ISSN 2616-5902 Vol. 3, Issue 7: 74-80, DOI: 10.25236/AJBM.2021.030713

Price versus Quantity in a Mixed Duopoly with Environmental Damage

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Abstract: *This paper examines the strategic choice of price and quantity in a mixed duopoly with a public firm and a private firm when environmental damage is included. The public firm needs to bear the harm caused by environmental damage. In contrast with Matsumura and Ogawa who find price is the unique equilibrium in a standard mixed duopoly, I show that dependent on the degree of environmental damage, all four types of choices of price and quantity can constitute strategy equilibria.*

Keywords: *mixed duopoly, public firm, private firm, environmental damage*

1. Introduction

The issue of endogenous strategic choices of competition modes in oligopolistic markets has been widely discussed. The earliest study, to the best of my knowledge, can be traced back to the seminal work by Singh and Vives (1984), who find that for two profit-maximizing private firms, the choice of quantity (price) is an equilibrium strategy when the goods are substitutes (complements). This topic has been extended from several aspects. For one thing, some external elements around market environments are included into the typical analytical framework and their effects on the equilibrium competition modes are revisited. Klemperer and Meyer (1986) and Reisinger and Ressner (2009) discuss the effect of demand uncertainty on duopolistic firms' strategic choices of price and quantity. For another, similar issue has been analyzed in alternative forms of industrial organizations. Tanaka (2001a; 2001b) discusses this issue with an oligopoly of multiple firms and with duopolistic vertical product differentiation. Chirco, Colombo and Scrimitore (2014) study how duopolistic firms' strategic choices of price and quantity are affected by their managerial or entrepreneurial organizational structures. Chang, Hu and Lin (2018) derive the choice of price and quantity for a subcontractor and an outsourcer in an outsourcing context. Ghosh and Mitra (2010) compare the Bertrand and Cournot models for a mixed duopoly with a profit-maximizing private firm and a social welfare-maximizing public firm. Based on Ghosh and Mitra (2010), Matsumura and Ogawa (2012) make the choice of price and quantity endogenous and find price is the unique equilibrium. Choi (2018) shows that the result of Matsumura and Ogawa still holds when the public firm is less efficient than the private firm. Nakamura (2017) discusses the effect of delegation and network externality on the mixed duopolistic firms' choices of price and quantity. Nakamura (2017) studies how the bargaining between an owner and a manager affects the endogenous choices of strategic contracts of price and quantity in a duopolistic setting.

This paper examines the endogenous strategic choice of price and quantity for a mixed environment-friendly duopoly with a public and a private firm. Compared with the typical mixed duopoly, a remarkable feature of such a mixed duopoly is that the public firm will have to bear the harm from environmental damage in its object function of social welfare (Bian, Guo and Li 2018). I find that the degree of harm from environmental damage drastically changes the public and the private firms' strategic choice of price and quantity. In particular, when the harm from environmental damage remains rather weak, the result of Matsumura and Ogawa (2012) is still valid and both firms would choose price. When the degree of the harm increases, the public firm chooses price and the private firm chooses quantity. As this harm becomes more serious, the public firm selects quantity and the private firm selects price. Finally, when this harm becomes rather strong, both firms would choose quantity.

The reminder of this paper is organized as follows. Section 2 gives and solves the model for the mixed duopoly with environmental damage. Section 3 derives the main results. Section 4 concludes this paper.

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2. The model

Consider a mixed duopoly with a public firm and a private firm in which environmental damage is taken into account. The private firm in this context behaves like a traditional private firm, whereas the public firm has to undertake the harm of environmental damage. The inverse demand functions for the public firm, denoted as firm 1, and the private firm, denoted as firm 2, take the form of

$$p_1 = a - q_1 - \gamma q_2, \quad p_2 = a - q_2 - \gamma q_1 \tag{1}$$

in which a is a positive constant, p_1 and p_2 are prices, q_1 and q_2 are quantities, and $\gamma \in [0,1)$ measures competition intensity. Both firms' profit functions are given by

$$\pi_1(q_1,q_2) = (a - q_1 - \gamma q_2 - c)q_1, \quad \pi_2(q_1,q_2) = (a - q_2 - \gamma q_1 - c)q_2$$

in which $c < a$ is both firms' marginal cost of production. Since consumer surplus is given by

$$CS(q_1,q_2) = \frac{q_1^2 + 2\gamma q_1 q_2 + q_2^2}{2},$$

social welfare, which is the sum of consumer surplus and both firms' profits, and incorporates the environmental damage from negative production externalities, takes the form of in which $e > 0$ measures the degree of environmental damage.

$$\begin{aligned} SW(q_1,q_2) &= \pi_1 + \pi_2 + CS - e(q_1+q_2) \\ &= (a - q_1 - \gamma q_2 - c)q_1 + (a - q_2 - \gamma q_1 - c)q_2 + \frac{q_1^2 + 2\gamma q_1 q_2 + q_2^2}{2} - e(q_1+q_2), \end{aligned}$$

The model is characterized by a two-stage game. Both firms would choose price or quantity in the first stage and engage in market competition in the second stage. Note that firm 1 aims at maximizing social welfare and firm 2 intends for maximizing its profit. There are four types of strategic choices in the first stage: both firms choose quantity (q–q game), firm 1 chooses quantity and firm 2 chooses price (q–p game), firm 1 chooses price and firm 2 chooses quantity (p–q game), and both firms choose price (p–p game).

2.1 Q–Q Game

In the q–q game, both firms choose quantity and their first-order conditions are given by

$$\frac{\partial SW}{\partial q_1} = a - c - e - q_1 - \gamma q_2, \quad \frac{\partial \pi_2}{\partial q_2} = a - c - \gamma q_1 - 2q_2$$

implying both firms' reaction functions

$$q_1 = r_1(q_2) = a - c - e - \gamma q_2, \quad q_2 = r_2(q_1) = (a - c - \gamma q_1)/2.$$

Therefore, both firms' equilibrium quantities are

$$q_1^{CC} = \frac{(2-\gamma)(a-c) - 2e}{2-\gamma^2}, \quad q_2^{CC} = \frac{(1-\gamma)(a-c) + \gamma e}{2-\gamma^2}$$

and their equilibrium prices are

$$p_1^{CC} = c + e, \quad p_2^{CC} = c + q_2^{CC} = \frac{(1+\gamma-\gamma^2)c + (1-\gamma)a + \gamma e}{2-\gamma^2}.$$

Firm 1's object function of social welfare is given by

$$SW^{CC} = \frac{2q_1^{CC}q_2^{CC} + \left(q_1^{CC}\right)^2 + (2\gamma-1)\left(q_2^{CC}\right)^2}{2}$$

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and firm 2's profit is given by $\pi_2^{CC} = \left(q_2^{CC}\right)^2$. Note that in order to ensure the public firm is active in the market, the condition $(a-c)/e > 2/(2-\gamma)$ must hold.

2.2 Q–P Game

In this case, firm 1 chooses quantity and firm 2 chooses price. One can derive from equation (1)

$$p_1(q_1, p_2) = (1-\gamma)a + \gamma p_2 - (1-\gamma^2)q_1, \quad q_2(q_1, p_2) = a - p_2 - \gamma q_1$$

which imply

$$\frac{\partial q_2}{\partial p_2} = -1, \quad \frac{\partial q_2}{\partial q_1} = -\gamma, \quad \frac{\partial p_1}{\partial p_2} = \gamma, \quad \frac{\partial p_1}{\partial q_1} = \gamma^2 - 1.$$

Since SW and π_2 are functions of (q_1, q_2) and q_2 is function of (q_1, p_2), one can obtain the first-order conditions

$$\frac{\partial SW}{\partial q_1} = (1-\gamma)(a-c-e) - (1-\gamma^2)q_1$$

$$\frac{\partial \pi_2}{\partial p_2} = \frac{\partial \pi_2}{\partial q_2}\frac{\partial q_2}{\partial p_2} = c - a + 2q_2 + \gamma q_1 = a + c - 2p_2 - \gamma q_1$$

and both firms' reaction functions

$$q_1 = r_1(p_2) = \frac{a-c-e}{1+\gamma}, \quad p_2 = r_2(q_1) = \frac{a+c-\gamma q_1}{2}.$$

At equilibrium, both firms' quantities are given by

$$q_1^{CB} = \frac{a-c-e}{1+\gamma}, \quad q_2^{CB} = \frac{a-c+\gamma e}{2(1+\gamma)}$$

and their prices can be calculated

$$p_1^{CB} = c + (1-\gamma)e + \gamma q_2^{CB} = \frac{\gamma a + (2+\gamma)c + (2-\gamma^2)e}{2(1+\gamma)}$$

$$p_2^{CB} = c + q_2^{CB} = \frac{a + (1+2\gamma)c + \gamma e}{2(1+\gamma)}.$$

One can derive firm 1's object function of social welfare and firm 2's profit

$$SW^{CB} = \frac{2q_1^{CB}q_2^{CB} + (1+2\gamma)\left(q_1^{CB}\right)^2 - \left(q_2^{CB}\right)^2}{2}, \quad \pi_2^{CB} = \left(q_2^{CB}\right)^2.$$

The condition $(a-c)/e > 1$ should be imposed on to ensure a positive quantity of the public firm.

2.3 P–Q Game

Considering that firm 1 chooses price and firm 2 chooses quantity, one can get from equation (1)

$$q_1(p_1, q_2) = a - p_1 - \gamma q_2, \quad p_2(p_1, q_2) = (1-\gamma)a + \gamma p_1 - (1-\gamma^2)q_2$$

which mean

$$\frac{\partial q_1}{\partial p_1} = -1, \quad \frac{\partial q_1}{\partial q_2} = -\gamma, \quad \frac{\partial p_2}{\partial p_1} = \gamma, \quad \frac{\partial p_2}{\partial q_2} = \gamma^2 - 1.$$

Because SW and π_2 are functions of (q_1, q_2) and q_1 is function of (p_1, q_2), both firms' first-order conditions are

$$\frac{\partial SW}{\partial p_1} = \frac{\partial SW}{\partial q_1}\frac{\partial q_1}{\partial p_1} = q_1 + \gamma q_2 - (a-c-e) = c + e - p_1$$

$$\frac{\partial \pi_2}{\partial q_2} = a - c - 2q_2 - \gamma q_1 + \gamma^2 q_2 = (1-\gamma)a - c + \gamma p_1 - 2(1-\gamma^2)q_2$$

and their reaction functions are

$$p_1 = r_1(q_2) = c + e, \quad q_2 = r_2(p_1) = \frac{(1-\gamma)a - c + \gamma p_1}{2(1-\gamma^2)}.$$

Hence, one can calculate both firms' quantities

$$q_1^{BC} = \frac{(1-\gamma)(2+\gamma)(a-c) - (2-\gamma^2)e}{2(1-\gamma^2)}, \quad q_2^{BC} = \frac{(1-\gamma)(a-c) + \gamma e}{2(1-\gamma^2)}$$

and prices

$$p_1^{BC} = c + e, \quad p_2^{BC} = c + (1-\gamma^2)q_2^{BC} = \frac{(1-\gamma)a + (1+\gamma)c + \gamma e}{2}.$$

Firm 1's object function of social welfare and firm 2's profit are thus

$$SW^{BC} = \frac{2q_1^{BC}q_2^{BC} + \left(q_1^{BC}\right)^2 + (2\gamma - 1)\left(q_2^{BC}\right)^2}{2} \text{ and } \pi_2^{BC} = (1-\gamma^2)\left(q_2^{BC}\right)^2.$$

In a similar way, the condition $(a-c)/e > (2-\gamma^2)/[(1-\gamma)(2+\gamma)]$ should be satisfied in order to guarantee a positive quantity of the public firm.

2.4 P–P Game

Since both firms would choose price, one can obtain their demand functions by equation (1)

$$q_1(p_1, p_2) = \frac{a(1-\gamma) - p_1 + \gamma p_2}{1-\gamma^2}, \quad q_2(p_1, p_2) = \frac{a(1-\gamma) - p_2 + \gamma p_1}{1-\gamma^2}.$$

Obviously,

$$\frac{\partial q_1}{\partial p_1} = -\frac{1}{1-\gamma^2}, \quad \frac{\partial q_1}{\partial p_2} = \frac{\gamma}{1-\gamma^2}, \quad \frac{\partial q_2}{\partial p_1} = \frac{\gamma}{1-\gamma^2}, \quad \frac{\partial q_2}{\partial p_2} = -\frac{1}{1-\gamma^2}.$$

Since SW and π_2 can be viewed as functions of (q_1, q_2) and meanwhile q_1 and q_2 can be viewed as functions of (p_1, p_2), one can derive both firms' first-order conditions

$$\frac{\partial SW}{\partial p_1} = \frac{\partial SW}{\partial q_1}\frac{\partial q_1}{\partial p_1} + \frac{\partial SW}{\partial q_2}\frac{\partial q_2}{\partial p_1} = \frac{(1+\gamma)q_1 - (a-c-e)}{1+\gamma} = \frac{(1-\gamma)(c+e) - p_1 + \gamma p_2}{1-\gamma^2}$$

$$\frac{\partial \pi_2}{\partial p_2} = \frac{\partial \pi_2}{\partial q_1}\frac{\partial q_1}{\partial p_2} + \frac{\partial \pi_2}{\partial q_2}\frac{\partial q_2}{\partial p_2} = \frac{c - a + (2-\gamma^2)q_2 + \gamma q_1}{1-\gamma^2} = \frac{(1-\gamma)a + c + \gamma p_1 - 2p_2}{1-\gamma^2}.$$

Both firms' reaction functions can be calculated as

$$p_1 = r_1(p_2) = (1-\gamma)(c+e) + \gamma p_2, \quad p_2 = r_2(p_1) = \frac{(1-\gamma)a + c + \gamma p_1}{2}.$$

At equilibrium, both firms' quantities and prices are

ISSN 2616-5902 Vol. 3, Issue 7: 74-80, DOI: 10.25236/AJBM.2021.030713

$$q_1^{BB} = \frac{a-c-e}{1+\gamma}, \quad q_2^{BB} = \frac{a-c+\gamma e}{(1+\gamma)(2-\gamma^2)}$$

$$p_1^{BB} = c+e+\gamma(q_1^{BB}-q_2^{BB}) = \frac{(1-\gamma)\gamma a+(2-\gamma)c+2(1-\gamma)e}{2-\gamma^2}$$

$$p_2^{BB} = c+(1-\gamma^2)q_2^{BB} = \frac{(1-\gamma)a+(1+\gamma-\gamma^2)c+(1-\gamma)\gamma e}{2-\gamma^2}.$$

One can derive firm 1's object function of social welfare and firm 2's profit

$$SW^{BB} = \frac{2q_1^{BB}q_2^{BB}+(1+2\gamma)\left(q_1^{BB}\right)^2-\left(q_2^{BB}\right)^2}{2}, \quad \pi_2^{BB} = (1-\gamma^2)\left(q_2^{BB}\right)^2.$$

Here, the assumption $(a-c)/e>1$ should be made to ensure a positive quantity of the public firm.

3. Main results

Note

$$\frac{2-\gamma^2}{(1-\gamma)(2+\gamma)} - \frac{2}{2-\gamma} = \frac{\gamma^3}{(1-\gamma)(4-\gamma^2)}.$$

Denote $(a-c)/e=\lambda$ and assume

$$\lambda > \frac{2-\gamma^2}{(1-\gamma)(2+\gamma)} = \lambda_0,$$

Which ensures that the public firm always has a positive quantity in four types of competition modes.

I now derive both firms' reaction functions in the first stage. When the private firm chooses quantity, if $SW^{CC} \geq SW^{BC}$, the public firm chooses quantity (Cournot competition), $R_1(C)=C$, and otherwise the public firm chooses price (Bertrand competition), $R_1(C)=B$. One can observe

$$SW^{CC}-SW^{BC} = \frac{e^2\gamma^2[(1-\gamma)\lambda+\gamma)](\lambda_1-\lambda)}{8(1-\gamma)(1-\gamma^2)(4-\gamma^2)(2-\gamma^2)^2} \quad \text{with} \quad \lambda_1 = \frac{8-4\gamma-4\gamma^2+\gamma^3}{(1-\gamma)(4-\gamma^2)}.$$

Hence, one can obtain

$$R_1(C) = \begin{cases} C, & \lambda \leq \lambda_1 \\ B, & \lambda > \lambda_1 \end{cases}.$$

When the private firm switches to chooses price, the public firm chooses quantity ($R_1(B)=C$) if $SW^{CB} \geq SW^{BB}$ and price ($R_1(B)=B$) if $SW^{CB} < SW^{BB}$. Simple calculations show

$$SW^{CB}-SW^{BB} = \frac{e^2\gamma^2(\lambda+\gamma)(\lambda_2-\lambda)}{8(4-3\gamma^2)(1+\gamma)^2(2-\gamma^2)^2} \quad \text{with} \quad \lambda_2 = \frac{8+4\gamma-4\gamma^2-\gamma^3}{4-3\gamma^2}.$$

One can find

$$R_1(B) = \begin{cases} C, & \lambda \leq \lambda_2 \\ B, & \lambda > \lambda_2 \end{cases}.$$

On the other hand, when the public firm chooses quantity, the private firm chooses quantity

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($R_2(C)=C$) if $\pi_2^{CC} \ge \pi_2^{CB}$ and price $R_2(C)=B$ if $\pi_2^{CC} < \pi_2^{CB}$. Because

$$\pi_2^{CC} - \pi_2^{CB} = \left(q_2^{CC} - q_2^{CB}\right)\left(q_2^{CC} + q_2^{CB}\right) = \frac{e\gamma^2(\lambda_3 - \lambda)\left(q_2^{CC} + q_2^{CB}\right)}{2(1+\gamma)(2-\gamma^2)} \text{ with } \lambda_3 = 2+\gamma,$$

one can derive

$$R_2(C) = \begin{cases} C, & \lambda \le \lambda_3 \\ B, & \lambda > \lambda_3 \end{cases}.$$

When the public firm chooses price, if $\pi_2^{BC} \ge \pi_2^{BB}$, the private firm chooses quantity ($R_2(B)=C$) and otherwise the private firm chooses price ($R_2(B)=B$). One can get

$$\pi_2^{BC} - \pi_2^{BB} = (1-\gamma^2)\left(q_2^{BC} - q_2^{BB}\right)\left(q_2^{BC} + q_2^{BB}\right) = \frac{e\gamma^2(\lambda_4 - \lambda)\left(q_2^{BC} + q_2^{BB}\right)}{2(2-\gamma^2)(1-\gamma)} \text{ with } \lambda_4 = (2-\gamma)/(1-\gamma)$$

and finally

$$R_2(B) = \begin{cases} C, & \lambda \le \lambda_4 \\ B, & \lambda > \lambda_4 \end{cases}.$$

One can compare the following threshold values

$$\lambda_4 - \lambda_1 = \frac{2\gamma^2}{(1-\gamma)(4-\gamma^2)}, \quad \lambda_1 - \lambda_2 = \frac{\gamma^6}{(1-\gamma)(4-\gamma^2)(4-3\gamma^2)}, \quad \lambda_2 - \lambda_3 = \frac{\gamma^2(2+\gamma)}{4-3\gamma^2}.$$

Moreover, one can derive

$$\lambda_3 - \lambda_0 = \frac{2-2\gamma^2-\gamma^3}{(1-\gamma)(2+\gamma)} > 0$$

if and only if $\gamma < 0.8393$. Hence, given $\gamma < 0.8393$, $\lambda_0 < \lambda_3 < \lambda_2 < \lambda_1 < \lambda_4$ holds.

Proposition 1. Given $\gamma < 0.8393$. With $\lambda \le \lambda_3$, both firms choose quantity; with $\lambda_3 < \lambda \le \lambda_2$, the public firm chooses quantity and the private firm chooses price; with $\lambda_1 < \lambda \le \lambda_4$, the public firm chooses price and the private firm chooses quantity; with $\lambda > \lambda_4$, both firms choose price.

Proposition 1 can be proved as follows. Both firms choose quantity or (C,C) is equilibrium strategy if and only if $R_1(C)=C$ and $R_2(C)=C$, equivalently, $\lambda \le \min\{\lambda_1, \lambda_3\} = \lambda_3$. The public firm chooses quantity and the private firm chooses price or (C,B) is equilibrium strategy if and only if $R_1(B)=C$ and $R_2(C)=B$, that are equivalent to $\lambda_3 < \lambda \le \lambda_2$. The public firm chooses price and the private firm chooses quantity or (B,C) is equilibrium strategy if and only if $R_1(C)=B$ and $R_2(B)=C$, that is, $\lambda_1 < \lambda \le \lambda_4$. Both firms choose price or (B,B) is equilibrium strategy if and only if $R_1(B)=B$ and $R_2(B)=B$, equivalently, $\lambda > \max\{\lambda_2, \lambda_4\} = \lambda_4$.

Recalling $(a-c)/e = \lambda$, one can conclude that the degree of harm from environmental damage alters both firms' strategic choices of price and quantity. Matsumura and Ogawa [7] show that the choice of price is the unique equilibrium strategy with a typical mixed duopoly. Proposition 1 reveals that although the result of Matsumura and Ogawa continues to hold for a rather weak harm from environmental damage, their result would be invalid when the harm becomes serious. Specifically, when environmental damage begins to become serious, the public firm chooses price and the private firm chooses quantity. As the severity of environmental damage further increases, the public firm chooses quantity and the private firm chooses price. When the harm from environmental damage becomes very strong, both firms would choose quantity.

Note that for $\gamma \geq 0.8393$, the public firm would be inactive in one of the four types of competition modes. In this situation, both firms' strategic choices of Proposition 1 make sense just for $\lambda > \lambda_0$. In other words, the qualitative result of Proposition 1 remains unchanged even though some equilibrium strategies may disappear.

4. Conclusion

This paper examines the effect of environmental damage on mixed duopolistic firms' strategic choices of price or quantity and finds that the result of Matsumura and Ogawa (2012) substantially changes. The model of this paper can be extended into several aspects that serve as further research directions. In the first place, one can attempt to introduce a general inverse demand function to validate whether the result of this paper still holds. Secondly, it is of interest to introduce the asymmetry between both firms' marginal costs to check the robustness of my result. Finally, one can examine the similar issue for other forms of mixed duopoly.

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