International Journal of Frontiers in Engineering Technology
ISSN 2706-655X Vol.5, Issue 11: 35-43, DOI: 10.25236/IJFET.2023.051106

Mechanistic Modeling and Optimal Design of a Heliostat Mirror Field

Ruize Liu[1,*], Shixuan Tang[1], Zijian Zhuang[1], Yijie Liu[1]

[1]*School of Physical Science and Technology, Lanzhou University, Lanzhou, China*
[*]*Corresponding author: liurz21@lzu.edu.cn*

Abstract: *The construction of a new power system mainly based on new energy is an important measure for China to realize the goal of "carbon peak" and "carbon neutrality". Tower solar photovoltaic power generation is a new type of low-carbon and environmentally friendly clean energy technology. In this paper, the position of the absorber tower, the size of each heliostat, the installation height and the position of the heliostat field of the tower-type photovoltaic power plant are investigated through the establishment of a model, and an optical efficiency model based on geometrical optics and coordinate transformation is established.*

Keywords: *Heliostat Mirror Field, Carbon Neutralization, Optical Efficiency Modeling, Genetic Algorithm*

1. Introduction

In order to actively respond to climate change, China has proposed carbon compliance and carbon neutral targets, vigorously develop clean energy, and promote comprehensive green transformation of economic and social development. Tower solar thermal power generation is a low-carbon and environmentally friendly new clean energy technology [1], through the use of a large number of heliostat mirrors to reflect the sun's rays directionally to the collector placed on the top of the tower, the sun's rays into the heat of the heat transfer medium, which generates high-temperature and high-pressure steam to drive the turbine to generate electricity. The cost of the mirror field of a tower power plant is very expensive, accounting for about 30% to 50% of the total investment of the entire power plant. As the energy input unit of the whole system, the integrated efficiency of the heliostat mirror field directly determines the highest performance of the power generation system. By optimizing the design of the heliostat mirror field to maximize the mirror output thermal power and effectively reduce the cost, it has a positive effect on the development of new energy in China.

Over the past years, the optimization design of tower-type solar thermal power plants has mostly focused on the optimization of the arrangement of the heliostat mirror field, and the literature [2-3] is based on the Monte Carlo algorithm for global optimization of the solar tower system, while the lack of relevant mechanism model is not conducive to understanding the physical process of the heliostat mirror light and heat collection. In this paper, a mechanistic modeling of the heliostat field is carried out based on geometrical optics, which reflects the physical process of heliostat light concentration and heat collection, as well as the effects of the sun's position and the parameters of the solar tower system on the heliostat field. On the basis of mechanism modeling, this paper establishes an optimization model and uses a heuristic optimization algorithm to optimize the design of the absorber tower's position coordinates, sizing of heliostat mirrors, mounting height, number of heliostats, and location of heliostats, so as to maximize the annual average output thermal power per unit mirror area.

2. Model Fundamentals

2.1 Sun-Earth position model

In an accurate model of the Sun-Earth position, the solar tensor angle is about 32′, as shown in Figure 1. The direct radiant light from the sun that strikes a point on the horizontal plane of the earth is not a parallel beam, but a cone of light with a cone angle of 9.3 mrad.

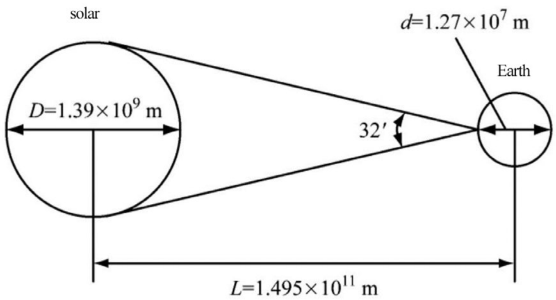


Figure 1: Diagram of solar tensor angle

2.2 Simulation of the position of the sun

Regardless of the time of day, the position of the sun is constantly changing due to the rotation and revolution of the earth, and the incident sunlight and the normal vector of the heliostat are always changing. The instantaneous optical efficiency of the heliostat field varies at each point in time due to the position of the sun. The solar position can be expressed by the solar altitude angle α_s and azimuth angle γ_s, which are calculated as follows.

$$\sin\ \alpha_s = \cos\ \delta\cos\ \varphi\cos\ \omega + \sin\ \delta\sin\ \varphi \tag{1}$$

$$\cos\ \gamma_s = \frac{\sin\ \delta - \sin\ \alpha_s\sin\ \varphi}{\cos\ \alpha_s\cos\ \varphi} \tag{2}$$

where φ is the local latitude, north is positive, ω is the solar time angle, and δ is the solar declination angle.

In this paper, we will use the ground coordinate system with the intersection of the central axis of the absorption tower and the heliostat field plane as the coordinate origin as the heliostat field coordinate system, and the geometric center of the base of the heat-absorbing tower is the origin of the coordinate system, O.

2.3 Rotated matrix coordinate transformation equations

The correspondence between a,b,c and φ,θ through the coordinate transformation matrix and a,b,c and φ,θ is shown below.

$$\begin{pmatrix} \cos\ \theta & 0 & -\sin\ \theta \\ 0 & 1 & 0 \\ \sin\ \theta & 0 & \cos\ \theta \end{pmatrix} \begin{pmatrix} \cos\ \varphi & -\sin\ \varphi & 0 \\ \sin\ \varphi & \cos\ \varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos\ \theta\cos\ \varphi & \cos\ \theta\sin\ \varphi & -\sin\ \theta \\ -\sin\ \varphi & \cos\ \varphi & 0 \\ \sin\ \theta\cos\ \varphi & \sin\ \theta\sin\ \varphi & \cos\ \theta \end{pmatrix} \tag{3}$$

3. Mechanistic modeling of the heliostat field

3.1 Modeling the optical efficiency of a fixed-heaven mirror field

Mirror field efficiency generally includes the reflectivity of the mirror surface η_{ref} (including the reflective properties of the mirror surface itself and the staining of the mirror surface exposed to air), the cosine loss η_{cos} due to the angle of the mirror surface to the incident sun line, the attenuation loss of the sunlight in the propagation process η_{at}, the shadow blocking of the light due to the interference between the heliostat mirrors η_{sb}, and the reflection to the absorber of the Sunlight spillover η_{trunc} and other various sub-efficiencies.

The instantaneous optical efficiency of a heliostat at a given moment can be calculated by the following formula.

$$\eta = \eta_{sb}\eta_{cos}\eta_{at}\eta_{trunc}\eta_{ref} \tag{4}$$

ISSN 2706-655X Vol.5, Issue 11: 35-43, DOI: 10.25236/IJFET.2023.051106

$$\eta_{sb} = 1 - \text{Loss of shadow cover}$$
$$\eta_{cos} = 1 - \text{Cosine loss} \tag{5}$$
$$\eta_{trunc} = \frac{\text{Collector receives energy}}{\text{Mirrored Total Reflective Energy} - \text{Shadows block lost energy}}$$

The mirror reflectivity η_{ref} can be taken as a constant, for example, 0.92. d_{HR} indicates the distance from the center of the mirror to the center of the collector (unit: m). The instantaneous optical efficiency of the heliostat mirror field is the result of the mutual influence of each single heliostat mirror in the mirror field, and the total efficiency of each heliostat mirror is affected by a variety of factors, so it is necessary to analyze the efficiency composition of the heliostat mirrors one by one.

3.2 Model for calculating cosine efficiency

Incident light and heliostat normal has a tilt angle θ, so that the heliostat received by the sun's incident light energy is only part of its perpendicular direction, resulting in a loss of energy received, known as the cosine loss. As shown in Figure 2.

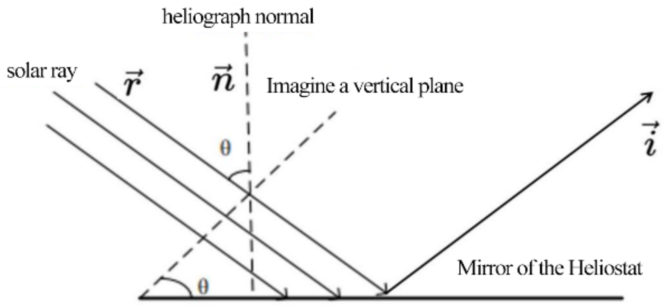


Figure 2: Schematic diagram of cosine efficiency

According to the assumptions made in this paper, the whole mirror surface is regarded as an ideal plane, and at this time, the normal vector of the mirror surface is the same for each reflection point on the mirror surface. Therefore, the cosine efficiency of the single sided mirror can be calculated by the following formula.

$$\eta_{cos} = \cos\ \theta = \vec{\iota} \cdot \vec{n} \tag{6}$$

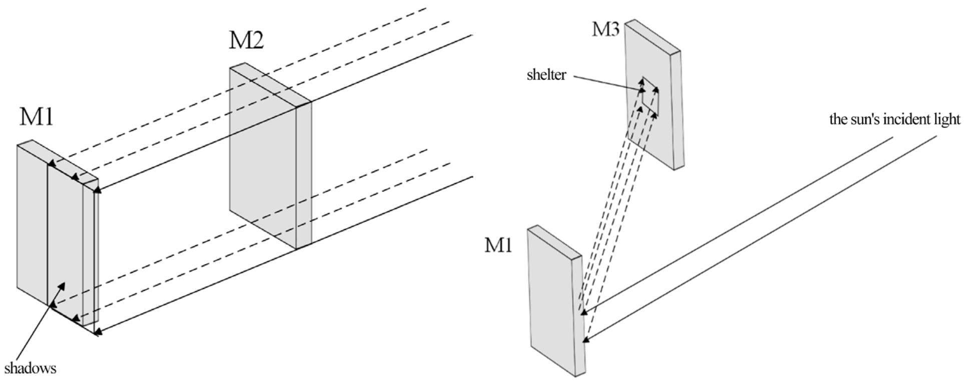
where $\vec{n}$ is the specular normal vector and $\vec{\iota}$ is the unit vector in the opposite direction of the incident light, i.e., the direction is directed from the point of specular reflection to the sun and is determined by the relative position of the sun in the geostrophic coordinate system, which can be expressed in terms of the sun's altitude angle and azimuth as follows.

$$\vec{\iota} = [-\cos(\alpha_s)\sin(\gamma_s), -\cos(\alpha_s)\cos(\gamma_s), -\sin(\alpha_s)] \tag{7}$$

3.3 A theoretical model of shadow masking efficiency

In a large heliostat field, there are a large number of heliostats in rows, so shadow masking may occur in the process of front and rear rows of heliostats in the process of concentrating light tracking. There are three types of shadow masking losses.

Especially in the morning and evening when the sun has just risen and is about to set, the sun's altitude angle is low, it is more likely to occur before and after the rows of shading and thus causing shadow loss. As shown in Figure 3a and Figure 3b.



a) Shadow loss *b) Shading losses*

Figure 3: Schematic diagram of shadow masking by a heliostat

Firstly, the shadow loss between the heliographs is analyzed. As shown in Figure 4, two coordinate systems o-xyz horizon coordinate system and o1-x1y1z1 image coordinate system (hereinafter referred to as o-system and o1-system) are illustrated.

Calculating the shadow loss is actually calculating the area of all fixed-sun mirrors that cannot receive the incident sunlight. It is easy to show that a heliostat of size $a * b$ rotates in a sphere of radius $r = \frac{\sqrt{a^2+b^2}}{2}$. If the distance between the line connecting the centers of adjacent heliostats and the incident sunlight is less than the radius r, the heliostat may have a shadow loss. Assume that the unit vector of incident sunlight is $\vec{V_0} = (a, b, c)$. The line connecting the centers of heliostat A and heliostat B is represented by the vector $\overrightarrow{n_a b} = (x_1, y_1, 0)$. $\vec{V_0} \times \overrightarrow{n_a b} < r$ is a necessary condition for the existence of shadow loss in a fixed-sun mirror. In this paper, the fixed-sun mirrors that do not satisfy this condition are excluded first, and then the remaining fixed-sun mirrors are computed iteratively, and the amount of computation is significantly reduced.

As shown in Figure 4, the sunlight direction vector $\vec{V}$, its projection on the ground is $\vec{V_d}$, and its direction is the direction of the rectangular shadow of the absorption tower. According to the collinear theorem, the shadow length h' of the absorption tower is calculated, and the width d' of the shadow of the absorption tower is equal to the width of the absorption tower d. In addition, taking into account that the heliostat may be located in the opposite direction of the shadow of the absorption tower, assuming that the vector with the starting point at the center of the absorption tower and the ending point at the center of the heliostat, the projection of the vector on the ground is $\overrightarrow{H_0}$. If it satisfies.

$$\begin{gathered} \overrightarrow{H_0} \cdot \overrightarrow{V_d} > 0 \\ k < \frac{d'}{2} + r \end{gathered} \tag{8}$$

Then it indicates that the heliostat is in the shaded region of the absorption tower.

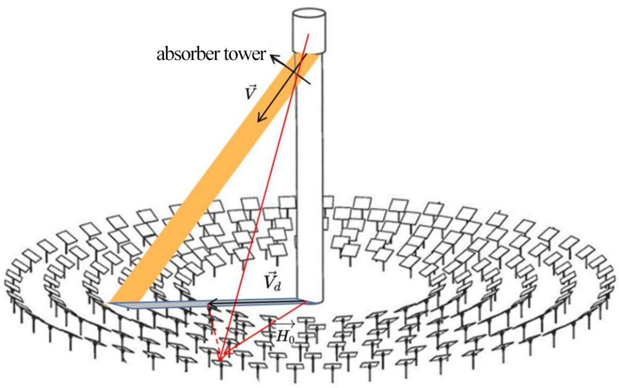


Figure 4: Shaded diagram of the absorption tower

3.4 Modeling of collector truncation efficiency

The cut-off efficiency is the percentage of energy received by the absorber to the solar energy reflected from the heliostat field.

The incident rays of the sun are cone-shaped and have a half-angle spread of 4.65 mrad; according to the reflection theorem, these rays are reflected back to the heat absorber, forming a diffuse cone-shaped light. The farther the distance between the collector and the heliostat, the larger the reflected spot. Manufacturing accuracy and surface shape errors of the heliostat affect whether the reflected spot is accurately projected to the target point and may cause it to be irregularly shaped. These factors cause part of the reflected spot to fall short of the heat absorber, resulting in some light spillage.

A ray tracing method is commonly used to calculate the optical efficiency of the mirror field spillover. The uniform tracking is done by dividing the steps in the half angle spreading direction by a uniform angle and also in the circumferential direction by a uniform angle as shown in Figure 5.

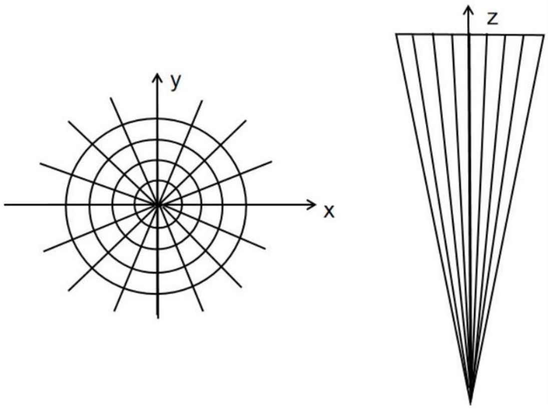


Figure 5: Angular homogenization divides the light cone into equal rays

In order to find the expression for one of the light rays in the light cone, a light cone coordinate system $O_s - X_s\ Y_s\ Z_s$ is first established, where Z' is along the direction of the main light rays in a cone-shaped beam and is directed toward the center of the solar disk. The X' axis is always parallel to the ground, and the Y' axis is perpendicular to both the Z' and the X' axes.

A beam of light cone consists of countless rays, assuming that the angle of any ray in the light cone with the Z' axis is σ and the angle with X' is τ, then the expression for any ray is.

$$\overrightarrow{S_s} = (\sin\alpha\cos\tau, \sin\sigma\sin\tau, \cos\sigma) \tag{9}$$

Find the coordinate representation of point P1 on the mirror coordinate system in the ground coordinate system.

$$P_1' = T \cdot P_1 + O_A = \begin{pmatrix} x_1' \\ y_1' \\ z_1' \end{pmatrix} \tag{10}$$

Find the vectorial representation of any ray in the light cone in the ground coordinate system.

$$\mathrm{T} = \begin{pmatrix} \sin\gamma & -\sin\alpha\cos\gamma & \cos\alpha\cos\gamma \\ -\cos\gamma & -\sin\alpha\sin\gamma & \cos\alpha\sin\gamma \\ 0 & \cos\alpha & \sin\alpha \end{pmatrix} \tag{11}$$

Transforming the vector $\vec{V_s} = (a, b, c)$ in the light-cone coordinate system to the ground coordinate system, we get:

$$\overrightarrow{V_{sl}} = T \cdot \vec{V_s} = (a_1, b_1, c_1) \tag{12}$$

Find the vector of the reflected rays in the ground coordinate system. Assuming the unit vector normal to the heliostat $\overrightarrow{V_N} = (u_0, v_0, w_0)$, find the vector of reflected rays of this light ray after reflecting through

ISSN 2706-655X Vol.5, Issue 11: 35-43, DOI: 10.25236/IJFET.2023.051106

the heliostat as $\overrightarrow{V_R}$. The angle between the two vectors is assumed to be θ, and the rest of the chords are computed as shown below.

$$\cos\theta = \frac{\overrightarrow{V_{sl}} \cdot \overrightarrow{V_N}}{\left|\overrightarrow{V_{sl}}\right|\left|\overrightarrow{V_N}\right|} = \overrightarrow{V_{sl}} \cdot \overrightarrow{V_N}$$
$$\overrightarrow{V_R} = 2\cos\theta\overrightarrow{V_N} - \overrightarrow{V_{sl}} = (m, n, l) \tag{13}$$

The equation of a reflected ray expresses.

$$\frac{x - x_1}{m} = \frac{y - y_1}{n} = \frac{z}{l} \tag{14}$$

Solve for the coordinates of the point of intersection of the rays of light according to the equation of the heat receiving surface of the collector. The equation of the reflected light has been obtained, combined with the equation of the receiving surface of the collector, the intersection of the reflected light on the collector can be obtained. The collector is known to be a cylinder, and its equation is expressed as follows.

$$\begin{cases} x^2 + y^2 = R^2 \\ z \in \left[-\frac{h}{2}, \frac{h}{2}\right] \end{cases} \tag{15}$$

Combined with the above formula, you can solve for the coordinates of light incident on the collector.

3.5 Calculation of average annual thermal power output

The solar radiation energy received at the heliostat site varies periodically with time as the orientation of the sun changes throughout the year. Normal direct radiation irradiance DNI (unit: $\mathrm{kW/m^2}$) is the solar radiation energy received per unit area of the plane on the earth perpendicular to the sun's rays, per unit time.

$$E_{\text{field}} = \mathrm{DNI} \cdot \sum_{i}^{N} A_i \eta_i \tag{16}$$

In order to calculate the thermal power output, the DNI needs to be calculated firstly, according to the approximate formula of DNI, combined with the sun's altitude angle and azimuth angle, the DNI value can be calculated at different moments of each day.

4. Optimized design of a heliostat mirror field

4.1 Fixed-day mirror field layout

4.1.1 radial staggered layout

In the radial conventional layout (shown in Figure 6), the heliostats are uniformly distributed on a number of concentric circles centered on the absorption tower with a certain regularity, and we assume that the coordinates of the bottom center of the absorption tower are (0, 0, 0). The layout method uses two parameters, radial spacing ΔR and circumferential spacing ΔAz, to represent the positions of the heliostats around the absorption tower.

ISSN 2706-655X Vol.5, Issue 11: 35-43, DOI: 10.25236/IJFET.2023.051106

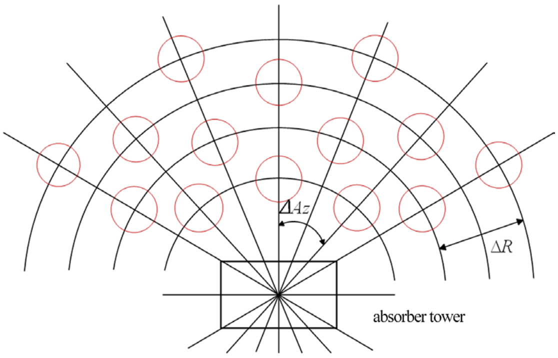


Figure 6: Traditional radial staggered layout

4.1.2 Campo Layout

Campo is a circular heliostat field arrangement method proposed by FJCollado et al. By this method, a flexible and regular radially staggered dense heliostat field can be generated [4].The Campo regular arrangement of the mirror field starts with the first row of the first arrangement area. The radius R_1 of the first row is calculated from the number of heliostats $Nhel_1$ in the first row. The second row of heliostats has the same number of mirrors as the first row, and is staggered with the first row of heliostats so that the characteristic circles of the neighboring rows of heliostats are tangent to each other. When the adjacent rows of heliostats are arranged in the densest way, the geometric relationship between the heliostats is shown in Figure 7.

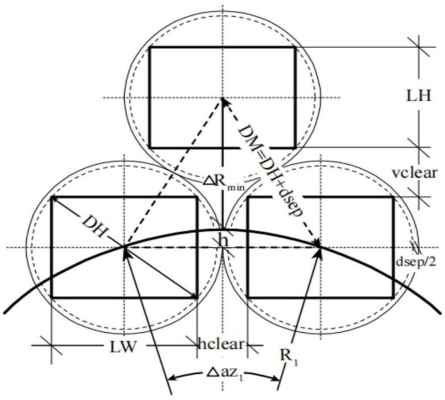


Figure 7: Geometric relations for the densest arrangement of heliostats

4.1.3 EB Layout

Based on the combination of the radial staggered layout and Campo layout, and optionally using the EB layout [5], the azimuthal spacing of the heliostats in each region of the mirror field increases as the radius of the mirror field increases. The geometric relationships are shown in Figure 8.

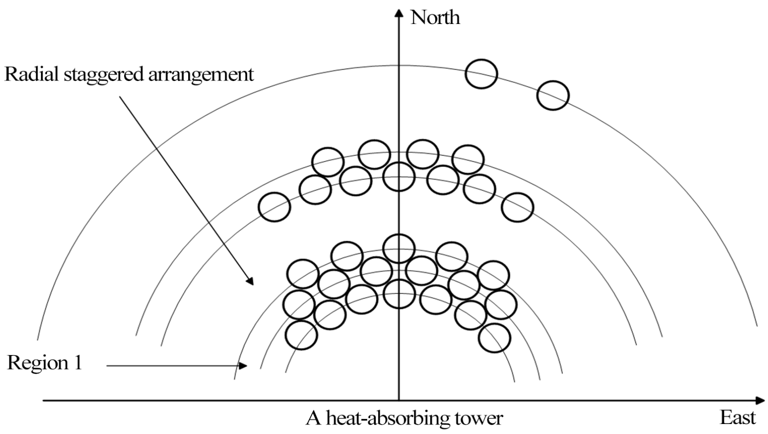


Figure 8: Schematic diagram of EB layout

ISSN 2706-655X Vol.5, Issue 11: 35-43, DOI: 10.25236/IJFET.2023.051106

4.2 Optimized design of heliostat mirror field

4.2.1 Optimization Modeling

In this paper, the dense heliostat field generated by the EB arrangement method is selected as the initial field before optimization, and then according to the relevant constraints, combined with the heuristic optimization algorithm, the maximum annual average thermal power output per unit mirror area is used as the evaluation criterion. The location of the absorption tower, the size of the heliostat, and the mounting height are also taken as the optimization variables. This method reduces the number of input variables to a large extent and greatly reduces the computational volume.

Decision variables: the location of the absorption tower, the size of the heliostat and the installation height.

Objective function: the objective is to find the maximum annual average output thermal power per unit mirror area of the heliostat field to achieve the rated annual average output thermal power constraints: the arrangement of the heliostat field is in accordance with the EB arrangement method, and satisfies:

$$\begin{cases} 2 \le d, l \le 8 \\ 2 \le h \le 6 \\ \dfrac{\sqrt{d^2 + l^2}}{2} \le h \\ d \le l \\ 100 \le \sqrt{x_i^2 + y_i^2} \le 350 \\ l + 5 < \sqrt{(x_i - x_i')^2 + (y_i - y_i')^2} \\ l + 5 \le \sqrt{(x_i - x_i')^2} \\ l + 5 \le \sqrt{(y_i - y_i')^2} \\ P \ge 60MW \end{cases} \tag{17}$$

Where, d is the mirror height of the heliostat, l is the mirror width of the heliostat, h is the mounting height of the heliostat x_i, y_i are the X-axis coordinates and Y-axis coordinates of any heliostat in the coordinate system of the mirror field, and x_i', y_i' are the X-axis coordinates and Y-axis coordinates of the heliostat neighboring to any heliostat in the coordinate system of the mirror field. P is the rated annual average thermal power output of the fixed-sun mirror field.

4.2.2 Solving the model

Due to the large number of decision variables and the large solution space, heuristic algorithms are used to realize the solution to them. Here the genetic algorithm is chosen, and the following is a brief introduction to the genetic algorithm.

Genetic algorithm is a computational model of biological evolution process that simulates the natural selection and genetics mechanism of Darwin's biological evolution theory, and searches for the optimal solution by simulating the natural evolution process. It simulates the phenomena of replication, crossover and mutation that occur in natural selection and reproduction.

The basic idea of genetic algorithm is an optimization method that performs a global search in the solution space in parallel and dynamically through the transformation, iteration and mutation of a large number of alternative solutions. Its main features are that it operates directly on structural objects without the qualification of derivation and function continuity; it has inherent hidden parallelism and better global optimization search capability; it adopts probabilistic optimization search method, which can automatically obtain and guide the optimized search space without the need of determining the rules, and adaptively adjusts the search direction.

The optimal design of the heliostat field can be obtained by following the above steps as shown in Figure 9.

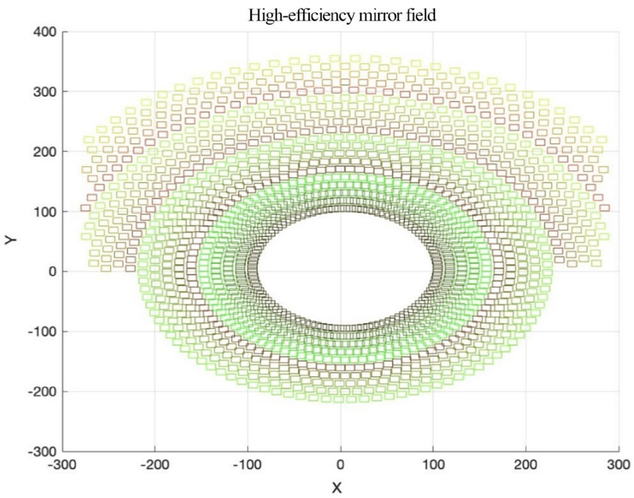


Figure 9: 2D visualization of the efficient mirror field

5. Conclusion

This paper establishes a mathematical model of sun-setting mirror concentrating light and heat collection based on geometric optics, and on the basis of the established mathematical model, the annual average optical efficiency, annual average output thermal power and annual average output thermal power per unit mirror area of the sun-setting mirror field are calculated. At the same time, considering the process boundary constraints, the optimization model is established, and the heuristic algorithm is used to optimize the design of each parameter of the heliostat mirror field, so as to maximize the annual average output thermal power per unit mirror area, achieve the purpose of cost reduction and efficiency, and provide a reference scheme for the arrangement of the tower-type photovoltaic power station.

References

[1] Zhang et al, Calculation method of optical efficiency of solar tower photothermal mirror field [J], Technology and Market, 2021, 28(6): 5-8.
[2] Sun Hao, Gao Bo, Liu Jianxing. Study on the layout of heliostat mirror field for tower solar power plant [J]. Power Generation Technology, 2021, 42(06): 690-698.
[3] Wei Xiudong, Lu Zhenwu, Lin Zi et al. Optimized design of mirror field for tower-type solar thermal power plant [J]. Journal of Optics, 2010, 30(09): 2652-2656.
[4] Nageb M M, El-Samahy A A, Rady M A, et al. An Integrated Design Approach of Local Control System of a Linear Drive Single Facet Heliostat[C]//ASME 2016 Power Conference collocated with the ASME 2016 10th International Conference on Energy Sustainability and the ASME 2016 14th International Conference on Fuel Cell Science, Engineering and Technology.2016.DOI:10.1115/POWER2016-59375.
[5] Cheng Xiaolong. Research on optimal design of tower power plant mirror field layout based on optical efficiency [D]. Hefei University of Technology, 2019.