

A Contour Interpolation Method Based on the Nearest Distance Method to Construct Auxiliary Lines

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Abstract: The regional interpolation method calculates the characteristic distance of the contour nodes to complete the node matching, divides the contour into multiple regions with consistent change trends, and constructs auxiliary lines equally within the region to complete the interpolation. Aiming at the uncertainty of the area divided by the regional interpolation method and the redundancy of the auxiliary line constructed, a contour interpolation method using the nearest distance method to construct the auxiliary line is proposed. Firstly, the Douglas-Peucker algorithm is used to extract the feature points of the contour line, and the nearest distance method is used to construct the auxiliary line for the feature points, and then the new contour line is interpolated. The nearest distance method constructs the auxiliary line based on the distance between the feature point and the feature point. The auxiliary line can reflect the change trend of the contour line, so that the new contour line can retain the curve features of the original contour line. Compared with the regional interpolation method, the method in this paper has a single uncertainty factor and the auxiliary lines constructed are simple and efficient, which improves the interpolation efficiency. Through experimental comparison, the effectiveness of the method is verified.

Keywords: Contour line, Auxiliary line, Nearest distance method, Douglas-Peucker, Feature point

1. Introduction

In geological maps, contour lines are used as linear elements to describe the actual landform. In the process of mapping, problems such as sparse contour lines and too large contour interval may be encountered. Contour interpolation refers to automatically interpolating a number of new contour lines that are consistent with the original contour line shape on the basis of the existing contour lines, which can be used to improve the accuracy of the regional map to meet the actual needs^[1]. Automatic contour interpolation is an important part of GIS research, and plays an important role in topographic map reduction, 3D terrain reconstruction and other applications.

With the digital evolution of maps, many contour interpolation methods have emerged. According to different data sources, they can be divided into two categories: one is based on discrete points, the other is based on original contours. There are two basic methods based on discrete points, the grid method and the triangular network method. These two methods are highly automated, but only consider the location attribute of the point, not the line attribute of the contour line. The interpolated contour line often intersects the bus, and the quality is poor^[1].

The method based on the original contour line cannot be separated from the auxiliary line, which can also be called the auxiliary line method. Common methods include the minimum angle maximum principle method, the nearest distance method, the inscribed circle method, the regional interpolation method, etc.^[2-9]. The principle of minimum angle and maximum angle is an important criterion of Delaunay triangulation, which can also be used to build a strip triangle network between contour lines to describe the terrain. The two sides of the triangle are auxiliary lines. This method is simple and efficient, but in areas where the contour changes rapidly or varies greatly, the constructed auxiliary line may intersect the bus, so the interpolated contour line may intersect the bus. The nearest distance method^[5] selects the zigzag to construct the auxiliary line. The relationship between the front and back vertices of the zigzag is that the next vertex is the nearest distance point of the previous vertex on the adjacent

contour line. Each point of the contour line at the bottom of the triangle area of the zigzag is connected with the vertex. This method is simple and fast, and the area with sharp contour changes will also have auxiliary lines intersecting with the bus.

Many scholars have proposed different solutions to the problem of the intersection between the area bus and the auxiliary line with rapidly changing contour lines. An Xiaoya et al. put forward a local interpolation algorithm based on the principle of minimum angle maximum in the literature [2], which uses the Doiglas-Peucker algorithm [10] (D-P algorithm for short) to extract the feature points of the contour lines, decomposes the contour lines by judging the similarity of the contour lines at the feature points, and decomposes two contour lines with low similarity into several simple contour lines for interpolation [4]. This method aims at the problem of poor interpolation quality of the minimum angle maximum principle method in the area with low contour similarity, and effectively reduces the intersection of auxiliary lines and buses, thus improving the interpolation quality. Qian Haizhong et al. proposed a method to obtain auxiliary lines by using inscribed circles to detect the spatial relationship between adjacent contour lines with the contour node as the center of the circle. The radius of the inscribed circle is the auxiliary line [6]. This method not only reduces the intersection between the bus and the auxiliary line, but also has high efficiency. However, the new contour lines interpolated by this method have distortion in the areas where the contour lines change rapidly or are far apart. Lin Chunfeng et al. proposed a regional interpolation method, which uses characteristic parameters to describe known contour nodes, calculates the characteristic distance of nodes to match contour nodes, and divides the contour into multiple regions with consistent change trends [8]. This method can effectively reduce the intersection of bus and auxiliary lines by building auxiliary lines uniformly in different regions. The quality of interpolated contour lines is good, but the process is complex and time-consuming.

To sum up, the auxiliary line method shall meet the following requirements: 1) The bus and auxiliary line shall not intersect as much as possible; 2) The contour terrain shall be taken into account to ensure the interpolation quality; 3) Satisfy quality while reducing time consumption.

2. The Regional Interpolation Method

The idea of regional interpolation method is to use the spatial attributes of contour nodes as characteristic parameters to describe nodes. The definition of node characteristic parameters is as follows:

$$P_i = \{X_i, Y_i, Z_i, \alpha_i, \beta_i\} \quad (1)$$

P_i is the characteristic parameter of the i th node on a contour line; X_i , Y_i and Z_i is the spatial coordinate value of the node; α_i and β_i is the angle value between the connecting line of the current node and the front and rear nodes and the horizontal direction respectively.

The feature parameters are used to match the nodes on two adjacent contour lines. The matching rule is: suppose the current node is P_c . Find the set R of M points closest to the point in the adjacent contour lines, and calculate the point P_c and each point P_i in set R_i Characteristic distance D_i ($P_i \in R, i \in [1, M]$), The calculation formula is:

$$D_i = (P_c \cdot \alpha - P_i \cdot \alpha)^2 + (P_c \cdot \beta - P_i \cdot \beta)^2 \quad (2)$$

Set distance threshold d , if $D_i > d$, judge the next point; Otherwise, P_i and P_c are matched successfully.

Select one of the two contour lines as the baseline, traverse each node on the baseline, find the matching node on the adjacent contour lines and connect them, as shown in Figure 1(a), there are cross and common points; Formulate rules to eliminate intersecting and common lines. The elimination results are shown in Figure 1(b). The remaining lines divide the bus into areas with consistent change trends.

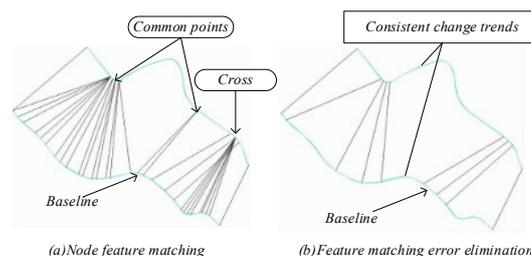


Figure 1: Contour division area.

Build multiple auxiliary lines in a fixed number and equally in each section area, as shown in Figure 2(a); Interpolate new contour lines on the basis of auxiliary lines, as shown in Figure 2(b); Using the bisection method to construct the auxiliary line will cause too many nodes on the new contour line. Use the D-P algorithm to simplify the new contour line to get the final result, as shown in Figure 2(c).

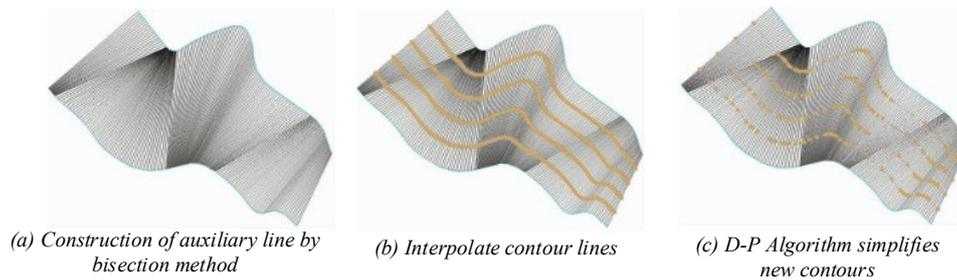


Figure 2: Building auxiliary line and Interpolating contours.

3. The Contour Interpolation Method Based on Nearest Distance

3.1 Methodology

The region division is the result of the node matching of the region division is the result of the node matching of the regional interpolation method. Whether the change trend of the divided region is consistent determines the final interpolation effect. If not, the interpolation contour may be distorted in the region. The multi segment contour area is intercepted on the topographic map to analyze the regional division of the sub regional interpolation method. Define the concepts of regional similarity and line segment change degree to classify the contour regions. If the consistency of the change trend of the two contours in the region is high, the regional similarity is high; otherwise, the regional similarity is low; The change degree of the line segment refers to the fluctuation degree of the contour line segment in the region, which is divided into two categories: smooth and rapid; Based on the comprehensive regional similarity and line segment change degree, regions are divided into four categories: high similarity, smooth change, high similarity, rapid change, low similarity, smooth change, and low similarity, rapid change, as shown in Figure 3.

In areas with high similarity, no matter whether the change is smooth or rapid, the nodes on the baseline can almost find the consistent point of change trend on the adjacent contour lines, as shown in Figure 3(a) and 3(c). However, the regions are highly similar, and most node connections do not help the interpolation results, resulting in redundancy; In addition, auxiliary line may have multiple lines at the same point or crossing, and redundant lines need to be removed to increase the time consumption.

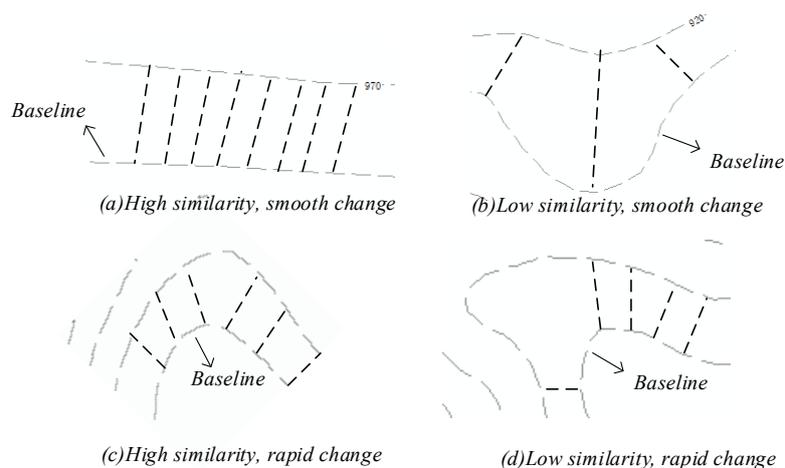


Figure 3: Four kinds of contour area partition.

In the regional interpolation method, one of the two contour lines is selected as the baseline, and the regions divided by the auxiliary lines with different baselines are different. The baselines of the three regions in Figure 3 are replaced, as shown in Figure 4. Different baselines will change the number of consistent points in the change trend. Some points that did not exist before the change may be added,

such as Figure 4(b) and Figure 4(c), or some points that existed before the change may be deleted, such as Figure 4(a). However, the increase or decrease of change trend consensus points has an uncertain impact on the final interpolation results. It has no impact on the interpolation results of regions with high similarity, while it may have no impact on regions with low similarity, as shown in Figure 4(a), or it may have an impact, as shown in Figure 4(c).

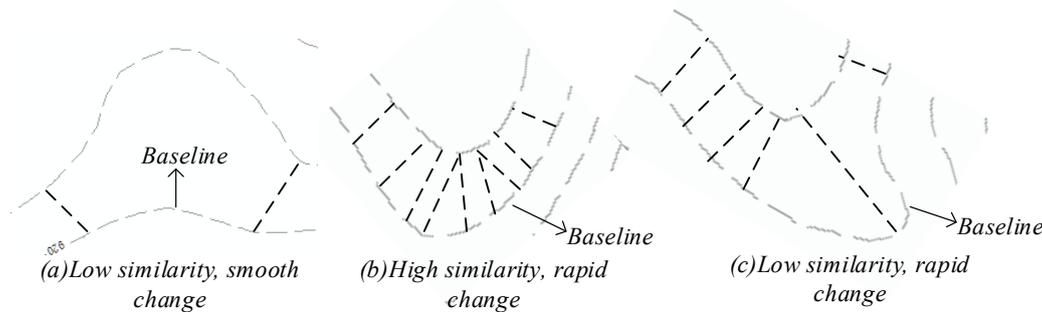


Figure 4: Contour area partition after baseline exchange.

To sum up, the regional interpolation method has the following limitations: 1) Uncertainty: the consistency point of change trend is judged by the size relationship between the characteristic distance D_i and the threshold value d , as well as the selection of potential targets from the nearest M points. The value of d and M will have an impact on the regional division, and the increase of M will increase the time consumption; Different choices of baselines may lead to different results of regional division. 2) Redundancy: the equal division method is used to construct auxiliary lines to make the auxiliary lines redundant in areas with high similarity and gentle changes. The change degree of each area can be calculated to determine the equal division to avoid redundancy, but it will increase the complexity of the method; 3) There are cross points and common points in the rapidly changing area node matching, so the matching results need to be filtered out according to rules, which will cause inconsistent change trends in partially matched areas, leading to local distortion of interpolated contours.

In view of the limitations of the regional interpolation method, a new region division strategy is proposed: the regions with high similarity and smooth change are divided by connecting lines at the contour turning point, and the original region is divided into multiple regions with single change trend, without redundant dividing lines, so as to avoid redundancy; In the area with low similarity and high similarity, the feature points of the bus are extracted and saved. Rules are made to connect the feature points. The interpolated new contour lines can retain the curve features of the bus and meet the interpolation requirements without using the bisection method to build auxiliary lines.

3.2 Method Flow

Based on the strategy of region division, a new contour interpolation method is proposed. First, D-P algorithm is used to simplify the contour lines to obtain nodes as feature points [11], and the two groups of feature points obtained are used to construct auxiliary lines using the nearest distance method to interpolate new contours.

3.2.1 D-P Algorithm to Extract Feature Points

In practical application, it may be to encrypt specific areas in the topographic map or to reduce the whole topographic map. In the former case, the D-P algorithm can be directly used in the designated area; If it is the latter, because there are a large number of closed contours in the topographic map, and D-P algorithm is applicable to the curve with starting point and ending point, it is necessary to segment the closed contours to use D-P algorithm. A feasible segmentation method suitable for D-P algorithm is designed, and the segmentation process is shown in Figure 5.

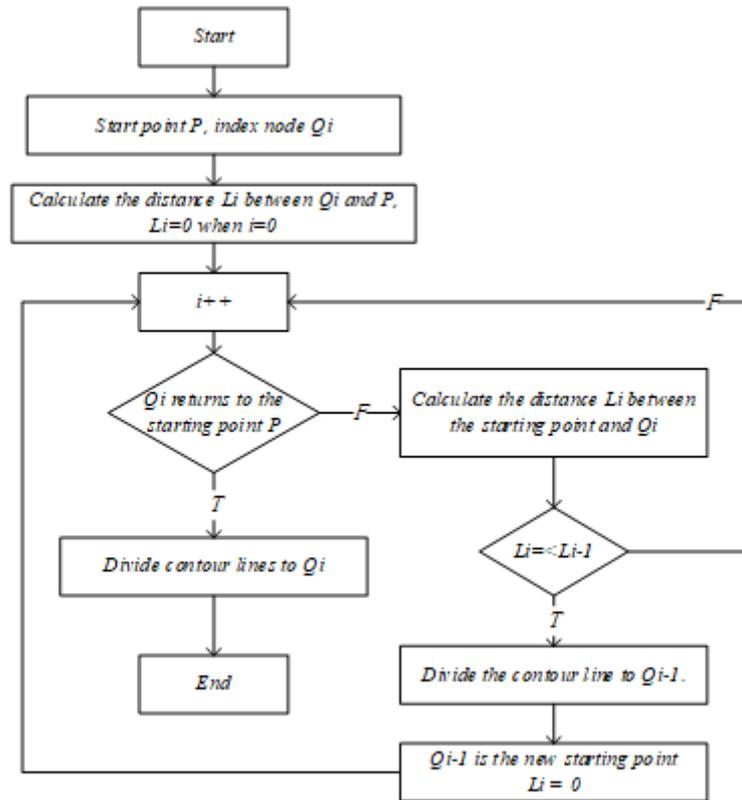


Figure 5: Closed contour segmentation process.

This segmentation method uses D-P algorithm to extract feature points for each segment of the region that is intercepted immediately. The segmentation time complexity is $O(n)$, while the time complexity of finding the farthest point on the ring to split the ring in two is $O(n^2)$.

The D-P algorithm is used to simplify the contour lines, which requires that the contour nodes can cover most of the turning points on the contour lines, as shown in Figure 6(a). The simplified contour can maintain the original curve shape to a certain extent and retain a small number of nodes, which are called feature points. The feature points are saved in order, as shown in Figure 6(b): the feature points of contour A are P_1 、 P_2 、 P_3 、...、 P_m . The feature point on contour B is Q_1 、 Q_2 、 Q_3 、...、 Q_n .

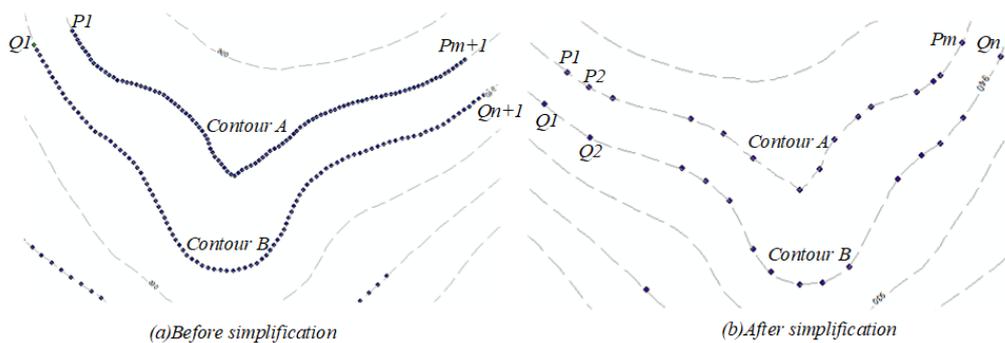


Figure 6: Extract contour feature points.

3.2.2 Nearest Distance Method to Construct Auxiliary Line

According to the strategy of region division, use the nearest distance method to build auxiliary lines for feature points. The method steps are as follows: set feature point P_i on contour A. Find the distance P_i on contour B Nearest point Q_j . Connection P_i and Q_j to get the auxiliary line $LP_i Q_j$. The above basic methods may have the error of intersection of auxiliary lines and auxiliary lines in areas with low similarity, as shown in Figure 7(a): $LP_{i+1}Q_{j-2}$ and $LP_{i+1}Q_{j-1}$ intersect $LP_i Q_j$. To solve this problem, it is necessary to find an auxiliary line intersecting it before building a new auxiliary line. If it does not

exist, it is necessary to directly build an auxiliary line; If it exists, it is necessary to replace the end point of the auxiliary line with the starting point of the intersecting auxiliary line from the original nearest distance point to avoid the intersection, as shown in Figure 7(b).

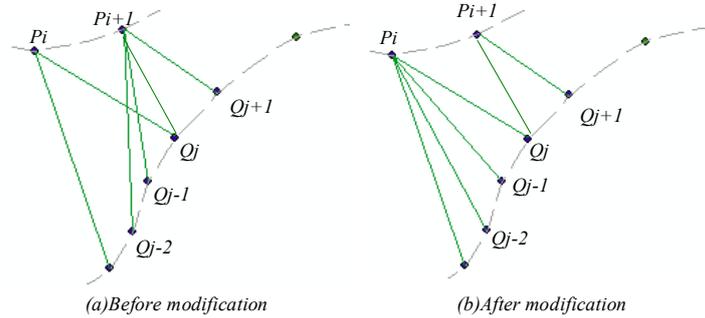


Figure 7: The auxiliary lines intersect.

According to the above steps, traverse the feature points on contour A to construct the auxiliary line with the feature point on contour A as the starting point, and then traverse the feature points on contour B to construct the auxiliary line with the feature point on contour B as the starting point, remove the duplicate auxiliary lines and connect the head and tail nodes of contour AB to obtain the final result, as shown in Figure 8.

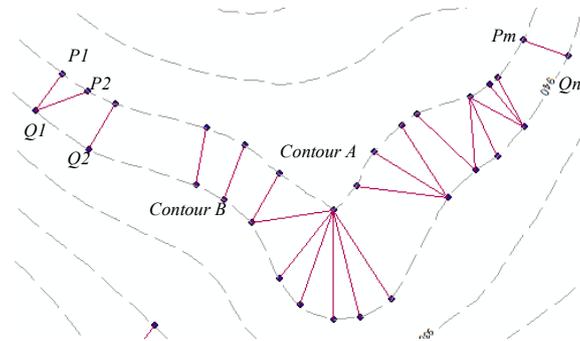


Figure 8: Nearest distance method to construct auxiliary line.

This method can ensure that there is an auxiliary line on the feature points of the bus, so that the new contour line can retain the curve features of the bus as much as possible; The divided areas are divided into two types: the area where the auxiliary lines have multiple points and the area adjacent to the auxiliary lines. The auxiliary lines separate the areas with different contour similarity, which can effectively reduce the intersection between the auxiliary lines and the bus; The area with high similarity and gentle change trend is the adjacent area of the auxiliary line. It is considered that the contour area of this section is paired to realize partial area matching; In the area with low similarity and the area with high similarity and rapid change, there will be multi line common point areas or multiple multi line common point areas connected, so that the interpolated contour lines can retain the bus characteristics in the area with low similarity and rapid change, which is more consistent with the interpolation requirements. However, the intersection of auxiliary lines may occur in areas with low similarity, and the intersection of auxiliary lines will lead to local distortion of new contour lines.

This method can not only reduce the intersection between auxiliary lines and buses, but also take into account the local features of contour lines. Besides, auxiliary lines are constructed by using the nearest distance method from two contour lines, avoiding the loss of feature points and the difference of area division results caused by different baselines, thus reducing the uncertainty.

3.2.3 Interpolated Contour

Contour interpolation can be performed after the auxiliary line between two contour lines is constructed. Let the coordinates of the two ends of each auxiliary line be (Xp, Yp, Zp) and (Xq, Yq, Zq) respectively, insert N contour lines between the two contour lines, and the calculation formula of the node coordinates of the interpolated N contour lines in the auxiliary line is:

$$\begin{cases} X = X_p + \frac{i}{N}(X_q - X_p) \\ Y = Y_p + \frac{i}{N}(Y_q - Y_p), i \in [0, N] \\ Z = Z_p + \frac{i}{N}(Z_q - Z_p) \end{cases} \quad (3)$$

Traverse the auxiliary lines in the order from the starting point to the ending point of the contour line, calculate the coordinates of the interpolation points, and connect the interpolation points to obtain a new contour line, as shown in Figure 9.

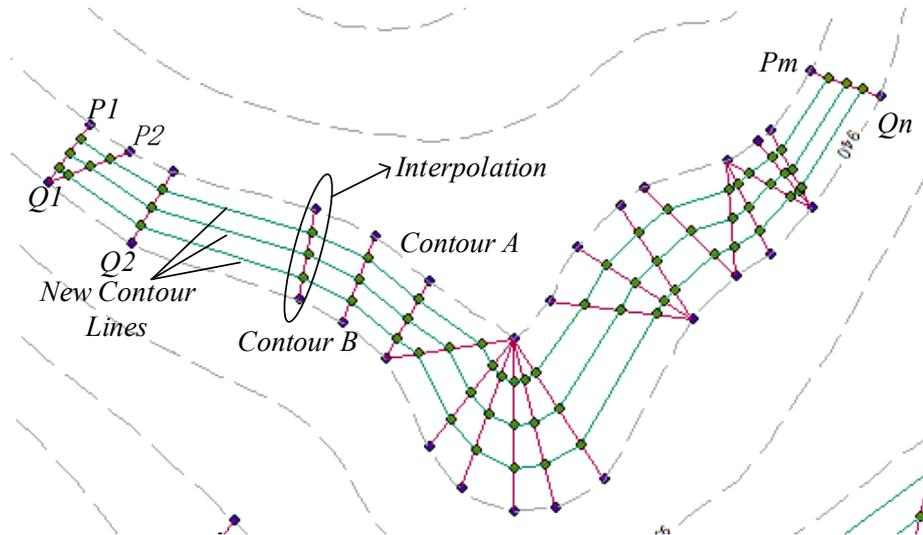


Figure 9: Interpolation Points and New Contour Lines.

4. Experiment and Analysis

The idea of this method is to construct auxiliary lines to divide regions with different similarity, and use them to complete interpolation. The key of the method lies in the construction of the auxiliary line. At present, there are many auxiliary line interpolation methods. In this paper, two methods are selected for experimental comparison: the minimum angle maximum principle method; The second is the regional interpolation method. By comparing the interpolation results of the three methods, the quality of contour lines and the time complexity of the interpolation are compared with other auxiliary line methods.

4.1 Interpolation Effect

Two local contours with different similarity are intercepted from the topographic map as the experimental object, and three interpolation methods are used for the two contours.

4.1.1 High Similarity

Figure 10 shows the interpolation effect of the three methods in a local area of a contour line with high similarity. It can be seen from the figure that the new contour lines interpolated by the three methods are roughly similar to the bus, which can accurately reflect the change trend of the bus and meet the basic requirements of contour line interpolation. However, the three methods are abnormal at the turning point, and the regional interpolation method has sharp angles on the two new contour lines near the inner side, which is more obvious. The minimum angle maximum principle method also has this problem, but it is more gentle than the regional interpolation method. The nearest distance rule has a turning point. Compared with the other two methods, the interpolation results in this area are more comfortable and natural, and conform to the change law of the curve.

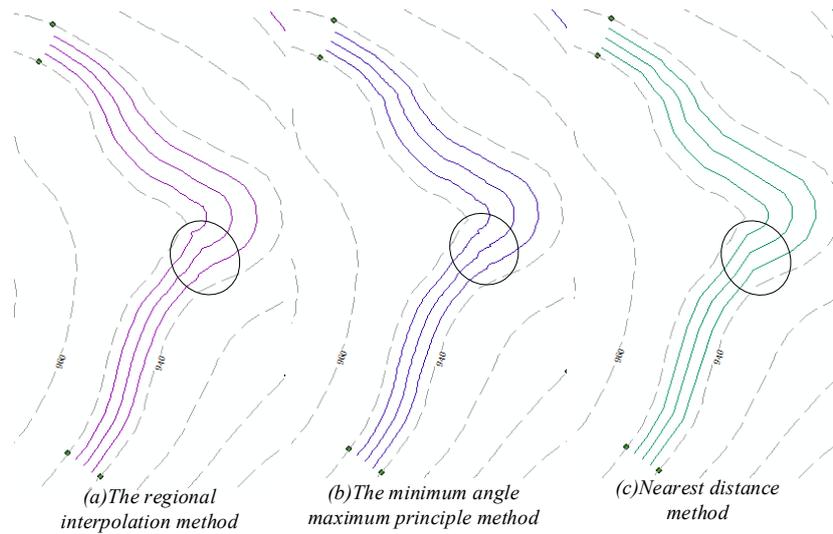


Figure 10: Contour comparison of interpolation in areas with high similarity.

4.1.2 Low Similarity

Figure 11 shows the interpolation effects of the three methods in a local area with low similarity. In contrast, the minimum angle maximum principle method is the most consistent with the interpolation requirements, followed by the nearest distance method. However, the regional interpolation method is significantly different from the original contour line. The minimum angle maximum principle method and the nearest distance method also have obvious shortcomings. The contour lines interpolated by the minimum angle maximum principle method have many sawteeth and uneven curves, while the nearest distance method has a few turns.

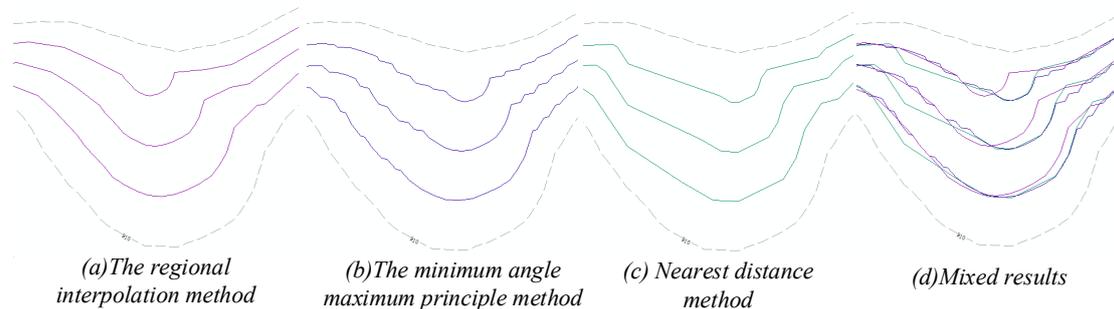


Figure 11: Contour comparison of interpolation in areas with low similarity.

4.2 Interpolation Time Complexity

In the process of local contour interpolation, the biggest difference between the three methods is to build an auxiliary line. The time complexity of the three methods is calculated around the core of building an auxiliary line, as shown in Table 1:

Table 1: Time complexity comparison.

	The regional interpolation method	The minimum angle maximum principle method	Nearest distance method
Extract feature points			$O(n^2)$
Building auxiliary lines	$O(2n^2)$	$O(2n * \text{LOG } 2n)$	$O(2n^2)$
Improve auxiliary lines	$O(n^2)$		

It can be seen from the results in the table 1 that the time complexity of the minimum angle maximum principle method is obviously better than the other two methods, but the nearest distance method uses D-P algorithm to extract feature points to greatly reduce data points, and the number of auxiliary lines is also significantly less than the other two methods. The time consumed by contour interpolation in the same area, whether it is the construction of auxiliary lines or the calculation of interpolation points, will

be significantly lower than the other two methods. The subarea interpolation method needs to calculate additional characteristic parameters for node matching, which will consume more time. It can be concluded that the nearest distance method has the shortest time consumption and the smallest amount of data, followed by the minimum angle maximum principle method.

The interpolation of contour lines by the nearest distance method has the following advantages:1) The regions with different contour similarity are divided to reduce the intersection between auxiliary lines and buses;2) The regions with high similarity and gentle change can be paired, and the regions with high similarity and rapid change have many auxiliary lines in common, so that the interpolated contour lines can retain the curve characteristics of the bus;3) The use of D-P algorithm greatly reduces the amount of data and operation time, and reduces the interpolation time;4) Compared with the subregional interpolation method, the uncertainty factor is less and unnecessary redundancy is avoided.

5. Conclusions

This paper discusses the existing contour line interpolation algorithms, especially the advantages and disadvantages of the algorithm with auxiliary line as the core, and proposes a contour line interpolation method using the nearest distance method to construct auxiliary line based on the regional interpolation method. Compared with other auxiliary line algorithms, this method is simple, easy to implement and less time-consuming, which improves the problem that some methods intersect the auxiliary line with the original contour line in the area where the contour line changes rapidly, and proves the effectiveness and rationality of the method through experiments. However, the possibility of distortion exists in the area with low similarity and there are a few transitions in the interpolated contour lines, which need to be improved; And the contour interpolation of complex terrain such as saddle needs to be expanded, which can be improved in future research.

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