

Optimized Decision Making for Vegetable Replenishment and Pricing Based on ARIMA Prediction and Nonlinear Programming Models

Chengxu Huang^{1,#,*}, Zhicheng Zhang^{1,#}, Qianjin Qu^{2,#}

¹*School of Information Science and Engineering, Wuhan University of Science and Technology, Wuhan, 430081, China*

²*School of Resources and Environmental Engineering, Wuhan University of Science and Technology, Wuhan, 430081, China*

[#]*These authors contributed equally.*

^{*}*Corresponding author*

Abstract: Vegetable products possess a limited shelf life and are susceptible to quality deterioration over time, necessitating the establishment of effective replenishment and pricing strategies. This study leverages historical sales and supply-related data from a superstore to conduct a comprehensive analysis. Firstly, a linear regression model is employed to explore the relationship between sales volume and cost pricing for each category across different time periods. Additionally, an ARIMA prediction model is developed to forecast the total daily sales volume with temporal sequence, with model smoothness validation conducted. Building on these analyses, predictions for the total sales volume from July 1 to July 7, 2023 are generated. Subsequently, a nonlinear planning model is constructed, with the objective of maximizing the superstore's revenue by treating the daily sales volume of six types of vegetables as the decision variable and establishing an objective profit function. The replenishment quantity for each category and the decision-making scheme for category unit price are determined through sophisticated problem-solving tools such as spsspro and MATLAB.

Keywords: ARIMA model, Nonlinear programming model, Replenishment strategy

1. Introduction

In fresh produce superstores, the perishable nature of vegetable items entails a need for daily restocking, as their quality diminishes over time. To effectively manage replenishment, merchants rely on historical sales and demand data for each commodity, despite the lack of specific individual item details and purchase prices. To ensure the superstore's profitability, the pricing of vegetables typically follows the "cost-plus pricing" method, often incorporating discounts to account for shipping losses and lower quality items. Thus, a robust market demand analysis plays a critical role in informing replenishment and pricing decisions.

In an effort to devise a strategic and efficient approach to replenishment and pricing, Tijen Fan's (2019) ^[1] research delves into the operational tactics employed by fresh produce retailers, presenting a dynamic programming model for revenue management that integrates consumer choice behavior. The study begins by formulating a dynamic pricing strategy for multiple batches of fresh produce, aligning with real-time freshness by means of solving the dynamic programming model. Subsequently, a replenishment strategy is developed, taking into account the freshness and inventory of the preceding batch of fresh products. Additionally, the study introduces four heuristic replenishment strategies to streamline the replenishment process. The findings reveal that the order quantity is contingent upon the freshness and inventory of the remaining fresh products when inventory falls below a certain threshold. Conversely, when inventory surpasses the threshold, the order quantity is solely influenced by freshness. Notably, the study highlights that order points and quantities diminish as initial freshness increases, with far-reaching implications for understanding the operational strategies of fresh produce retailers and the ramifications of consumer choice behavior on revenue management. Nevertheless, the study is not without limitations, such as the absence of an exhaustive literature review and a less detailed methodological description.

Wenchong Chen (2015) ^[2] emphasized in their study that market demand for agricultural products is

susceptible to uncertainties stemming from factors like weather, temperature, and consumer preferences, rendering the traditional inventory model unfit for application within an integrated agricultural supply chain. To address this challenge, an optimal replenishment strategy is proposed for ascertaining an integrated agricultural supply chain with stochastic demand. In this EOQ/EPQ model, shortages and backlogs are permitted, with the primary objective being to minimize the total cost of the supply chain over the planning horizon, encompassing ordering costs, holding costs, shortage costs, and purchasing costs. The study constructs a system dynamics (SD) simulation model to determine the optimal lot size and replenishment intervals, taking into consideration the nonlinear relationships and dynamic forces within the model. Furthermore, a sensitivity analysis of the simulation model is conducted through an example, revealing a 16.27% reduction in total cost with the new replenishment strategy compared to the traditional approach. The findings of the study demonstrate that the newly proposed replenishment strategy, referred to as the "intelligent method," plays a pivotal role in facilitating the supply chain to make informed and strategic decisions. It effectively simulates stochastic demand and resolves the intricate and mathematically intractable replenishment problems. The study offers valuable insights and methodologies for agricultural supply chain management, carrying significant theoretical and practical implications for enhancing efficiency and cutting down on supply chain costs. Nonetheless, certain limitations within the study, such as the reasonableness of the model assumptions and the necessity for further exploration and research on practical application validation, need to be addressed.

Hengyu Liua (2017) [3] delved into the procurement and inventory management of perishable seasonal agricultural products in their research. They underscored the need for storage of perishable products such as apples, pears, and potatoes post-procurement during the harvest season, and their subsequent retrieval during the marketing period, which holds substantial implications for wholesalers. The study formulated a finite-cycle inventory model for a single product, taking into consideration storage costs, under- and overstocking, future price and demand prospects, and product spoilage. In this model, the wholesaler procures the product in the first period, and subsequent sales are subject to fluctuating and uncertain product prices. The study aims to maximize the anticipated total profit, offering a comprehensive depiction of the optimal purchasing and inventory recovery policy, and conducts numerical experiments to scrutinize the sensitivity of various parameters and gain managerial insights. Additionally, the study compares the performance of the optimal policy with two approximate policies, demonstrating the model's value through a case study. The results highlight the ability of the optimal policy to boost expected profit by 22.4% in 2014 and decrease expected loss by 10.2% in 2015. The study provides valuable theoretical and practical guidance for the procurement and inventory management of perishable seasonal agricultural products, bearing considerable application value.

This paper examines the correlation between total sales and cost-plus pricing. Initially, the data underwent preprocessing, and a comparative analysis was conducted on the data in Dataset. It was observed that some vegetables listed in Dataset were not included in the wholesale data of Dataset, therefore these vegetables were excluded from the analysis. To investigate the relationship between total sales and pricing, this study established linear regression equations for pricing and sales for each category of vegetables across different quarters, distinguishing between pre and post-discount periods. The resulting linear regression equations provide insights into the association between total sales and cost-plus pricing.

Furthermore, by analyzing the daily sales data of each vegetable category from July 2020 to July 2023, it was observed that the data exhibits a time-series nature. Consequently, an ARIMA time-series model was developed to forecast the total daily sales for each category within the forthcoming week. This forecasting can aid in the formulation of replenishment and pricing strategies. The anticipated total daily sales for all vegetables over the course of 7 days, both pre and post-discount, were computed using MATLAB. While Dataset furnishes pricing and attrition rates for individual items relative to their sales, the forecast applies to the total daily replenishment and pricing strategy for the vegetable category. Therefore, weighted pricing and attrition rates before and after discounting were determined by factoring in the individual items within each vegetable category based on their sales volume.

Subsequently, the objective function for maximizing profit, expressed as "profit = pricing × sales volume - cost," was derived. Constraints were then formulated based on the relationship between linear regression and the range of values for each category, to calculate the replenishment program and pricing strategy that maximize profit, both pre and post-discount.

2. The structure of the Vegetable Pricing and Replenishment Model

2.1 Data pre-processing

The dataset in "www.mcm.edu.cn" was selected for comparative analysis, and vegetable numbers that did not match the table in some datasets were found, and the vegetable data with the single product numbers of 102900011011782, 102900005116776, 102900011032145, 102900005116042, and 102900011023648 were excluded.

2.2 Relationship between total sales and cost-plus pricing

In order to find out the relationship between total sales volume and pricing, we establish linear regression equations between pricing and sales volume for each category of vegetables under different quarters. The following linear regression equations were established for six categories of vegetables with sales volume under different quarters as the independent variable and the corresponding pricing in that quarter as the dependent variable, as shown in Table 1.

Table 1: Linear regression equations for different categories of vegetables

kind	Linear regression equations for sales and pricing
Cauliflower	$y = a_1x + b_1$
Philodendron	$y = a_2x + b_2$
Capsicum	$y = a_3x + b_3$
Aquatic rhizomes	$y = a_4x + b_4$
Edible fungi	$y = a_5x + b_5$
eggplant	$y = a_6x + b_6$

We fitted a linear regression by matlab to the relationship between sales and pricing of each category of vegetables before and after discounting, and obtained the results as shown in Tables 2 and 3:

Table 2: Relationship between pricing and sales volume for each category of vegetables before discounting

kind	Linear regression equations for sales and pricing
Cauliflower	$y = 11.189 - 0.001x$
Philodendron	$y = 7.236 - 0.0001x$
Capsicum	$y = 11.224 - 0.0003x$
Aquatic rhizomes	$y = 12.559 - 0.0009x$
Edible fungi	$y = 9.875 - 0.0002x$
eggplant	$y = 10.541 - 0.0009x$

Table 3: Relationship between pricing and sales volume of each category of vegetables after discounting

kind	Linear regression equation for sales volume and pricing
Cauliflower	$y = 7.795 - 0.0004x$
Philodendron	$y = 5.129 - 0.0006x$
Capsicum	$y = 8.195 - 0.0048x$
Aquatic rhizomes	$y = 9.581 - 0.0022x$
Edible fungi	$y = 11.993 - 0.0050x$
eggplant	$y = 10.5409 - 0.0009x$

2.3 Develop replenishment totals and pricing strategies

By analyzing the daily sales of each category from July 2020 to July 2023, we found that it is time-series in nature, and we can build a time-series model to predict the total daily sales of each category from July 1 to July 7, 2023, which will facilitate the development of replenishment totals and pricing strategies^[4].

2.4 The establishment of Arima time series model

For a smooth time series of total daily sales, the series can be predicted using the Arima model, let z_1, z_2, \dots, z_6 be the predicted total daily sales of all single-category vegetables from July 1 to July 7, 2023, and y_1, y_2, \dots, y_6 are the predicted single-day sales of the six categories of vegetables^[6]. The model structure is

$$z_t = c + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \dots + \phi_p X_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \dots - \theta_q e_{t-q} \quad (1)$$

Where z_i is the total daily sales time series data, c is the constant term, e_t is the white noise sequence, p, q is the order of the model, and ϕ_p, θ_q are the autoregressive and moving average coefficients.

The total daily sales of all vegetables for 7 days undiscounted versus discounted is obtained by matlab and the prediction graph for undiscounted is shown in Fig. 1.

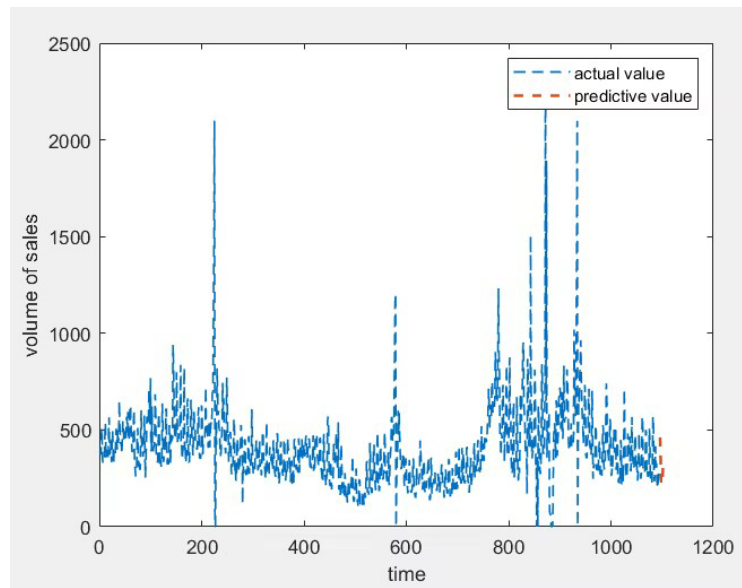


Figure 1: Forecast of total daily sales of undiscounted 7-day vegetables

The predicted data for the total daily sales of all vegetables not discounted for 7 days is shown in Table 4.

Table 4: Forecast of total daily sales of undiscounted vegetables for 7 days

date	Forecasted total sales volume (kg)
July 1, 2023	462.628
July 2, 2023	369.518
July 3, 2023	228.491
July 4, 2023	220.480
July 5, 2023	251.775
July 6, 2023	262.626
July 7, 2023	342.377

3. Results

3.1 The establishment of Single-objective planning model

Based on the relationship between the pricing and attrition rate of individual items given in the dataset and the sales volume, we ask the question of how to maximize the profit and finally arrive at the predicted total daily replenishment and pricing strategy for the vegetable category. So to start with, the individual items in each category of vegetables are weighted through the volume size relationship and the weighted pricing and attrition rates are shown in Table 5.

Table 5: Weighted pricing and attrition rates

kind	Weighted attrition rate		weighted wholesale price	
	Before the discount	After the discount	Before the discount	After the discount
Cauliflower	11.253	10.056	4.447	4.447
Philodendron	5.902	10.603	5.772	5.772
Capsicum	9.422	9.019	7.694	7.694
Aquatic rhizomes	9.373	6.305	5.690	5.690
Edible fungi	7.483	6.097	6.147	6.147
eggplant	9.880	7.255	8.090	8.090

Let the profit be L , the total cost be C , the weighted attrition rate be ε_n , and the weighted purchase price be \tilde{X} . In order to ensure that the merchant can maximize the profit, it is assumed that the sales volume is equal to the replenishment volume.

Based on the cost formula "total cost = purchase price \times sales volume \times (1 + wastage rate)", substituting the weighted purchase price and the weighted wastage rate, we get the formula for calculating the cost.

$$C = \tilde{x}_n y_n (1 + \varepsilon_n) \tag{2}$$

Define the decision variables as x_1, x_2, \dots, x_6 , where x_1 is foliar, x_2 is cauliflower, x_3 is pepper, x_4 is eggplant, x_5 is edible mushroom, and x_6 is aquatic rootstock.

Based on the profit formula "Profit = Pricing \times Sales - Cost"^[5], the objective function to maximize profit is as follows:

$$\max_{x_n, n \geq 1} L = \sum_{n=1}^6 [x_n - \tilde{x}_n y_n (1 + \varepsilon_n)] \tag{3}$$

z_1, z_2, \dots, z_6 is the predicted total daily sales of vegetables, y_1, y_2, \dots, y_6 is the predicted single-day sales of six categories of vegetables, and let x_1, x_2, \dots, x_6 be the predicted daily sales pricing of the vegetable category.

Combining the relationship between z_i and x_n in 1.3.1 and based on the regression relationship between pricing and sales in 6.2, the following constraints are obtained, and the undiscounted constraints are shown in Equation (2).

$$\left. \begin{array}{l}
 y_1 = 11.189 - 0.001x_1 \\
 y_2 = 7.236 - 0.0001x_2 \\
 y_3 = 11.224 - 0.0003x_3 \\
 y_4 = 12.559 - 0.0009x_4 \\
 y_5 = 9.875 - 0.0002x_5 \\
 y_6 = 10.541 - 0.0009x_6 \\
 z = x_1 + x_2 + \dots + x_6 \\
 x_n > 0 \\
 31.298 < x_1 \leq 1265.473 \\
 0 < x_2 \leq 186.155 \\
 6.066 < x_3 \leq 604.231 \\
 0 < x_4 \leq 118.931 \\
 3.012 < x_5 \leq 511.136 \\
 0.926 < x_6 \leq 296.792
 \end{array} \right\} \text{s.t.} \quad (4)$$

Similarly, for discounted vegetables, the following constraints are obtained based on the regression relationship between pricing and sales of discounted vegetables in 1.2, and the constraints for discounted vegetables are given in Equation (3).

$$\left. \begin{array}{l}
 y_1 = 7.795 - 0.0004x_1 \\
 y_2 = 5.129 - 0.0006x_2 \\
 y_3 = 8.915 - 0.0048x_3 \\
 y_4 = 9.581 - 0.0022x_4 \\
 y_5 = 11.993 - 0.0050x_5 \\
 y_6 = 10.541 - 0.0009x_6 \\
 z = x_1 + x_2 + \dots + x_6 \\
 x_n > 0 \\
 0 < x_1 \leq 99 \\
 0 < x_2 \leq 55.997 \\
 0 < x_3 \leq 52.492 \\
 0 < x_4 \leq 34.096 \\
 0 < x_5 \leq 66 \\
 0 < x_6 \leq 59.52
 \end{array} \right\} \text{s.t.} \quad (5)$$

Using Spsspro, this paper calculates the results of the total daily replenishment of each type of vegetable at the time of maximum profit before discounting, as shown in Table 5.

3.2 Analysis of experimental results

Table 6 predicts the total amount of replenishment and the maximum profit of each category of vegetables for each day of the week, and then combined with the "relationship between pricing and sales of each category of vegetables before the discount" can be calculated before the discount of each category of vegetables before the daily pricing strategy, the results of the calculation are shown in Table 7.

Table 6: Total replenishment scenario when profits are maximized for the week without discounting

Date	Sales by category (kg)						Maximum profit value
	x_1	x_2	x_3	x_4	x_5	x_6	
7.1	31.3	21.6	109.9	0	3.0	296.8	4928.5
7.2	31.3	0	38.4	0	3.0	296.8	3887.3
7.3	31.3	0	6.1	0	3.0	188.1	2211.9
7.4	31.3	0	6.1	0	3.0	180.1	2114.2
7.5	31.3	0	6.1	0	3.0	211.4	2495.3
7.6	31.3	0	6.1	0	3.0	211.4	2626.7
7.7	31.3	0	6.1	0	3.0	222.3	3583.7

Table 7: Pricing strategy when profits are maximized for the week without discounting

Date	Sales by category (kg)						Maximum profit value
	y_1	y_2	y_3	y_4	y_5	y_6	
7.1	7.233	11.169	11.194	10.541	9.875	8.914	4928.5
7.2	7.233	11.189	11.223	10.541	9.875	8.914	3887.3
7.3	7.233	11.189	11.224	10.541	9.875	9.018	2211.9
7.4	7.233	11.189	11.224	10.541	9.875	9.112	2114.2
7.5	7.233	11.189	11.224	10.541	9.875	9.114	2495.3
7.6	7.233	11.189	11.224	10.541	9.875	9.114	2626.7
7.7	7.233	11.189	11.224	10.541	9.875	9.113	3583.7

4. Conclusions

Due to the perishable nature of vegetable products, their quality is prone to deterioration, necessitating the establishment of effective replenishment and pricing strategies. This study aims to maximize the profit of a superstore in the upcoming week, under the premise that the store devises a replenishment plan for each vegetable category. To achieve this, we initially utilized MATLAB to derive linear equations representing the relationship between sales and pricing for various vegetable categories. Subsequently, leveraging the temporal sequence of replenishment data, we constructed an ARIMA time series model to forecast the total daily sales for the forthcoming week. Finally, we developed a planning model with profit optimization as the sole objective, and employed SPSSpro to determine the daily total replenishment quantity for each vegetable category in the upcoming week, along with the corresponding decision to achieve maximum profit. This will help merchants to clarify and maximize the profitability of wholesale vegetables and their quantities for subsequent ongoing operations. And reasonable vegetable replenishment and pricing can also avoid certain vegetable stagnation and avoid certain vegetable waste. Due to the specific analysis of real data and the establishment of scientifically rigorous models, the future practical feasibility of application is highly promising.

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