garratt & Lee's use of a random walk model as a reference model against the other models.

**2.2. Volatility forecasting models**

For the first three bivariate economic-meaningful models, this paper applies a constant conditional correlation (CCC) multivariate GARCH model developed by Bollerslev (1990)<sup>[6]</sup>. By using CCC-GARCH to model the variables in  $Z_t$ , people can better integrate the correlations between these variables into the volatility forecasting exercises of exchange rate returns. The general form of the CCC-GARCH model can be described as follows given the above mean equations (1):

$$\mu_t = H_t^{1/2} \Psi_t \tag{2}$$

Where  $\Psi_t$  is a vector of i.i.d random variables with mean zero and variance one;  $H_t^{1/2}$  is the conditional covariance matrix  $H_t$ . And  $H_t$  is given by:

$$H_t = D_t R D_t \tag{3}$$

Where:  $D_t$  is a diagonal matrix with elements  $h_{i,t}^{1/2}$ , the conditional standard deviations of the individual series;  $R$  is a time-invariant correlation matrix. Each element  $h_{i,t}$  of the diagonal matrix  $D_t$  follows a univariate GARCH(1,1) process:

$$h_{i,t} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1} \tag{4}$$

Where  $\omega_i > 0$ ;  $\alpha_i \geq 0$ ;  $\beta_i \geq 0$  and  $\alpha_i + \beta_i < 1$ .

The major feature of the CCC-GARCH model is that the conditional correlation of variables of interest remains constant over time, and there can be separable conditional variance forecasts for each variable. This makes the modeling exercise more straightforward while considering the correlation in the analysis.

For the univariate conditional volatility process, the GARCH (1,1) is the most appropriate model (see Boffelli & Urga, 2016)<sup>[4]</sup>. Therefore, this paper adopts the univariate GARCH(1,1) model to fit the sample and make forecasts for the last two models involving only the exchange rate return. A GARCH(1,1) model developed by Bollerslev (1986) can be written in the following form<sup>[5]</sup>:

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \tag{5}$$

Where  $h_t$  is the conditional variance;  $\omega > 0$ ;  $\alpha \geq 0$ ;  $\beta \geq 0$  and  $\alpha + \beta < 1$ .

### 2.3. Evaluation methods

This paper applied two evaluation methods for the volatility forecasting performances: a more traditional QLIKE loss function method that is based on purely statistical grounds and a more practical Value at Risk (VaR) method that is applied in the financial risk management field. The QLIKE loss function belongs to the same class of model evaluation methods as the mean squared error (MSE), the latter commonly used to evaluate the accuracy of either conditional mean or conditional variance forecasts. However, the QLIKE loss function is less affected by extreme observations in the sample and the volatility of returns (see Boffelli & Urga, 2016)<sup>[4]</sup>. Therefore, in this paper, author adopts the QLIKE loss function to conduct the comparison. The QLIKE loss function can be defined as follows:

$$L(\sigma^2, h) = \log h + \frac{\sigma^2}{h} \tag{6}$$

When  $\sigma^2$  is the true latent variance and  $h$  is a variance forecast. The model with the lowest QLIKE statistic is considered to give the most accurate volatility forecasts in terms of statistical criteria.

Value at Risk (VaR), the second method, is deemed the most famous measure of risk, introduced by J.P. Morgan in the early 1990s (see Boffelli & Urga, 2016)<sup>[4]</sup>. VaR has various other variants, such as conditional VaR. Here, the paper applies only the classic form of VaR, which can be described as follows:

$$VaR_\alpha(r) = \mu + h^{\frac{1}{2}} q_\alpha \tag{7}$$

When  $r$  is the exchange rate return;  $\mu$  is the conditional mean given by mean equation model (1);  $\alpha$  is significant level;  $h$  is the conditional variance given by the corresponding volatility forecasting models in section 2.2;  $q_\alpha$  is the  $\alpha$  th quantile of the standardized distribution of  $r$ . The VaR gives the maximum loss that could happen to an asset in a given time horizon (typically one day or one month) and at a given significance level  $\alpha$  (typically 0.01 or 0.05). After the variances are forecasted and VaR is calculated daily for the out-of-sample period, the backtesting procedure is required to compare the forecasting performance in terms of the VaR method. One of the most famous procedures is the unconditional coverage (UC) test introduced by Christoffersen (1998)<sup>[8]</sup>. First, a VaR violation is defined as:

$$I_t = \begin{cases} 1 & \text{If } L_t < -VaR_t(\alpha) \\ 0 & \text{Otherwise} \end{cases} \tag{8}$$

The VaR violation sequence is a sequence of dummy variables taking 1 when the actual out-of-sample exchange rate return is lower than the VaR and taking 0 otherwise. Next, the UC test can be conducted using this sequence, and the null hypothesis is that the sequence  $I_t$  in (8) is i.i.d Bernoulli with parameter  $(1 - \alpha)$  against the alternative that the Bernoulli parameter is equal to the empirical ratio  $\pi$ , which is defined as:

$$\pi = \frac{\text{number of violations}}{\text{total number of forecasts}} \tag{9}$$

If the VaR performs well, then the UC test must be statistically significant. The likelihood-ratio test for the UC can be written as:

$$LR_{uc} = 2\{\ln L(z;\pi) - \ln L(z;p)\} \sim \chi_1^2 \tag{10}$$

Where  $L(z;p)$  is the likelihood function of a Bernoulli variable  $z$  with parameter  $P$ :

$$L(z;p) = (1 - p)^{\text{total number of forecasts} - \text{number of violations}} p^{\text{number of violations}} \tag{11}$$

### 3. Data

The empirical work of this paper uses level logarithms of the data of interest from the US (domestic) and the UK, including the US/GBP exchange rate, one-month forward exchange rate, CPI for the US and UK, money supply for the US and UK, etc., and all of them are monthly data. The data sources for each

variable are in Appendix A, and all the variables are fully described in Section 2.1. The in-sample period is from January 1990 to December 2014 for model estimation, and the out-of-sample period (after the vertical dashed line in the figures introduced in the next paragraph) extends from January 2015 to June 2021-for model evaluation.

Figures 1-5 plot the nominal spot exchange rate, percentage change of nominal exchange rate (return), one-month forward exchange rate, Monetary fundamentals term  $x$  that relates to the MF model in section 2.1, and the ratio of US to UK’s consumer price index that relates to the PPP model. Figure 1 shows that the exchange rate is rather volatile without a trend, and time-varying means and variances indicate its non-stationarity, which is confirmed by the unit root tests in the later paragraph. Figure 2 shows that the exchange rate return looks stationary but still with a time-varying volatility. Figure 3 shows that the forward exchange rate is highly correlated with the spot rate, even displaying a close to 1-to-1 relationship. Significant jumps during the financial crisis of 2008 can be observed in both the spot and forward exchange rate time series. Figures 4 and 5 suggest that the log levels of the other two variables are not stationary and present a similar high volatility during the financial crisis as the exchange rate level, which indicates that some information is contained in these variables that people can model.

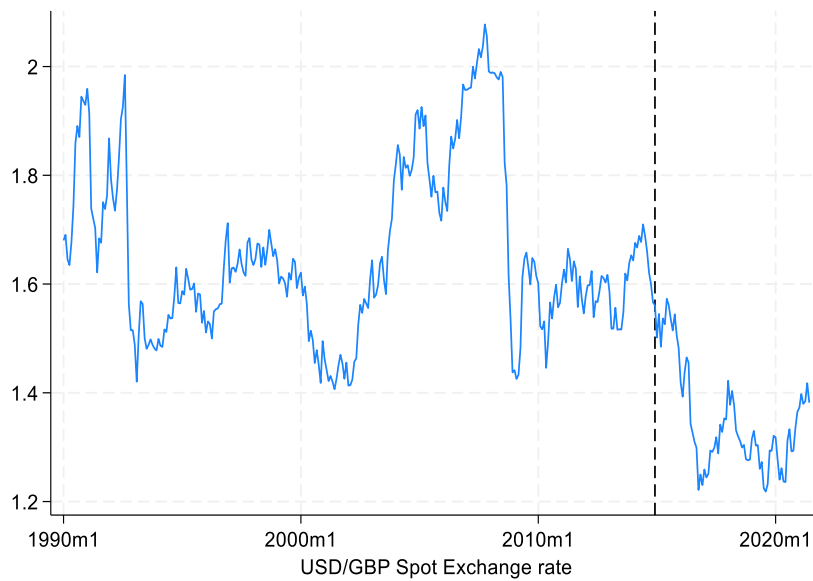


Figure 1: SD/GBP spot exchange rate (The dashed line separates the in-sample and out-of-sample period).

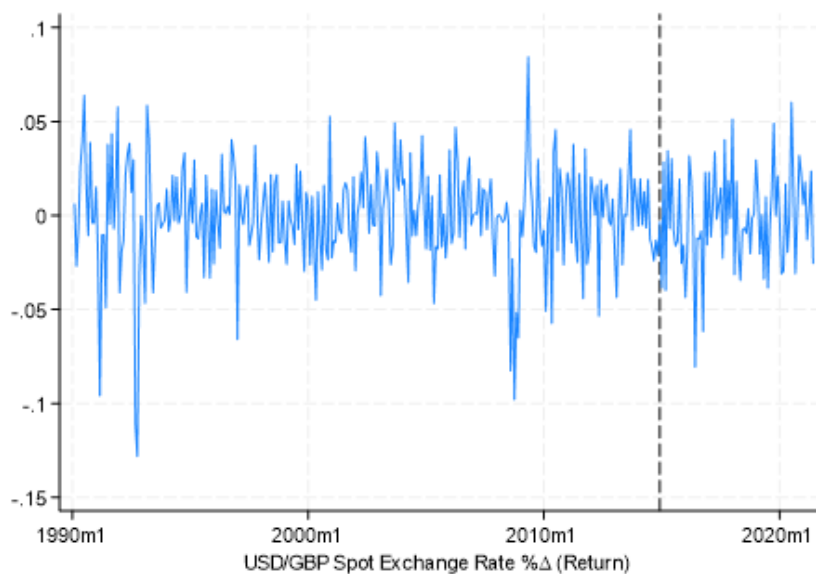


Figure 2: USD/GBP spot exchange rate change (percent).

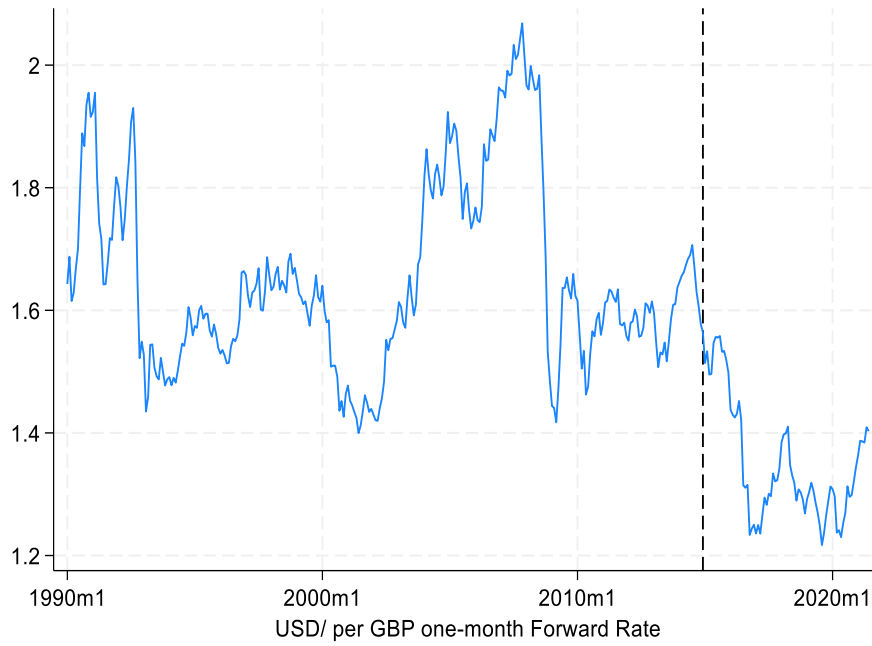


Figure 3: USD/GBP one-month forward exchange rate.



Figure 4: Monetary fundamental term in MF model.

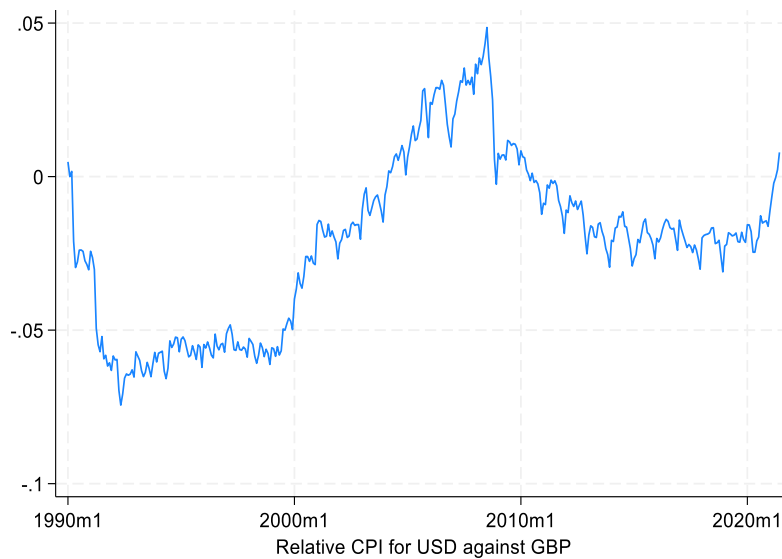


Figure 5: Relative CPI of USD against GBP.

#### 4. Model estimation and evaluation

##### 4.1. Model estimation and forecasting

The analysis of this paper, which begins by testing the variables in  $Z_t$ , shows that they are I(1), so the vector error correction model can be estimated. This paper conducted the Phillips-Perron test for unit roots, and the result failed to reject the null in levels but rejected the null in first differences, confirming all the variables are I(1) (The test output is omitted here to save space and is available on request). Next, as Garratt and Lee (2010) have tested and confirmed that a lag length of 12 is adequate and appropriate for the models in question, this paper also chooses a lag length of twelve for all the models under investigation to remove lag length uncertainty and focus on the comparison of economic and non-economic models<sup>[12]</sup>.

The next analysis, the Johansen's cointegration tests, provides mixed evidence of a long-run relationship for the three pairs of variables ( $e_t$  with  $f_t$ ,  $x_t$ , and  $p_t^*$ ). In EMH, there is a cointegration rank of one identified by the tests at both the 0.05 and 0.01 significance levels. In MF, the cointegration rank of zero is selected by the test at both 0.05 and 0.01 significance levels. In PPP, the cointegration rank of zero is selected at 0.02 and 0.01 (the output for the statistic is available on request to save space as the paper analysis focuses only on volatility forecasts evaluation). However, This paper ignores the result from this test by proceeding to build the same VECM model as Garratt and Lee's (2010) did in their forecasting exercise, which is the purpose of this study<sup>[12]</sup>. By fitting VECM models using the three pairs of variables, the cointegration equation parameters (cointegration vector  $\beta$ ) estimates for the EMH and MF models are closer to a one-to-one relationship, and they are both statistically significant. In contrast, the cointegration vector for the PPP model is far from a one-to-one relationship and is not statistically significant (See Table 1). The estimates for the mean equation in (1), upon which the paper builds volatility models, can be seen in Table 2. The R squared is rather low for all equations for exchange rate returns, which aligns with what Garratt and Lee (2010) found. Interestingly, the other equations for the variables integrated with exchange rate returns have much higher R square<sup>[12]</sup>. This means that the equations for  $\Delta f_t$ ,  $\Delta x_t$ , and  $\Delta p_t^*$  have higher explanatory power than the ones for  $\Delta e_t$ . The post-estimation diagnostic tests for VECM models suggest mixed results as well. All three models meet eigenvalue stability conditions. However, only EMH and MF models passed the Lagrange-multiplier test, while PPP exhibits massive autocorrelation up to the seventh lag. Again, the analysis of this paper only focuses on the volatility forecasts, and the exact output table is omitted here and is available on request.

After the VECM is fitted using in-sample data, this paper establishes bivariate CCC GARCH on the three theoretical VECMs. Table 3 summarizes the primary output for CCC-GARCH estimation with the GARCH (1,1) process, assuming the corresponding variables follow a joint t-distribution. The table also contains the two univariate GARCH (1,1) models for the AR and N models in (1). The coefficient results suggest that, generally, long-run persistence exists in that three out of five model's coefficients for GARCH terms are statistically significant at the 0.05 significance level, while all constant terms and ARCH terms are statistically insignificant except for the N model. After the MGARCH and GARCH (1,1) models are fitted, post-estimation procedures are conducted to determine the appropriateness of the volatility models. All models are well-behaved in terms of the lack of autocorrelation of the standardized residuals, and all models are not significantly different from the assumed t-distributions. However, the MF model failed the LM test for autoregressive conditional heteroskedasticity indicating that the ARCH effect is not completely captured by the fitted model, while the other models passed this test (the output is also omitted here).

In the next step, this paper makes one-step-ahead static forecasts of conditional variances for the out-of-sample period. The term "static" means that when making each forecast, although the model is not estimated using new observed data as a moving-window forecast is, the information set does update. For example, when making a forecast for conditional variances of exchange rate returns at time  $t + \kappa$ , namely  $h_{t+\kappa}$ , the corresponding information set is always  $I_{t+\kappa-1}$  for all  $\kappa$ . Figures 6-10 show the forecasts for the conditional variances from the five models in both the in-sample period and the out-of-sample period, with additional squared exchange rate return time series as references. It can be seen that when the return is in more volatile periods, the variance forecasts are higher for that period, and vice versa. It is obvious that the MF model gives very poor forecasts, which is expected given the earlier poorly behaved MGARCH model.

After the out-of-sample conditional variance forecasts are required for each model, the VaRs could be calculated using the method specified in (7). Figures 11-15 show the one-day VaRs at the 0.01 significance level for each model in both the in-sample and out-of-sample periods, with exchange rate returns as references. The VaRs give the minimum exchange rate returns (equivalent to maximum losses of the price of the foreign currency) each day at the 0.01 significant level. The 0.01 significant level infers that there is a 1% probability that the actual return could be lower than the corresponding VaR. From a visual inspection, the conditional volatility forecasts of EMH, PPP, AR, and N models perform relatively well compared to the MF model, as the forecasts are relatively high during volatile periods and low during more stable periods.

Table 1: Coefficients for co-integration equation.

EMH model			MF model			PPP model		
Variables	Coefficient	p-Value	Variables	Coefficient	p-Value	Variables	Coefficient	p-Value
$e_t$	1	.	$e_t$	1	.	$e_t$	1	.
$f_t$	-1.004075	0.000	$x_t$	-1.419073	0.002	$\frac{p}{p^*}$	15.93446	0.278

Table 2: Vector error correction models.

EMH				MF				PPP			
Equation	parameters	R <sup>2</sup>	P>Chi <sup>2</sup>	Equation	parameters	R <sup>2</sup>	P>Chi <sup>2</sup>	Equation	parameters	R <sup>2</sup>	P>Chi <sup>2</sup>
$\Delta e_t$	23	0.12	0.04	$\Delta e_t$	23	0.14	0.01	$\Delta e_t$	23	0.09	0.24
$\Delta f_t$	23	0.54	0	$\Delta x_t$	23	0.22	0.00	$\Delta \frac{p_t}{P_t^*}$	23	0.17	0

Table 3: Coefficients of CCC-MGARCH models and GARCH (1,1) models.

	EMH		MF		PPP		AR		N	
	Coefficients	P-value	Coefficients	P-value	Coefficients	P-value	coeffecient	P-value	Coefficients	P-value
constant	0.0001084	0.084	0.0007177	0.068	0.0000672	0.191	0.0000919	0.182	0.0006754	0
ARCH coefficient	0.0598645	0.064	0.0630941	0.260	0.0563066	0.118	0.0854155	0.075	0.2016301	0.044
GARCH coefficient	0.7482671	0.000	-0.2157502	0.721	0.8281516	0.000	0.7697081	0.000	-0.1753473	0.138
Corr. with the other var.	0.8759261	0.025	-0.0048432	0.938	0.0943885	0.123	.	.	.	.

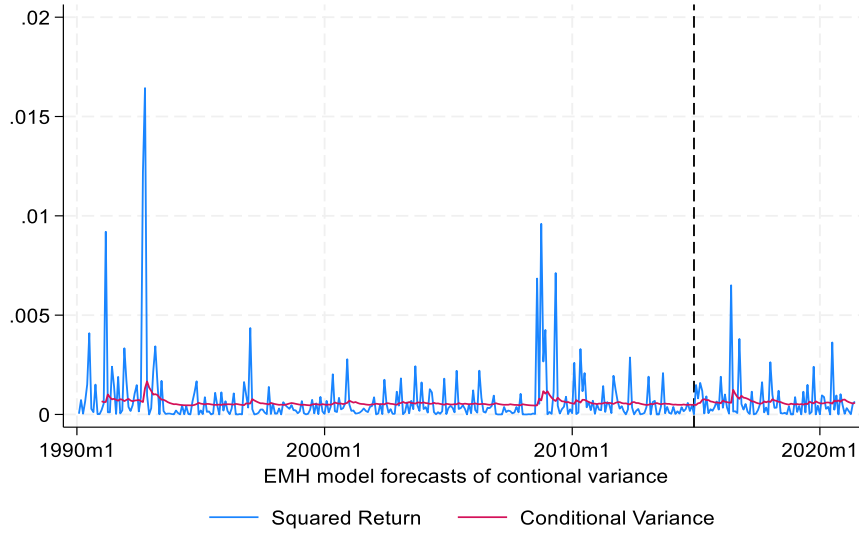


Figure 6: EMH model forecasts of conditional variances.

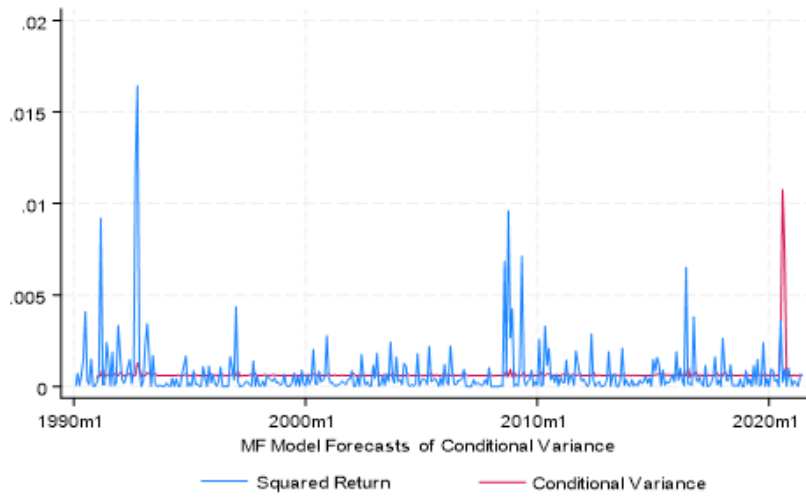


Figure 7: MF model forecasts of conditional variances.

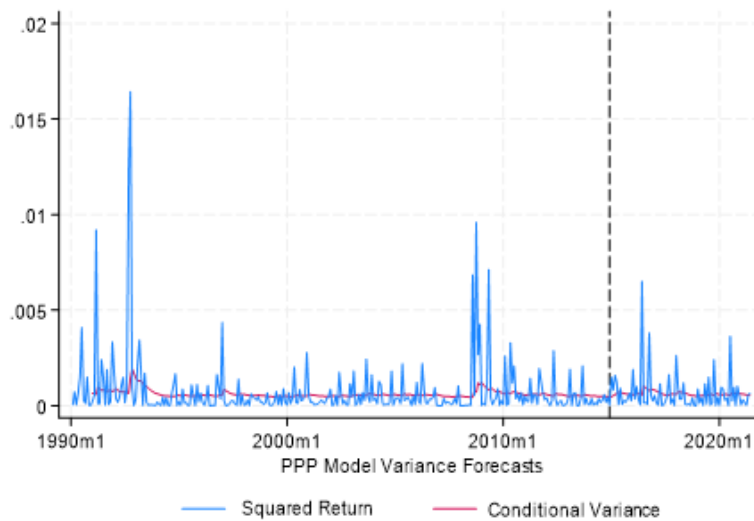


Figure 8: PPP model forecasts of conditional variances.

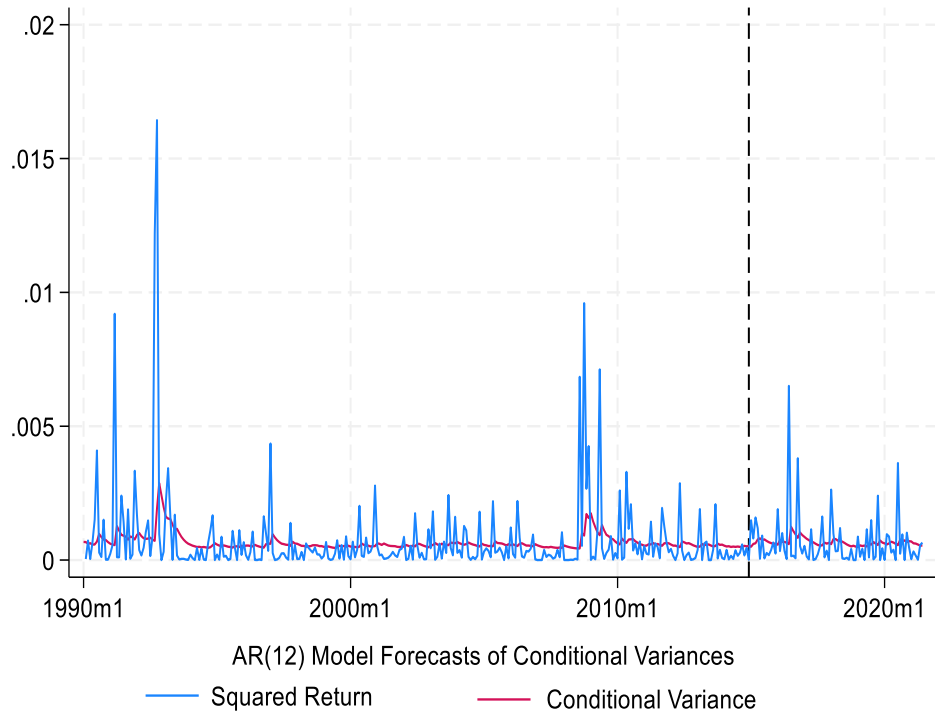


Figure 9: AR model forecasts of conditional variances.

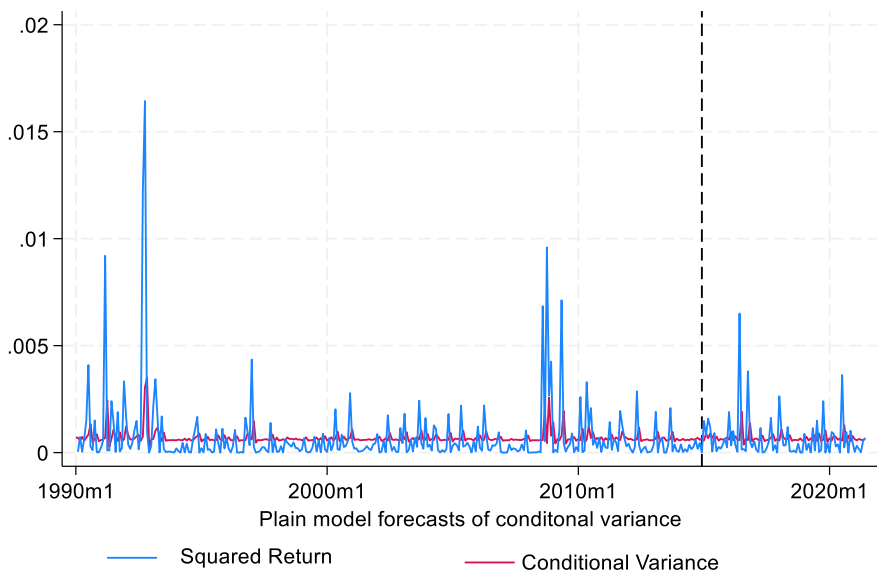


Figure 10: Plain model forecasts of conditional variances.

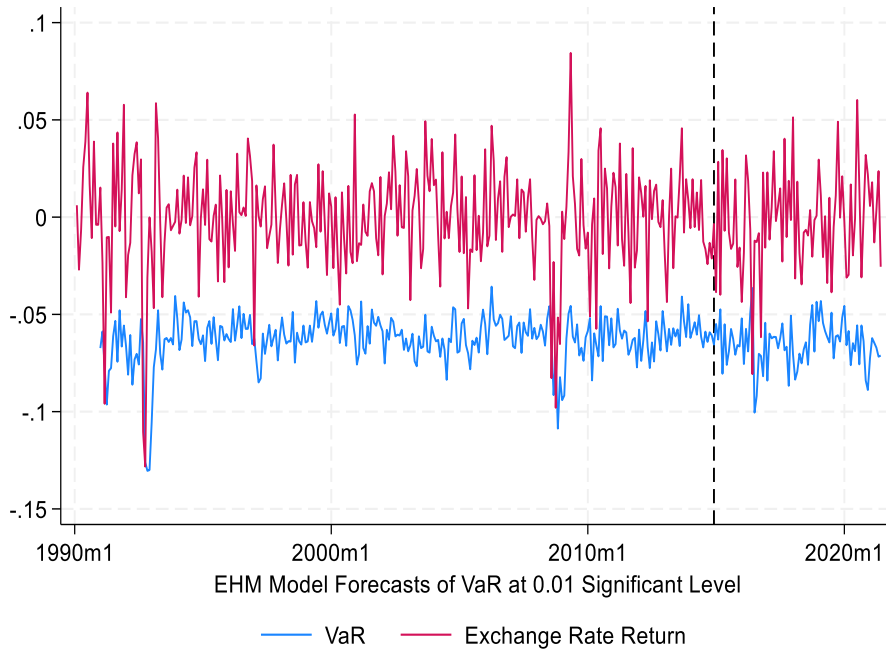


Figure 11: EMH model forecasts of VaR.

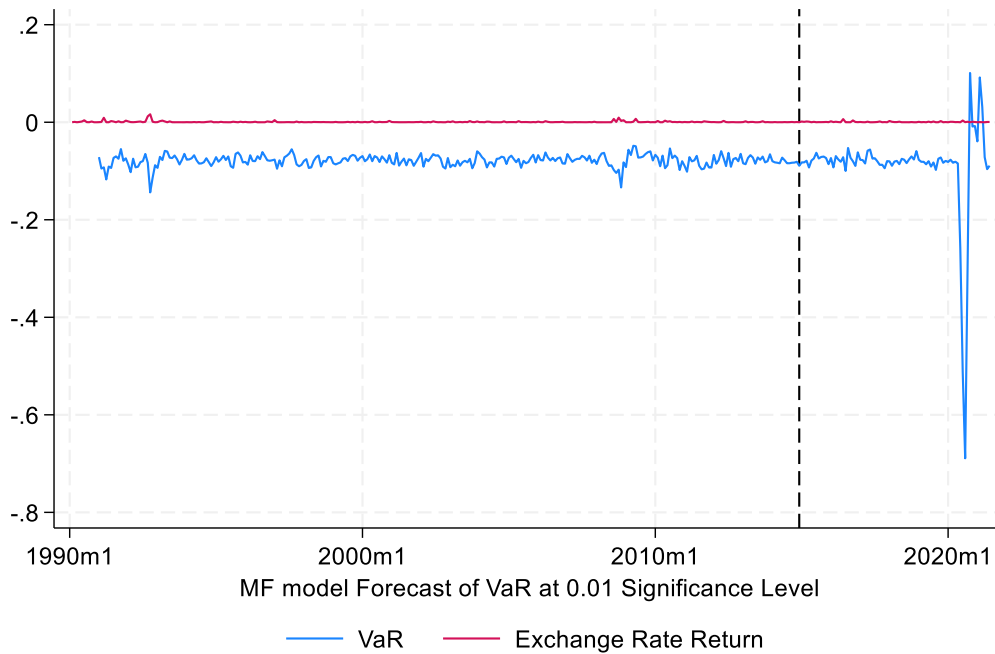


Figure 12: MF model forecasts of VaR.

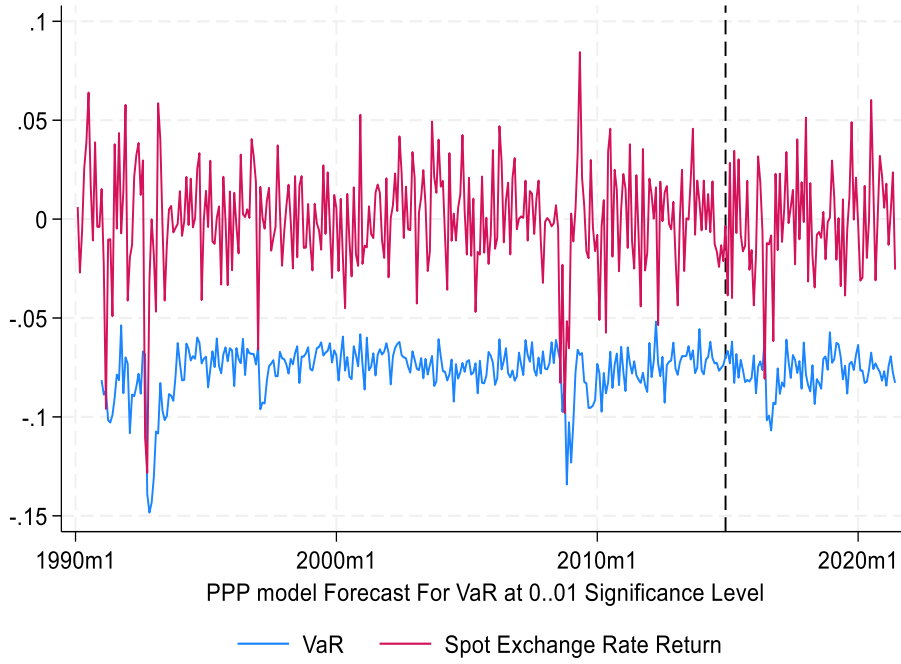


Figure 13: PPP model forecasts for VaR.

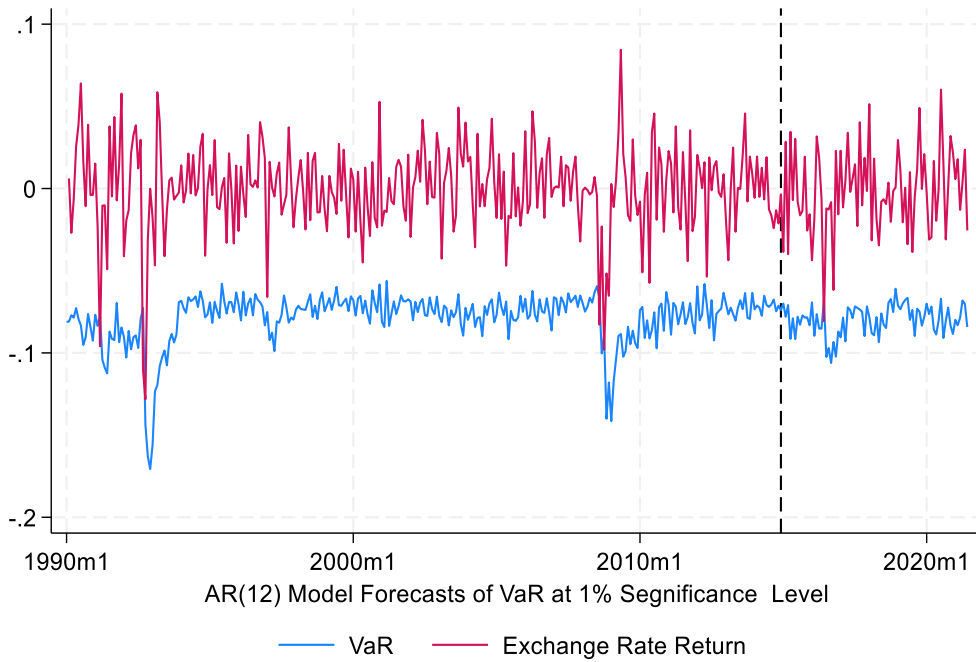


Figure 14: AR model forecasts of VaR.

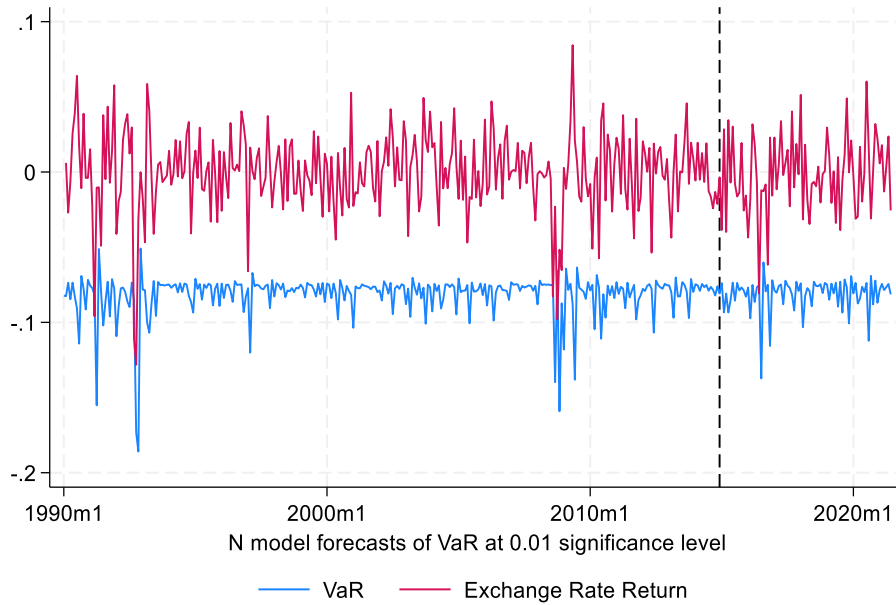


Figure 15: N model forecasts for VaR.

4.2. Model evaluation

4.2.1 Statistical criterion

In the next step of this paper’s analysis, author compare the models using the statistical criterion specified in (6), the QLIKE loss function. All evaluation exercises are conducted using out-of-sample forecasts against out-of-sample observed data. Table 4 displays the QLIKE statistic for each model, respectively, and the model with the smaller statistic is considered to be more accurate in volatility forecasting in terms of the statistical criterion. Surprisingly, this method suggests that the MF model provides the most accurate variance forecasts in terms of statistical criteria. The second-best model using this evaluation method is the EMH model, while the worst one is the N model. Using this statistical criterion, all three economically meaningful models outperform the two theoretical models.

Table 4: QLIKE loss function evaluation.

Quasikelihood variance statistic					
model	EMH	MF	PPP	AR	N
statistic	110.7514	109.66	110.9218	111.2358	113.6548

Table 5: UC test evaluation of VaR forecasts (p-value indicates the statistical significance of the equivalence of empirical violation ratio to theoretical ratio 0.01).

uc test for VaR forecasts at 0.01 significance level					
	EMH	MF	PPP	AR	N
Empirical violation ratio	0.0128	0.051	0.0128	0.0128	0.0128
P-value	0.810	0.009	0.810	0.810	0.810

4.2.2 Value at Risk backtesting criterion

Following the QLIKE loss function evaluation for each model, the VaR backtesting criterion specified in (8), (9), and (10) is conducted. Table 5 displays the empirical violation rates and the results of the UC tests for each model. Except for the MF model, all models perform well in a practical risk management environment in that their empirical ratios are very close to the theoretical ratio of 0.01, which is confirmed by the UC test, and the economic models have the same performance as pure econometric models. In the UC test, this paper failed to reject the null hypothesis that the empirical ratio

is equivalent to the theoretical one for EMH, PPP, AR, and N models. Using this practical criterion, incorporating economic theories into traditional econometric models does not provide additional benefits. However, this could be attributed to the small number of monthly out-of-sample forecasts evaluated. In future research, a study involving more forecasts and higher-frequency daily data could be conducted using the methods in this paper.

## 5. Conclusion

This research project investigates two evaluation methods on two distinct grounds, with the first one being purely statistical and the second one being more practical. It explores how these two methods yield different results on the accuracy of volatility forecasting for a series of multivariate macroeconomic-meaningful models and univariate standard econometric models. To conclude, the three economic models (EMH, MF, and PPP) produce more accurate conditional variance forecasts by statistical criteria. However, when these models are applied in a practical environment, the economic-theory-based models did not outperform the econometric models (AR and N), at least for a mid-term out-of-sample period. Therefore, integrating economic relationships into the volatility forecasting models does not add much economic value in practice. This result is similar to Garratt and Lee's (2010) findings that economic models outperform the AR models using one evaluation method but underperform using the other<sup>[12]</sup>. This evaluation exercise has the same research philosophy in the related field as what Garratt and Lee (2010) have proposed. Whenever possible, the financial forecasting models should not only be judged on a single criterion, especially a statistical one, but they should also be judged on more purpose-oriented methods such as the VaR method, which is used in financial institutions daily, or the investment environment articulated in Garratt and Lee's (2010) research<sup>[12]</sup>. The incremental contribution of this study is introducing a new VaR approach to evaluate the models instead of using only statistical criteria (such as RMSE) that are commonly used in research in this area. Moreover, this study incorporates economic theories into volatility forecasting of exchange rates, which no other study has done before.

## 6. Limitations

Due to limited computational capacity, this research does not use a more appropriate rolling-window forecasting for the conditional variances. Using a rolling-window forecasting approach, the coefficient for each model is re-estimated iteratively using the same sample size but with an updated dataset. In this study, the coefficient is only estimated once using in-sample data. Another drawback of this study is the small out-of-sample dataset that is not persuasive enough when comparing the models using the VaR approach as the VaR gives the same evaluation result for the majority of models in this study. The evaluation results based on VaR could have potentially been different from each other if the out-of-sample dataset had been larger.

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#### Appendix A. Data sources

- From International Financial Statistics: Monthly Industry production indices for US and UK in logarithm ( $Y_t$  and  $Y_t^*$ ), Monthly Consumer Price Index for US and UK in logarithm ( $P_t$  and  $P_t^*$ ).
- From Bank of England: Nominal Spot Exchange Rate for Dollars against Sterling in Logarithm ( $e_t$ ), Nominal One-Month Forward Rate for Dollars against Sterling in Logarithm ( $f_t$ ), Monthly Narrow Money Supply for UK seasonally adjusted in Logarithm ( $m_t^*$ )
- From Federal Reserve Economic Data: Monthly Narrow Money Supply for US seasonally adjusted in Logarithm ( $m_t$ ).